Internet-Based CAD Tool for Design of Gripper Jaws
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ABSTRACT
We describe an Internet-based CAD tool that automatically designs gripper jaws to align a given part from an initial orientation to a desired final orientation. We describe algorithmic improvements in computational efficiency, frictional analysis and form closure analysis. We describe the interface and give examples. We then apply the design tool to identify conditions under which jaw designs exist, leading to an analytic condition for parts that cannot be aligned. The CAD tool is online at www.ieor.berkeley.edu/~goldberg/sa-gripper/.

1. INTRODUCTION
Geometric algorithms for design, simulation, and manufacturing are often available only to a small number of researchers and engineers. The Internet makes it possible for public to share resources. Interactive design is essential because we often need to check if the part designed is easy to produce or if we have proper tools to handle it. CAD software with public accessibility can provide rapid feedback to designers.

In this paper, we describe a Java-based CAD software to design robot gripper jaws. As illustrated in Figure 1, users can draw a part, define the COM and range of friction coefficients, and choose the initial and the final orientations; the applet computes, ranks, and displays the resulting jaw designs. Online CAD tools such as this can allow designers to rapidly check manufacturability and modify designs to avoid handling problems on the assembly line.

2. RELATED WORK
Computer-Aided Design for manufacture is an active research topic. Since the introduction of Java in 1996, a number of geometric algorithms have been made available online. The Geometry Center at University of Minnesota maintains an extensive collection of interactive geometric algorithms at [23]. Interactive geometric software is also available at [24] to find Delaunay triangulation using 3D Convex Hulls. The Berkeley Remote Interactive Optimization Testbed (RIOT) includes several online optimization algorithms [25].

Several researchers have developed online CAD software for manufacturing. Wright et al. [14][5] develop Cybercut [27]; a network CAD/CAM service for design and fabrication of mechanical parts on the Internet. Piccinocchi et al. [11] present a Java applet to compare different nonholonomic motion planning algorithms. Goldberg and colleagues [4][18][2] provide an online fixture design system that finds all modular fixtures for a given part consisting three locators and one clamp. Cheng et al. [17][6] describe online CAD tools for cam and planar mechanical system design at [26]. Berretty et al. [3] describe an online CAD tool for design of traps for the tracks of vibratory bowl feeders. Smith et al. [15] develop a Java applet to compute and rank all pairs of parallel-jaw grip points for polygonal parts.

The CAD tool in this paper computes a class of gripper jaws based on rapidly-machineable trapezoidal modules. An optimal design is an arrangement of trapezoidal jaw modules that maximizes contacts between the gripper and the part at its desired final orientation in a form-closure grasp.

Several authors address grasping in the vertical (gravitational) plane. Trinkle and Paul [16] show how to align parts in the gravitational plane by lifting them off work-surface using a planar gripper with two pivoting jaws. Abell and Erdmann [1] study how a planar polygon can be rotated while stably supported by two frictionless contacts. Rao et al. [13] give a planar analysis for picking up polyhedral parts using 2 hard point contacts with a pivoting bearing, allowing the part to pivot under gravity to rotate into a new configuration. Lynch [7][8] derives sufficient
mechanical conditions for toppling parts in term of constraints on contact friction, location, and motion and we built on his analysis. Zhang et. al [19] introduce geometric functions to represent the mechanics of toppling and develop several algorithms for gripper design in [20,22].

3. ALGORITHM

We assume the part can be treated as a rigid extrusion of a polygon, both the part and the jaws are rigid, part geometry, location of the COM and the jaws are known, and that part motion is sufficiently slow to apply quasi-static analysis.

We consider the following design problem. The input is: the vertices of an n-sided convex projection of an extruded polygonal part, its center of mass (COM), initial and desired orientations, a vertex clearance radius \( \epsilon \), and bounds on the part-surface friction coefficient \([0, \mu_{t,\text{max}}]\) and on the part-gripper friction coefficient: \([\mu_{s,\text{min}}, \mu_{s,\text{max}}]\). The output is a report that no solution can be found or a gripper jaw design, specified by an arrangement of trapezoidal jaw modules that is able to topple the part to its final orientation and grasp it in form closure (see Figure 1).

Each jaw module is defined by an accessible segment that is the portion of the module that makes contact with the part at its desired orientation. Our objective is to find a set of accessible segments with maximum total length.

