Exotic Options for Interruptible Electricity Supply Contracts

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Abstract

This paper presents the design and pricing of financial contracts for the supply and procurement of interruptible electricity service. While the contract forms and pricing methodology have broader applications, the focus of this work is on electricity market applications, which motivate the contracts structures and price process assumptions. In particular we propose a new contract form that bundles simple forwards with exotic call options that have two exercise points with different strike prices. Such options allow hedging and valuation of supply curtailment risk while explicitly accounting for the notification lead time before curtailment.

The proposed instruments are priced under the traditional GBM price process assumption and under the more realistic assumption (for electricity markets) of a mean reverting price process with jumps. The latter results employ state of the art Fourier transforms techniques.

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1. Introduction

Sweeping changes in the electric power industry have been directed at increasing competition in the generation of electricity. This has been accompanied by a commoditization of electricity with the emergence of spot markets, along with forward and derivative markets. On the operations side, standard financial instruments can be used to emulate various contracts that serve purposes such as risk management and improving efficiency. This paper focuses on the pricing and hedging of such contracts under standard financial instruments such as forwards and options using different price process specifications that increasingly capture realistic aspects of observed prices.

One efficiency-motivated contract known as an interruptible service contract (see Chao and Wilson (1987)) involves giving firms a discount on the forward price of electricity for a particular delivery date while providing suppliers with the option of curtailing supply to these customers when the price is above some cutoff value specified by the customer. This allows suppliers to provide electricity to those customers who are willing to pay the highest prices in times of scarcity and gives customers lower rates. If the time of notification of curtailment is only a few minutes before delivery, the supplier can substitute these demand side measures for reserve capacity that it must have on-hand for reliable operation of the grid, thus reducing costs of operation and need for expansion of capacity.

However, it is expected that not many customers, particularly industrial ones, will provide viable cutoffs as the costs of unexpected shutdowns may be very high. On the other hand including another earlier date, perhaps even a few months before delivery, will provide incentive for these firms to plan shutdowns in a period of forecasted scarcity, such as when there is a weak rainy season for a hydro-dominated grid, or planned shutdowns of some major electricity plants. We will call this contract an interruptible service contract with early notification. Another type of contract known as a dispatchable Independent Power Producer (IPP) supply contract, involves the customer paying a premium for the option to cancel purchase in the event the spot price is below the contracted price. This caps the price at which the customer buys electricity.

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All of the above contracts have one feature in common - they involve forward contracting with some optionality built into the forward contract. One can therefore use standard financial instruments like forwards and options to price these contracts.

Section 2 describes the formulation of these contracts using standard financial instruments. Section 3 presents pricing formulae and hedging portfolios associated with these contracts using geometric brownian motion (GBM) and an affine diffusion process (AD) that incorporated mean-reversion in the price process. Section 4 uses an affine jump-diffusion process to model electricity prices and uses transform analysis to price these contracts. Concluding remarks and a discussion of further research are presented in section 5. Proofs are presented in an Appendix.

2. Formulation

As in any commodity market, financial instruments can be used in the electricity market to hedge against and diversify risk. The simplest hedge is a forward contract that guarantees a fixed price for power regardless of the spot price. The call (put) option, gives its holder the right but not the obligation to buy (sell) electricity at a pre-determined "strike" price. The call (put) option will be excercised only when the spot price is higher (lower) then the strike price. More complex payoff structures can be simulated by holding portfolios of these basic securities. These contracts can be settled physically or financially. In any commodity market the volume of traded commodity contracts usually exceeds by far the volume of actual physical deliveries (by a factor of ten or more).

The proliferation of financial derivatives has led to many developments in techniques for modeling these instruments (see Cox and Rubinstein (1985), Hull and White (1998)). The literature dealing specifically with financial modeling in electricity markets is more recent (see Cater (1995), Oren (1996), Kaminsky (1997) and Deng (1999)). Earlier papers use geometric brownian motion (GBM) to model electricity prices. It has been observed, however, (see Kaminsky (1997)) that electricity prices show characteristics such as mean-reversion due to production characteristics and seasonality effects, jumps and spikes (or regime switching) due to non-storability and unpredictable outages, and stochastic volatility due to uncertain weather and demand patterns. Hence the GMB assumption produces systematically erroneous results for pricing of electricity contracts and derivatives. Deng (1999) was the first to use affine jump-diffusions, introduced in Duffie and Kan (1996,) to describe electricity spot prices and price electricity contracts. This specification can capture the aformentioned characteristics of the price process. Further, one can use transform analysis (see Duffie, Pan and Singleton (1998)) to compute the prices and hedging portfolios of various derivatives.

