1.24. It can be shown that
\[ v(y) = -cy + p\mathbb{E} \min[D, y] = (p - c) - L(y) \]
where \( L(y) = (p - c)\mathbb{E}(D - y)^+ + c\mathbb{E}(y - D)^+ \). It follows that in the event of overstocking \((y > D)\) the gain per unit of stocking less inventory is \(c\), while in the case of understocking, the gain per unit of stocking additional inventory is \(p - c\). Hence, we may regard the newsvendor model using the overage/underage formulation with \(c_o = c\) and \(c_u = p - c\).

1.25
(a) Assuming that \(y\) bookings are made and \(\Phi(0) = 0, \Phi(n) = 1\) where \(\Phi\) is the distribution of \(Z\), expected return is
\[
v(y) = p\mathbb{E} \min(n, y - Z) - \pi\mathbb{E} \max[n - Z - y, 0]
\]
\[
= np - p\mathbb{E} \max[n + z - y, 0] - \pi\mathbb{E} \max[y - z - n, 0]
\]
\[
= np - \pi \int_{\xi=y-n}^{n} \xi \phi(\xi) d\xi - \pi \int_{\xi=0}^{y-n} (y - n - \xi) \phi(\xi) d\xi
\]
\[
= np - \pi \int_{\xi=y-n}^{n} \xi \phi(\xi) d\xi - p(n - y)\left[1 - \Phi(y - n)\right]
\]
\[
- \pi (y - n)\Phi(y - n) + \pi \int_{0}^{y-n} \xi \phi(\xi) d\xi.
\]
Since \(Z\) is assumed to be continuous and takes values in \((0, n]\), we restrict ourselves to \(y \in [n, 2n]\).

(b)
\[
v'(y) = p - (p + \pi)\Phi(y - n)
\]
\[
v''(y) = -(p + \pi)\phi(y - n)
\]
\(v''(y) < 0\) for \(y > n\), so \(v(y)\) is concave on \(y > n\). Optimal \(y\) is solution of
\[
\Phi(y - n) = \frac{p}{p + \pi}.
\]
\(\Phi(0) = 0\) and \(\Phi(n) = 1\) implies that this value of \(y\) is strictly larger than \(n\) and strictly smaller than \(2n\) when \(p, \pi > 0\).

(c) \(\mathbb{P}[\text{empty seat}] = \mathbb{P}[y - Z < n] = \mathbb{P}[Z > y - n] = 1 - \Phi(y - n) = \frac{\pi}{p + \pi} \).
1.26

(a)

\[ v(y) = -cy + p\mathbb{E} \min[D, y] \]
\[ = -cy + p\mathbb{E} \{ y + \min[D - y, 0] \} \]
\[ = (p - c)y - p\mathbb{E} \max[y - D, 0] \]
\[ = (p - c)y - py\Phi(0) - p \int_{0}^{y} (y - \xi)\phi(\xi)d\xi \]
\[ = (p - c)y - py\Phi(0) - py[\Phi(y) - \Phi(0)] + p \int_{y=0}^{y} \xi\phi(\xi)d\xi \]
\[ = (p - c)y - py\Phi(y) + p \int_{\xi=0}^{y} \xi\phi(\xi)d\xi \]

Same as case when \( \Phi(y) = 0 \).

(b)

\[ v'(y) = p - c - p\Phi(y). \]

If

\[ v'(0) = p - c - p\Phi(0) \leq 0 \iff \Phi(0) \geq \frac{p - c}{p} \]

then optimal capacity is \( S = 0 \). Otherwise, optimal capacity is solution of the equation:

\[ v'(y) = 0 \iff \Phi(y) = \frac{p - c}{p}. \]

(c) Let \( S \) be the optimal capacity. From question 1.9

\[ v(S) = (p - c)\mathbb{E}[D \mid D \leq S] \]

so it’s profitable so long as \( \mathbb{E}[D \mid D \leq S] > 0 \). That is, if \( S > 0 \) or (from part (b)) if

\[ \Phi(0) < \frac{p - c}{p} \]