![Figure 2 A part in initial orientation with gripper jaws (trapezoidal jaw modules in black).](image)

As shown in Figure 2, the part sits on a flat work-surface at a stable initial orientation. Without loss of generality, we assume the part rotates counterclockwise. We define the World frame, \( W \), to be a Cartesian coordinate system originating at pivot point \( P \) with X-axis on the surface pointing right, Z-axis vertical to the surface pointing up. The COM is a distance \( \rho \) from the origin and angle \( \eta \) from the +X direction.

The gripper jaws make contact with the part at two points initially. The pushing tip \( A' \) contacts edge \( e_i \) to the left of pivot point \( P \) at a distance \( z_{x_i} \) from the surface; the toppling tip \( A \) contacts edge \( e_j \) to the right of \( P \) at a distance \( z_{x_j} \) from the surface. Starting from the pivot, we consider each edge of the part in counter-clockwise order, namely \( e_1, e_2, ..., e_n \). An edge \( e_k \), with vertices \( v_k \) at \((x_{v_k}, z_{v_k})\) and \( v_{k+1} \) at \((x_{v_{k+1}}, z_{v_{k+1}})\), is in direction \( \psi_k \) from the X-axis.

Let \( F_r, F_J, \) and \( F_s \) denote the contact force at \( A, A' \), and the work-surface, respectively, and \( f_r, f_J, \) and \( f_s \) denote the direction of the corresponding contact force. We also denote the direction at the left edge of the toppling (i.e. pushing / surface) friction cone as \( F_p / f_p \) and the right edge as \( F_r / f_r \). Let \( f_j \) denote the vertical line through the part’s COM. Let \( I_{a,b} \) denote the intersection of line \( a \) and line \( b \), where \( a \) and \( b \) can be anyone of these directions.

Let \( \theta \) denote the rotation angle of the part; initially \( \theta = 0 \) and finally \( \theta = \theta_f \). Let \( \theta_f \) denote the rotation angle where the COM is right above \( P \).

In [21], we gave an \( O(n^3) \) algorithm to find the optimal jaw design; in this paper we improve the algorithm to handle bounded friction coefficients and faster form closure analysis. Here we report details on the resulting \( O(n^3 \log n) \) algorithm and its implementation in an online Java applet.

3.1 Bounded Friction Coefficients

In [21] we assumed exact friction values were given. In this section we relax that assumption and derive the toppling conditions given upper and lower bounds on the friction coefficients.

The pushing condition to prevent jamming does not depend on the friction coefficients. It is easy to see that we only need to consider the upper bounds of \( \mu_s \) and \( \mu_r \), i.e. \( \mu^*_s = \mu_{s,\text{max}} \) and \( \mu^*_r = \mu_{r,\text{max}} \), to satisfy the pushing conditions to prevent rolling and jamming (see [22] for details).

The rolling function and jamming function describe the mechanics of toppling. Given \( z_{x_i} \), the toppling tip makes contact with edge \( e_i \), and the range of friction coefficients, the rolling function \( H_j(\theta) \) is the minimum height of \( A \) guaranteed to cause the part to rotate at orientation \( \theta \) and the jamming function \( J_j(\theta) \) gives the minimum height of \( A \) that is guaranteed not to cause jamming. \( H_j(\theta) \) is defined in the range \([0, \theta_f]\).

For each value of \( \theta \), the geometric functions span a range of values depending on the range of friction coefficients. To guarantee toppling for all the friction values in the range, we take the maximum values of \( H_j(\theta_f) \) or \( J_j(\theta_f) \) over the given bounds. At each rotational angle \( \theta \), the maximum \( H_j(\theta) \) or \( J_j(\theta) \) corresponds to the critical friction coefficients denoted by \( \mu^*_s \) and \( \mu^*_r \). We first consider the functions at each rotational angle \( \theta \) to derive \( \mu^*_s \); then we find \( \mu^*_r \) for given \( \mu^*_s \) and \( \theta \).