Gedra (1991) and Gedra and Varaiya (1993) introduce the concepts of "callable forwards" and "putable forwards" and provide a thorough analysis of their efficiency properties and their potential as substitutes to interruptible service contracts and dispatchable Independent Power Producer (IPP) supply contracts, respectively. As explained above, these contracts have optionality in them which translates into a discount or premium of the forward price paid by the holder of the option.

In a "callable forward", the customer is "long" 1 forward contract and "short" 1 call option which has a customer-selected strike price. The supplier holds opposite positions and can exercise the call option whenever the spot price exceeds the strike price, effectively canceling the forward contract at the time of delivery. The discount that the customer gets on the forward price is the option price at the time of contracting scaled to the delivery date.

In a "putable forward" the customer is "long" 1 forward contract and "long" 1 put option which has a supplier-selected strike price, while the supplier holds opposite positions. The customer can exercise the put option whenever the spot price is below the strike price, effectively canceling the forward contract at the time of delivery. The premium that the customer pays on the forward price is the option price at the time of contracting scaled to the delivery date.

Oren (1996) considers another instrument that captures an early notification option in a "callable forward" contract. The call option embedded in the forward contract now has an additional, earlier date at which the supplier can cancel the forward contract. For a forward contract with delivery at time T_2, two strike prices are specified by the customer, k_{T_1} and k_{T_2}, for times T_1 and T_2, respectively. T_1 is an intermediate time when the supplier has the option of notifying the customer that he will be curtailed at time T_2. It is expected that the T_1 strike price will be lower than the one at T_2.

Oren shows that the strike prices specified by a rational customer will be its reservation prices (from shortage costs) and the instrument will be able to replicate an efficient outcome that would reflect decisions taken by a benevolent central planner with perfect information. More specifically, curtailment will occur at time T_2 only if the electricity spot price is above the reservation price, k_{T_2}, of the customer (and if curtailment has not occurred at T_1). The payoff to the supplier at time T_2 is therefore exactly like a call option on the forward with strike price k_{T_2} expiring at T_2. On the other hand, notification of curtailment will occur at time T_1 if the payoff to the supplier from curtailment (say, by entering into a forward contract with another party) is greater than the value of the time-T_2 call option that is killed by the notification of curtailment. Thus, there will be a region above the T_1 strike price where the supplier will not curtail the first customer. The payoff of the call option at the two time points is described in Figure 1. The discount that the customer gets on the forward price is the price of this double-call option at the time of contracting scaled forward to the date of delivery.

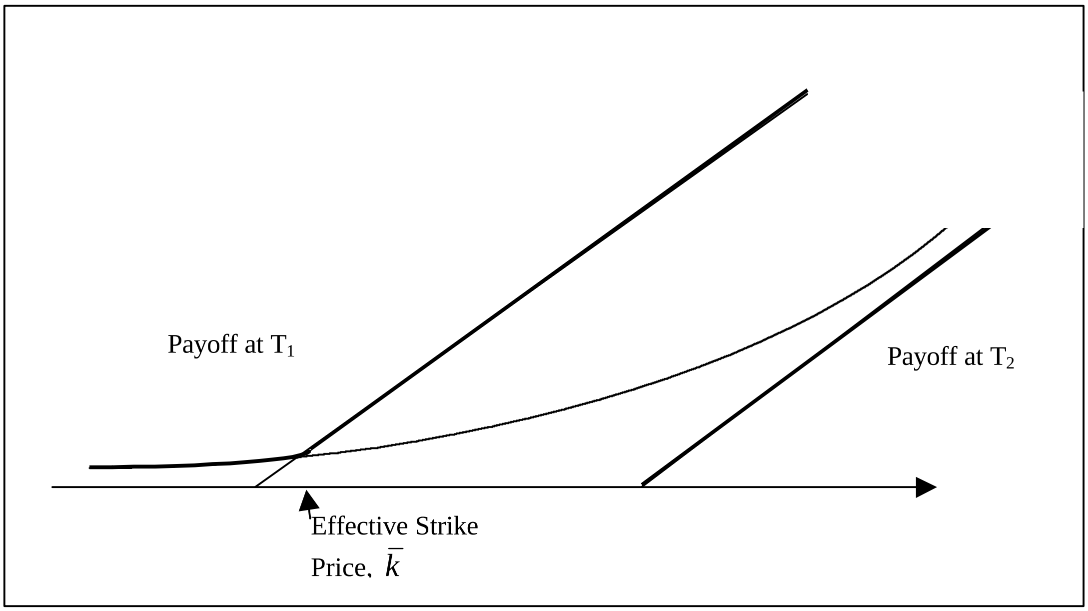

Figure 1. Payoff of the double-call option at time T_1 and T_2.

