Due date: Thursday November 7.

1. An energetic salesman works every day of the week. He can work in only one of two towns A and B on each day. For each day he works in town A (or B) his expected reward is $r_A$ (or $r_B$, respectively). The cost for changing towns is $c$. Assume that $c > r_A > r_B$ and that there is a discount factor $\alpha < 1$.

(a) Show that for $\alpha$ sufficiently small, the optimal policy is to stay in the town he starts in, and that for $\alpha$ sufficiently close to 1, the optimal policy is to move to town A (if not starting there) and stay in A for all subsequent times.

(b) Solve the problem for $c = 3$, $r_A = 2$, $r_B = 1$, and $\alpha = 0.9$ using policy iteration.

(c) Use a computer to solve the problem in part (b) by value iteration, with and without the error bounds (which were discussed in class).

2. An unemployed worker receives a job offer at each time period, which she may accept or reject. The offered salary takes on of $n$ possible values $w^1, \ldots, w^n$ with given probabilities, independently of preceding offers. If she accepts the offer, she must keep the job for the rest of her life at the same salary level\(^1\). If she rejects the offer, she receives unemployment compensation $c$ for the current period and is eligible to accept future offers. Assume that income is discounted by a factor $\alpha < 1$.

(a) Show that there is a threshold $\bar{w}$ such that it is optimal to accept an offer if and only if its salary is larger than $\bar{w}$, and characterize $\bar{w}$.

(b) Consider the variant of the problem where there is a given probability $p_i$ that the worker will be fired from her job at any period if her salary is $w^i$. Show that the result of part (a) holds in the case where $p_i$ is the same for all $i$. Analyze the case where $p_i$ depends on $i$.

3. Let $\bar{J} : S \to \mathbb{R}$ be any bounded function on $S$ and consider the value iteration method with starting function $J : S \to \mathbb{R}$ of the form:

$$J(x) = \bar{J}(x) + r, \quad x \in S$$

\(^1\)yikes!
where $r$ is some scalar. Show that the bounds $(T^k J)(x) + c_k$ and $(T^k J)(x) + \bar{c}_k$ of Proposition 1.3.1 in the textbook are independent of $r$ for all $x \in S$. Show also that if $S$ consists of a single state $\hat{x}$ (i.e. $S = \{\hat{x}\}$), then:

$$(TJ)(\hat{x}) - \xi_1 = (TJ)(\hat{x}) + \bar{c}_1 = J^*(\hat{x})$$