Due date: Thursday October 31.

1. The purpose of this problem is to show that shortest path problems with a discount factor make little sense. Suppose that we have a graph with a nonnegative length \( a_{i,j} \) for each arc \((i,j)\). The cost of a path \((i_0,i_1,\ldots,i_m)\) is \(\sum_{k=0}^{m-1} \alpha^k a_{i_k,i_{k+1}}\), where \(\alpha\) is a discount factor from \((0,1)\). Consider the problem of finding a path of minimal cost that connects two given nodes. Show that this problem need not have a solution.

2. Consider a problem similar to the standard infinite horizon problem

\[
J^*(x) = \min_{\pi \in \Pi} \mathbb{E}_{w_k} \left\{ \sum_{k=0}^{N-1} \alpha^k g(x_k, \mu_k(x_k), w_k) \right\}
\]

\[
x_{k+1} = f(x_k, \mu_k(x_k), w_k)
\]

except that when we are at state \(x_k\), there is a probability \(\beta\), where \(0 < \beta < 1\), that the next state \(x_{k+1}\) will be determined according to \(x_{k+1} = f(x_k, \mu_k(x_k), w_k)\) and a probability \(1 - \beta\) that the system will move to a termination state, where it stays permanently thereafter at no cost. Show that even if \(\alpha = 1\), the problem can be put into the discounted cost framework.