3. Pricing and Hedging Portfolios - GBM and AD

Oren (1996) uses Geometric Brownian Motion (GBM) to describe the electricity price process and derives prices of the three contracts under this canonical form. This section revisits the pricing problem under GBM and shows that the formulae can be easily extended to the case where the spot price is mean-reverting. Section 4 considers jump behavior in the spot price process and uses transform analysis to arrive at almost-closed form solutions of the option price.

3.1 Geometric Brownian Motion (GBM)

Consider the spot price process:

$$\frac{dS_t}{S_t} = m_t dt + s \ dB_t \tag{1}$$

where S_t is the spot price, m_t and s (we treat s as a constant) are parameters and B_t is standard brownian motion. It can be shown (by an application of Girsanov's theorem (see Oksendal

(1995))) that under the equivalent martingale measure or the risk-neutral measure, the process will follow:

$$\frac{dS_t}{S_t} = rdt + s \ dB_t^Q \qquad (2)$$

where B^Q is brownian motion under the risk-neutral measure and r is the interest rate. We can write down the process for lnS_t by applying Itô's lemma (see Oksendal (1995)). Observe that lnS_T given S_t will be Normally distributed.

$$d \ln S_t = (r - \tfrac{1}{2} s^2)dt + s \ dB_t^Q \qquad (3)$$

The above implies that $\ln \frac{S_T}{S_t} \sim N((r - \frac{1}{2} s^2)(T-t), s^2(T-t))$ and thus the spot price is lognormal at time T. This is the usual brownian motion model used to model stock prices.

3.1.1 Forward Prices

Consider a forward contract for delivery at time T_2. The forward[2] price of an underlying is defined as the price paid at delivery that sets the price of a derivative with the payoff - $(S_T - f_{t,T2})^+$ - at time t to zero. As the values of all derivative securities will be martingales (after discounting) under the risk-neutral measure (see Harrison and Kreps (1979), Duffie (1996)) the forward price can be expressed as:

$$f_{t,T_2} = E_t^Q[S_{T_2}] = E_t^Q[\exp\{\ln S_{T_2}\}] \qquad (4)$$

As the forward price is an expectation of a time-T_2 random variable it will be a martingale under the risk neutral measure before time T_2. Using Itô's lemma:

$$\begin{aligned} df_t &= s \ f_t \, dB_t^Q \\ \therefore f_{T_2} &= f_t \exp\{-\tfrac{1}{2} s^2 (T_2 - t) + s\,(B_{T_2}^Q - B_t^Q)\} \end{aligned} \qquad (5)$$

Forward prices are also lognormal under this model.

3.1.2 Option Pricing

As described before, a call option expiring at time T_2, on a forward contract for delivery at the same time is a security with a payoff of $(f_{T_2} - k_{t_2})^+$, where f_{T_2} is the price of the underlying at the time of expiration, and k_{t_2} is the strike price. Using the valuation principle described in the section above, the call option can be valued as follows:

$$C_t(k_{T_2} \mid f_t) = E^Q[e^{-r(T_2 - t)}(f_{T_2} - k_{t_2})^+ \mid f_t] \qquad (6)$$

[2] Under constant interest rates, futures prices will equal forward prices at the time of contracting.

Evaluating the above expectation using equation 5 for the forward price at T_2, one can derive the option price (see Figure 2 for a plot of option prices at various times before expiration). The resulting formula is also known as Black's formula (see Black (1976)).

Proposition 1 The option value in a "callable forward with early notification" after time T_1 is the price of a simple call option on the forward. The discount on the forward price in a "callable forward" is e^{rt_2} times the option price given by:

$$C_t(k_{T_2} \big| f_t) = e^{-rt_2}[\, f_t \; N(d_1) - k_{T_2} \; N(d_2)] \tag{7}$$

where, f_t is forward price at time t; k_{T_2} is the strike price of the option;

$$d_1 = \frac{\ln(f_t / k_{T_2}) + \frac{1}{2} s^{2}(GBM, t, T_2)}{s\,(GBM, t, T_2)}; d_2 = d_1 - s\,(GBM, t, T_2);$$

$t_2 = T_2 - t$ and $s^{2}(GBM, l_1, l_2) = s^{2}(l_2 - l_1)$.

