Power Systems Analysis - Homework 1

1. Consider the circuit:

(a) Write a second-order differential equation describing $i(t)$.
(b) Explain why $i(t)$ can be written as the sum of a vanishing term and a sinusoidal term.
(c) Find $i(t)$ for $R = L = C = 1$.

2. In the following figure, assume that

$$V_1 = 1 \angle 0^\circ$$
$$Z_{line} = 0.01 + j0.1$$
$$S_{D1} = 0.5 + j0.5$$
$$S_{D2} = 0.5 + j0.5$$

Pick $Q_{G2}$ so that $|V_2| = 1$. In this case what are $Q_{G2}$, $S_{G1}$ and $\angle V_2$?

3. The system shown in the following figure is balanced. Find $V_{a'n}$, $V_{b'n}$, $V_{c'n}$, and $V_{a'b'}$.

4. Draw the power circle diagram in the case $|V_1| = 1.05$, $|V_2| = 0.95$, $Z_{line} = 0.1 \angle 85^\circ$. Find
(a) $P_{12}^{\text{max}}$
(b) $\theta_{12}$ at which we get $P_{12}^{\text{max}}$
(c) $-P_{21}^{\text{max}}$
(d) $\theta_{12}$ at which we get $-P_{21}^{\text{max}}$
(e) Active power loss in the line when $\theta_{12} = 10^\circ$

5. Assume that Bus 0 is connected to Buses 1, $\cdots$, $n$ in such a way that $V_i = 1 \angle \theta_i$ for $i = 0, \cdots, n$.
(a) Find $P$ and $Q$ in terms of $z_1, \cdots, z_n$ and $\theta_0, \cdots, \theta_n$.
(b) Assume that $|\theta_i - \theta_0|$, $i = 1, \cdots, n$, is so small that

$$
\sin(\theta_i - \theta_0) \simeq \theta_i - \theta_0 \\
\cos(\theta_i - \theta_0) \simeq 1.
$$

Find a good approximation of $P$ and $Q$ as linear functions of $\theta_0, \cdots, \theta_n$. 

\[ S = P + Qj \]