Due date: Thursday September 19.

1. Question 1.10 from text.

2. Question 2.1 from text.

3. We considered in class the problem of finding the ‘most likely state sequence’ $\hat{X}_N = \{\hat{x}_0, \cdots, \hat{x}_N\}$ of a Markov chain given noisy observations $Z_N = \{z_1, \cdots, z_N\}$; that is, the solution $\hat{X}$ of the maximization problem:

$$\max_{X_N} P[X_N | Z_N] = \max_{X_N} \frac{P[X_N, Z_N]}{P[Z_N]}.$$ 

It was shown that by repeated application of simple conditioning arguments on $P[X_N, Z_N]$, this problem can be reformulated as a deterministic shortest path problem that can be solved using forward dynamic programming (i.e. the ‘Viterbi Algorithm’).

It was pointed out in class that this reformulation can also be carried out by applying conditioning arguments to the right hand side of the following equation:

$$P[X_N, Z_N] = P[Z_N | X_N] P[X_N]. \quad (1)$$

Try this yourself. i.e. apply conditioning arguments to the RHS of (1) and express it in terms of $p_{ij}$ and $r(z; i, j)$ (see class notes, or chapter 2.2, for notation).

4. We considered in class the problem of linear quadratic control. In the example we considered, however, we assumed that the conditional variance of $x_{k+1}$ (given $x_k$ and $u_k$) was independent of the choice of $u_k$.

In many applications, this is an unrealistic assumption. For example, suppose that $x_k$ is the total value of your investment portfolio at period $k$, while $u_k$ (an $m$-dimensional vector) corresponds to the value of your position in each of $m$ stocks at time $k$. Clearly, in this example, the variance of $x_{k+1}$ (conditional on $x_k$ and $u_k$) depends on $u_k$ since more exposure to stocks (typically) increases the volatility of your portfolio. Therefore, control problems of
the following type (in which the control decision \( u_k \) affects the variance of \( x_{k+1} \)) are of interest:

\[
\min_{u_k} E\left\{ \sum_{k=0}^{N-1} (x'_k Q x_k + u'_k R u_k) + x'_N H x_N \right\}
\]

Subject to:

\[
x_{k+1} = Ax_k + Bu_k + w_k D u_k
\]

\[
x_0 \text{ given.}
\]

In this case, assume that \( x_k \in \mathbb{R}^n \), \( u_k \in \mathbb{R}^m \), that \( w_0, \ldots, w_{N-1} \) are independent, scalar valued, independent random variables with \( E w_k = 0 \) and \( E w_k^2 = \sigma_k^2 \), \( D \in \mathbb{R}^{n \times m} \), and \( A, B, Q, R, H \) are of appropriate dimension with \( H \geq 0 \), \( Q \geq 0 \) and \( R > 0 \). Use dynamic programming to find the optimal policy and value function.