The premium on the forward price in a "putable forward" is e^{rt_2} times the price of a put option given by:

$$P_t(k_{T_2} \big| f_t) = e^{-rt_2}[\, f_t \; N(-d_1) - k_{T_2} \; N(-d_2)] \tag{8}$$

Proof : see Appendix or Black (1976) for an alternative derivation.

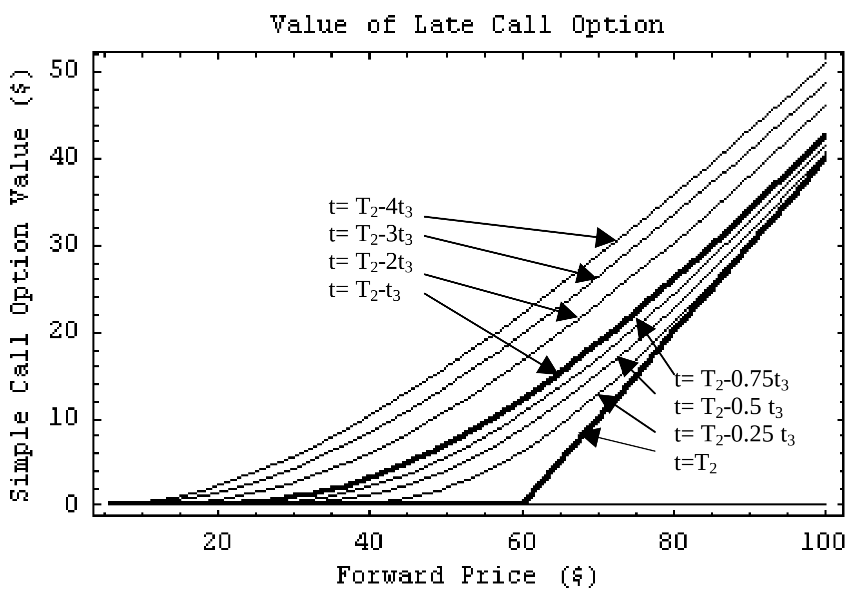

Figure 2. Value of the Simple Call Option under GBM ($t_3 = T_2 - T_1$).

3.1.3 The Callable Forward with Early Notification

The holder of a double call option emulating an interruptible contract with early notification will exercise it at time T_1 if the forward price is above the effective strike price $\bar{k}$. If early exercise is not optimal, the holder will exercise the option at T_2 only if the spot price is above the T_2 strike price. Figure 3 shows an example where the forward price at T_1 is \$50 and the spot price at T_2 is \$40.

