Power Systems Analysis - Homework 5

1. Consider a synchronous generator described by the swing equation:

\[ M \ddot{\delta}(t) + D \dot{\delta}(t) + P_e(\delta(t)) = P_m \]

where the mechanical power \( P_m \) is fixed and the electrical power \( P_e(\delta(t)) \) is given as

\[ P_e(\delta(t)) = a \sin(\delta(t)) + b \sin(2\delta(t)) \]

(a) Given an equilibrium point \( \delta_0 \) (i.e., \( P_m = P_e(\delta_0) \)), study its local stability (the stability condition can depend on \( M, D, a, b, P_m \) and \( \delta_0 \)).

(b) Consider the energy function

\[ V(\delta, \dot{\delta}) = \frac{1}{2} M \dot{\delta}^2 + \int_{\delta_0}^{\delta} P_e(u)du \]

Prove that the energy of the system is preserved if \( D = 0 \) and reduces if \( D > 0 \).

2. Consider a short transmission line with series impedance \( z = r + jx \). Show that if \( |V_1| = |V_2|, \ r \ll x \) and \( \theta_{12} \) is small, then the line loss varies quadratically with transmitted active power, i.e.,

\[ P_L \sim \frac{r}{|V_1|^2} P_{12}^2 \]

(Hint: Use second-order approximations for \( \sin \theta_{12} \) and \( \cos \theta_{12} \)).

3. Consider a wind turbine with a diameter of 4 meters. Wind is blowing at the speed of 20 miles per hour, with the following power:

\[ P_{ava} = A \times 6 \times 10^{-4} V^3 \]

(units: KW for \( P_{ava} \) and meter/sec for the wind speed \( V \)). The turbine converts wind energy to electrical energy with the efficiency of 30% of the maximum theoretical limit. Find the output power.