We consider the case where the COM is to the right of \( P \) and \( A' \), i.e., \( \theta < \theta_f \) and \( x_{v_i} < 0 \).
Following Mason [9], we construct triangle \( P_0P_1P_2 \) as shown in Figure 2 if \( \psi_i > 2\pi - (\frac{\pi}{2} - \alpha_i) + \theta \), i.e., \( \psi_i > \pi + \alpha_i - \alpha_i \), where \( P_0(x_{p0}, z_{p0}) \) is \( I_{d, \alpha} \), \( P_1(x_{p1}, z_{p1}) \) is \( I_{d, \alpha} \), and \( P_2(x_{p2}, z_{p2}) \) is \( I_{d, \alpha} \). The closed polygon defined by these critical points is called the critical polygon, which in Figure 3 is triangle \( P_0P_1P_2 \). If all the forces within the toppling friction cone generate clockwise moments about the critical polygon, the part will be rotated instantaneously.

![Figure 3 Rolling condition.](image)

As a conservative solution, we find the contact height sufficient to roll the edge by projecting lines from \( P_0, P_1, \) and \( P_2 \) at the angle of \( \theta_f \) until they intersect the edge of the part. The intersection with the maximum height of those three is the minimum height sufficient to roll the part.

To relax the exact friction requirement, we note that as \( \mu_s \) decreases, \( P_0 \) moves up along \( r_{02} \) and \( H_f(\theta) \) will not increment. It is sufficient to consider only the upper bounds of \( \mu_s \), i.e., \( \mu_{s, max} = \mu_{s,max} \), to get \( H_f(\theta) \).

Given \( \mu_{s, max} \) and \( \theta \), the rolling function is a function of \( \mu_s \), i.e., \( H(\mu_s) \). As \( \mu_s \) decreases, \( P_2 \) moves down along \( r_{12} \) but \( P_0 \) moves up along \( r_{01} \). We sample \( \mu_s \) in the given range, compute the rolling function for each \( \mu_s \), and then find \( \mu_s^{*} \) for \( H_f(\theta) \) using numerical search.

### 3.2 Form Closure Analysis

We require that the resulting set of the accessible segments generates a form-closure grasp on the part.

We reorder the endpoints of the corresponding accessible segments in counter-clockwise. Starting from the one closest to \( P \), we rename the endpoints \((x_1, z_1), (x_2, z_2), \ldots, (x_m, z_m)\). Let \( \mathbf{V}_i \) denote the unit normal inward vector at contact \((x_i, z_i)\), \( \mathbf{V}_{kx}(\mathbf{V}_{kz}) \) denote the \( X \) (\( Z \))-axis projection of the unit normal inward vector at contact \((x_i, z_i)\), and \( T_k \) denote the torque of \( \mathbf{V}_k \) relative to \( P \). The Wrench matrix \( \mathbf{W} = \begin{bmatrix} V_{1x} & V_{1z} & V_{2x} & V_{2z} & \ldots & V_{mz} \\ V_{1z} & V_{2z} & V_{3z} & \ldots & V_{mz} \end{bmatrix} \). The Wrench space is a 3-dimensional manifold defined by \( V_x, V_z \), and \( T \). Each column of \( \mathbf{W} \) represents a point in the wrench space. Mishra et. al [10] prove that a form-closure grasp is guaranteed if the origin of the wrench space lies strictly in the convex hull of these points.

The algorithm checks the form-closure of the accessible segment set:

1. Compute wrench matrix \( \mathbf{W} \);
2. Define the set of points, \( \mathbf{Q} \), in wrench space by each column of \( \mathbf{W} \) and the origin of wrench space \( \mathbf{O} \);
3. Find the convex hull of \( \mathbf{Q} \), \( \text{CH}(\mathbf{Q}) \);
4. If \( \mathbf{O} \notin \text{CH}(\mathbf{Q}) \), the set of the accessible segments achieve form-closure on the part.

Since it takes time \( O(m \log m) \) to construct the 3D convex hull of \( m \) points [12] and \( m \leq 2n \), the running time to check the form-closure of the set of accessible segments is \( O(n \log n) \). Thus, the total running time of the improved algorithm is \( O(n^2 \log n) \), an improvement over the \( O(n^3) \) of the previous algorithm.

### 4. IMPLEMENTATION

We have implemented this algorithm into an online Java-based CAD tool. Users can draw a convex polygonal part (up to 10 sides) with a mouse, drag it from the initial resting pose to the desired orientation, and view the resulting designs (if any exist) ranked by their total contact area.