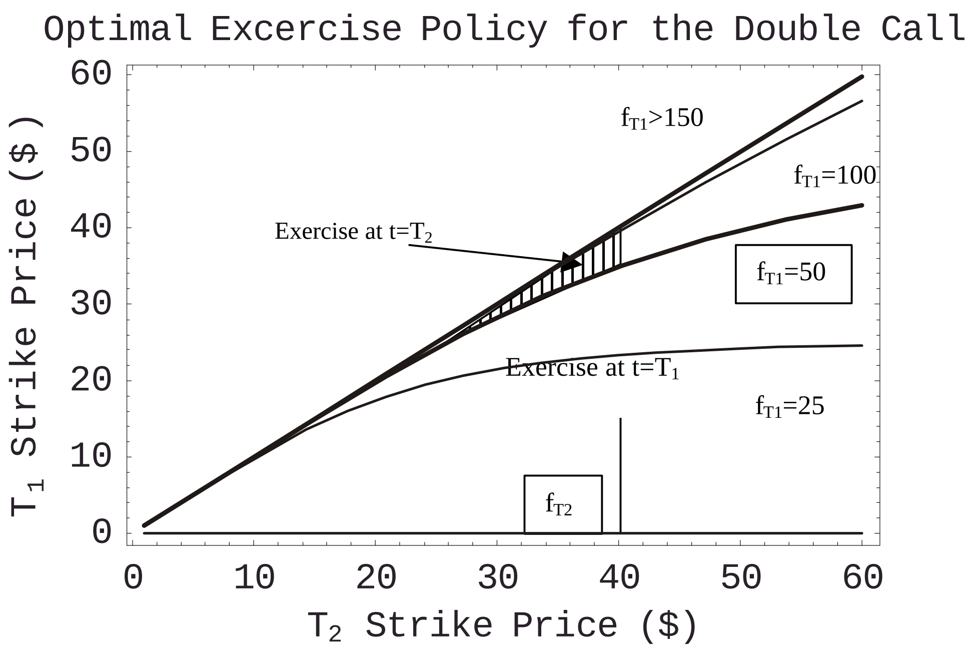

Figure 3. Optimal Exercise Policy for the Double-call Option.

The double-call option will be excercised at T_1, if the T_1 strike price is below the indifference curve corresponding to f_{T_1} = \$50. Additionally, if the T_1 strike price is above this curve, but the T_2 strike price is below the T_2 spot price of \$40 (shaded area in the example), the option will be excercised at T_2.

To calculate the discount given to a customer in a "callable forward with early notification" we need to price the double-call option embedded in the forward contract. We begin by analyzing the payoff of the double-call at time T_1. Excercising the double-call option at T_1 means that the time T_2 option embedded in it is killed. The double-call will be excercised only when the payoff to the supplier, say, by entering into a new forward contract at time T_1 is higher then the value of the killed option. One can therefore calculate a forward price $\bar{k}$, which is the effective strike price at which the double-call is excercised at $T_{1.}$

One can now proceed by breaking up the payoff of the double-call at T_1 into payoffs of simpler derivatives. Observe that the payoff can be seen as that of a special call option (as actual payoff at T_1 does not accrue until T_2 the slope of the effective payoff is not equal to 1) expiring at T_1 with strike price $\bar{k}$, plus the value of the later call option given that the forward price is $\bar{k}$, less the price of a compound put option with strike price $e^{-rt_3}(\bar{k} - k_{T_1})$ (see Figure 4).

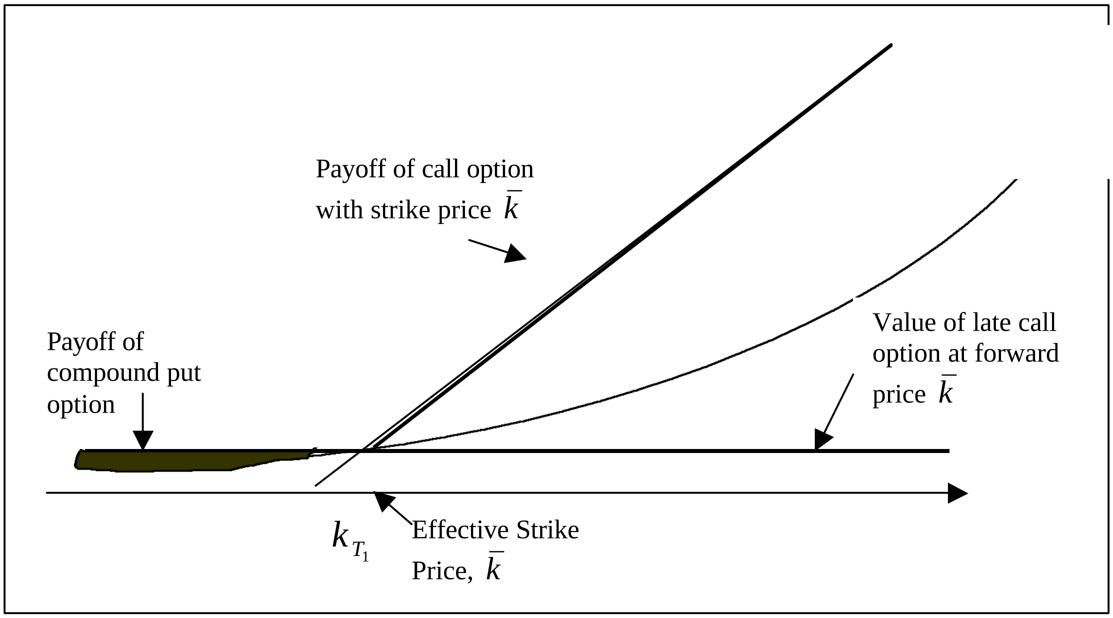

Figure 4. Analyzing the Payoff of the Double-call Option at time T_1.

