1. Find a second-order approximation of the function \( f(x) = \cos(x_1 - x_2) + \sin(x_1 + x_2) + \sin(x_1) \cos(x_2) \) around the point \((0, 0)\).

2. Consider the function \( f(x) = x_1^2 + e^{x_1 + x_2} + x_1 x_3 \). Find all descent directions for this function at the point \( x = [1, 1, -2] \).

3. Consider a function \( f(x) : \mathbb{R}^2 \to \mathbb{R} \) with the property that its Gradient and Hessian are equal to \([0, 0] \) and \( \begin{bmatrix} 2 & 4 \\ 4 & 5 \end{bmatrix} \) at some point \( \bar{x} \).
   
   a) Find a direction \( \Delta x \) such that the relation \( f(\bar{x} + \varepsilon \Delta x) < f(\bar{x}) \) holds for sufficiently small and positive values of the scalar \( \varepsilon \).
   
   b) Find a direction \( \Delta x \) such that the relation \( f(\bar{x} + \varepsilon \Delta x) > f(\bar{x}) \) holds for sufficiently small and positive values of the scalar \( \varepsilon \).

4. Run three iterations of the Gradient algorithm for the problem

\[
\min_{x \in \mathbb{R}^2} -(x_1 + x_2)e^{-x_1 - x_2} + x_1
\]  

by starting at the point \((0, 1)\) and using the fixed step size \( t = 0.01 \). Compute the value of the function at every iteration.