As illustrated in Figure 6, there are three numbers in the right lower corner of the applet: Angle is the degree of the part’s rotational angle; \( (X, Z) \) is the location of your cursor. To use the applet, start by clicking Design Part button, then use your mouse to draw a convex polygon above the black horizontal line (counterclockwise). The polygon represents the part and the horizontal line represents the work-surface. You can also drag the part’s Center of Mass around using your mouse. The default value of friction cone half-angles is 10 degree. Next, click Next Stable Pose, the part will align with the work-surface in one of its stable orientation. Click the button again, the part will rotate to the next stable orientation. Then, you may click Drag to Desired Pose OFF. The button will change to Drag to Desired Pose ON and a gray image of the part in the initial orientation will display. Now, you can drag the part to rotate it to the desired orientation and Angle shows the current rotational angle. We can run the jaw design algorithm by clicking Design Jaws. An optimal jaw design will appear. The algorithm requires 7–10 seconds to find and rank jaw designs on a 266MHz Pentium II PC. Black trapezoids represent jaw modules. Move your
Cursor close to a vertex of any jaw module, *Angle* disappears and the $(X, Z)$ location of the vertex displays.

To study the sensitivity of the input parameters, we ran the Java applet with varying friction coefficients, part geometry, center of mass, and desired orientation. With friction coefficients of $10^\circ$, Table 1 shows simulation results of triangular parts and Table 2 shows those of some more complicated parts. The jaw modules are in black and the arrows indicate the pushing tip and the toppling tip.

![Table 1 Simulation results: triangular parts.](image)

<table>
<thead>
<tr>
<th>Initial Orientation of Parts</th>
<th>Final Orientation with Optimal Jaw Design</th>
<th>Running Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Triangular part" /></td>
<td><img src="image" alt="Optimal jaw design" /></td>
<td>9</td>
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<tr>
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</tbody>
</table>

Table 2 Simulation results: complicated parts.

<table>
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<th>Final Orientation with Optimal Jaw Design</th>
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</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Complicated part" /></td>
<td><img src="image" alt="Optimal jaw design" /></td>
<td>9</td>
</tr>
<tr>
<td><img src="image" alt="Complicated part" /></td>
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<tr>
<td><img src="image" alt="Complicated part" /></td>
<td><img src="image" alt="Optimal jaw design" /></td>
<td>10</td>
</tr>
</tbody>
</table>

5. **EXISTENCE OF SOLUTIONS**

Some parts, like the last case in Table 1, have no jaw design solution. To characterize such parts, we consider the class of triangular parts shown in Figure 4. Without loss of generality, we assume that edge $e_1$ is of unit length from $(0,0)$ to $(1,0)$. We assume the part has uniform density.

![Figure 4: A triangular part.](image)

We are interested in the location of $v_3$ at $(x_3, z_3)$ that makes the triangular part impossible to topple. Note that $x_3 \in [-1, 2]$ to keep the part stable and $e_j$ on the work-surface.

As shown in Figure 3, that lower $A'$ and smaller the friction coefficients will lead to larger feasible contact area for $A$. Therefore we consider the extreme condition where the pushing tip $A'$ makes contact with $v_j$ at $(0, 0)$ with zero friction, i.e., $\mu_t = 0$, $\mu_s = 0$, and $\varepsilon = 0$. By geometric relationship, we have:

\[
\tan (\pi - \omega) = z_j / (1-x_j) \Rightarrow \omega = \pi - \arctan(z_j / (1-x_j)),
\]

\[
\tan (\psi - \pi) = z_j / x_j \Rightarrow \psi = \pi + \arctan(z_j / x_j),
\]

\[
\tan \eta = \frac{z_j / 3}{(1+x_j)/3} \Rightarrow \eta = \arctan(z_j / (1+x_j)),
\]

\[
\rho = \sqrt{\left(\frac{1+x_j}{3}\right)^2 + \left(\frac{z_j}{3}\right)^2}.
\]

In this case, the critical polygon is in gray as shown in Figure 4. We find the height sufficient to roll the edge by projecting lines from $P_0$ and $P_2$ at the angle vertical to edge $e_j$ until they intersect the edge. The intersection with the maximum height of those two is the minimum height sufficient to roll the part.

The height of the intersection corresponding to $P_0$ can be shown to be $\frac{z_j(1-x_j)}{x_j^3 - 2x_j + z_j^3 + 1}$; the corresponding $P_2$ can be shown to be $\frac{z_j(1-x_j)}{x_j^3 - 2x_j + z_j^3 + 1}$. 