The price of the call option before time T_1 can therefore be written as:

$$\hat{C}_t(k_{T_1}, k_{T_2} \mid f_t) = C'_{t,T_1}(\bar{k} \mid f_t) + e^{-r(t-T_1)} C_{T_1}(k_{T_2} \mid f_{T_1} = \bar{k}) - P^{co}_{t,T_1}(e^{-rt_3}(\bar{k} - k_{T_1}) \mid f_t) \quad (9)$$

where,

The first term is the value of a special call option expiring at T_1 with strike price $\bar{k}$;

The second term is the discounted sure value of the later call option at forward price $\bar{k}$;

The third term is the price of a compound put option that allows the holder to sell a call option for time T_2 at strike price $e^{-rt_3}(\bar{k} - k_T)$. The payoff of this put at time T_1 is $[e^{-rt_3}(\bar{k} - k_{T_1}) - C_{t,T_1}(k_{T_2} \mid f_{T_2})]^+$. Figure 5 gives the value of the double call option under GBM.

Proposition 2 The discount on the forward price is e^{rt_2} times the price of the double-call option given by:

$$\hat{C}_t(k_{T_1},k_{T_2}\,|\,f_t)=e^{-rt_2}[\,f_t\,N(d_3)-\bar{k}\,N(d_4)]+e^{-rt_2}[\,\bar{k}\,N(d_1)-k_{T_2}\,N(d_2)]-P^{co}_{t,T_1}(k_1\,|\,f_t) \qquad (10)$$

where, f_t is the forward price at time t;
$t_1 = T_1 - t$; $t_2 = T_2 - t$; $t_3 = T_2 - T_1$ and

$$d_1=\frac{\ln(f_t/k_{T_2})+\frac{1}{2}s^2(GBM,t,T_2)}{s(GBM,t,T_2)};\, d_2=d_1-s(GBM,t,T_2);$$

$$d_3=\frac{\ln(f_t/\bar{k})+\frac{1}{2}s^2(GBM,t,T_1)}{s(GBM,t,T_1)};\, d_4=d_3-s(GBM,t,T_1);$$

$$s^2(GBM,t_1,t_2)=s^2(t_2-t_1)$$

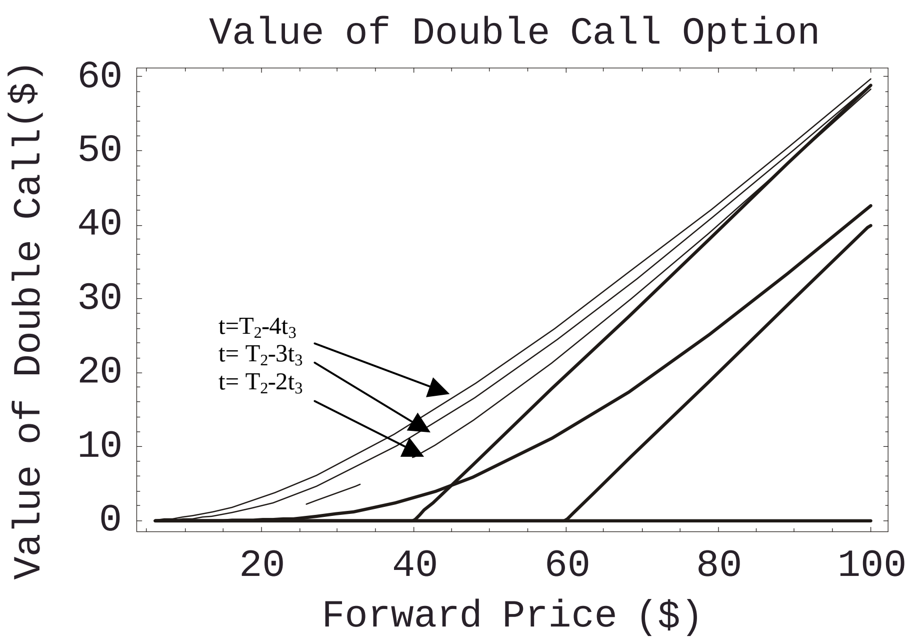

Figure 5. Value of the Double Call Option under GBM ($t_3 = T_2 - T_1$).