The rolling function $H = \max \left( \frac{z_3(1-x_i)}{x_i^2 - 2x_i + z_3^2 + 1}, \frac{z_3(1-x_i)}{x_i^2 - 2x_i + z_3^2 + 1} \right)$. Thus, the rolling function $H = \frac{z_3(1-x_i)}{x_i^2 - 2x_i + z_3^2 + 1}$. If $H > z_3$, there is no jaw design will be able to initialize the toppling process. There will be no jaw design solution if $H$ is higher than $h = z_3$. As shown in Figure 5, a triangular parts will have no jaw solution if its vertex $v_3$ is in the critical area where the rolling function $H$ protrudes from the plane $h = z_3$.

No jaw design exists if $v_3$ is within the boundary of the critical area. The boundary is represented by the intersection of $H$ and $z_3$. We consider each term in $H$, and want to prove that $H$ is determined by $\frac{z_3(1-x_i)}{x_i^2 - 2x_i + z_3^2 + 1}$. That is equivalent to the following inequality:

$\frac{z_3(1-x_i)}{x_i^2 - 2x_i + z_3^2 + 1} \geq \frac{z_3(1-x_i)}{x_i^2 - 2x_i + z_3^2 + 1} - \frac{3z_3 |1+x_i|}{3(x_i^2 - 2x_i + z_3^2 + 1)}$

Proof:

$(x_3 - 1)^2 + z_3^2 \geq 0$ and $z_3 \geq 0$

$\Rightarrow x_3^2 - 2x_3 + z_3^2 + 1 \geq 0$ and $z_3 \geq 0$

$\Rightarrow 0 \geq \frac{z_3 |1+x_i|}{3(x_i^2 - 2x_i + z_3^2 + 1)}$

$\Rightarrow z_3(1-x_i) \geq \frac{z_3(1-x_i)}{x_i^2 - 2x_i + z_3^2 + 1} - \frac{3z_3 |1+x_i|}{3(x_i^2 - 2x_i + z_3^2 + 1)}$

So the boundary can be determined by:

$\max \left( \frac{z_3(1-x_i)}{x_i^2 - 2x_i + z_3^2 + 1}, \frac{z_3(1-x_i)}{x_i^2 - 2x_i + z_3^2 + 1} \right) = z_3$

$\Rightarrow z_3(1-x_i) = z_3$

$\Rightarrow x_i^2 - x_i + z_3^2 = 0$

$\Rightarrow (x_i - 0.5)^2 + z_3^2 = 0.5^2$

Since $z_3$ is a nonnegative number, the above equation indicates a half-circle as shown in Figure 6. This result implies that any triangular part with the interior angle at $v_3$ greater 90 degree will be certainly unable to find a jaw design to topple the part.

![Figure 6 Boundary of $v_3$ to guarantee no jaw design solution.](Image)

We can also interpret this result by geometric construction as shown in Figure 7. $E$ is the intersection of the half-circle and the toppling edge in the initial orientation. In case (a), $v_3$ is on the circle. The contact force at $v_3$ passes exactly through one vertex of the critical polygon in gray and makes counterclockwise moment respective to the critical polygon. Thus, a toppling tip at height $z_3$ will initialize the toppling process. In case (b), $v_3$ is out of the circle and the interior angle at $v_3$ is less than 90 degree. The contact force at $E$ passes exactly through one vertex of the critical polygon in gray and makes counterclockwise moment respective to the critical polygon. Thus, a toppling tip at any height between $E$ and $v_3$ will initialize the toppling process. In case (c), $v_3$ is within the circle and the interior angle at $v_3$ is greater 90 degree. Thus, no contact can topple the part and no solution exists.

![Figure 7 Geometric interpretation of the boundary.](Image)

6. DISCUSSION

We describe a new Java-based CAD tool that designs gripper jaws that align a given part from an initial
orientation to a desired final orientation. We describe algorithmic improvements in computational efficiency, frictional analysis and form closure analysis. We describe a Java-based graphical interface and give examples. We then apply the CAD tool to identify conditions under which jaw designs exist, leading to a formal characterization of a class of triangular parts for which no gripper design exists. Online CAD tools such as this can allow designers to rapidly check manufacturability and modify designs to avoid handling problems.

REFERENCES


[27] http://cybercut.berkeley.edu/