$\bar{k}$ *solves* :

$$\bar{k}-k_{T_1}=e^{rt_3}C_{T_1}(k_{T_2}\,|\,f_{T_1}=\bar{k})$$

$$P^{co}_{t,T_1}(k_1\,|\,f_t)=e^{-rt_2}k_{T_2}N_2(-a_2,b_2;-\sqrt{\tfrac{t_1}{t_2}})-e^{-rt_2}\,f_tN_2(-a_1,b_1;-\sqrt{\tfrac{t_1}{t_2}})+e^{-rt_1}k_1N(-a_2)$$

where $N_2(a, b; r)$ is the bivariate Normal distribution with correlation r; $k_1 = e^{-rt_3}(\bar{k}-k_{T_1})$

$$a_1 = \frac{\ln(f_t/k) + \frac{1}{2} s^2(j,t,T_1)}{s(j,t,T_1)}; b_1 = \frac{\ln(f_t/k_{T_2}) + \frac{1}{2} s^2(j,t,T_2)}{s(j,t,T_2)} \text{ and } r = -\sqrt{\frac{t_1}{t_2}};$$

$$a_2 = a_1 - s(j,t,T_1) \text{ and } b_2 = b_1 - s(j,t,T_2)$$

Proof: see Appendix.

3.2 Affine Diffusion (Mean reverting)

A class of processes which incorporates many characteristics useful for modeling electricity spot prices are Affine Jump-Diffusions (AJDs), first introduced in Duffie and Kan (1996). One can incorporate various aspects of the price process such as mean reversion, stochastic volatility and jumps using this class of processes. An advantage in calculating option prices using AJDs is that the Fourier transform of the distribution of the underlying is known - for some cases up to the solution of ODEs.

In this section we consider a special case - an Affine Diffusion (AD) - for the log spot price under the risk-neutral measure. This incorporates mean reversion in the spot price process and results in time dependent volatility for the forward price. We show that spot and forward prices remain lognormal in the mean-reverting case with different volatility than the GBM case. As the forward price will converge in value and variance to the spot price at the delivery date, pricing formulae under an affine diffusion will have the same form as the GBM case with a different variance term.

Proposition 3 Option prices, and hence discounts in the forward contracts under an affine diffusion will have the same form as the GBM case with the variance term replaced by $s^2(MR, t_1, t_2) = \frac{s^2}{2k}[1 - \exp\{-2k(t_2 - t_1)\}]$.

Proof: see Appendix.

Figure 6 shows the value of a simple call option under a mean-reverting spot price. As information about mean reversion is included in the forward price, the only noticeable difference between option prices under GBM and the affine diffusion is the lack of time value in the affine diffusion case for longer time periods before expiration i.e., the value converges rapidly to a steady state. Figure 7 which shows the optimal excercise policy for the double call option under this mode, while Figure 8 shows the value of the double-call option under the affine diffusion.

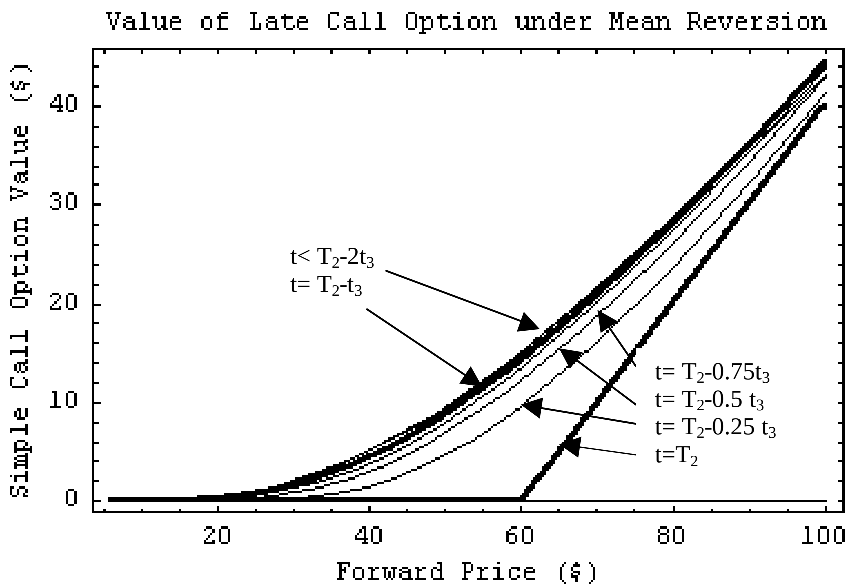

Figure 6. Value of the Simple Call Option under an Affine Diffusion (Mean Reverting) ($t_3 = T_2 - T_1$).

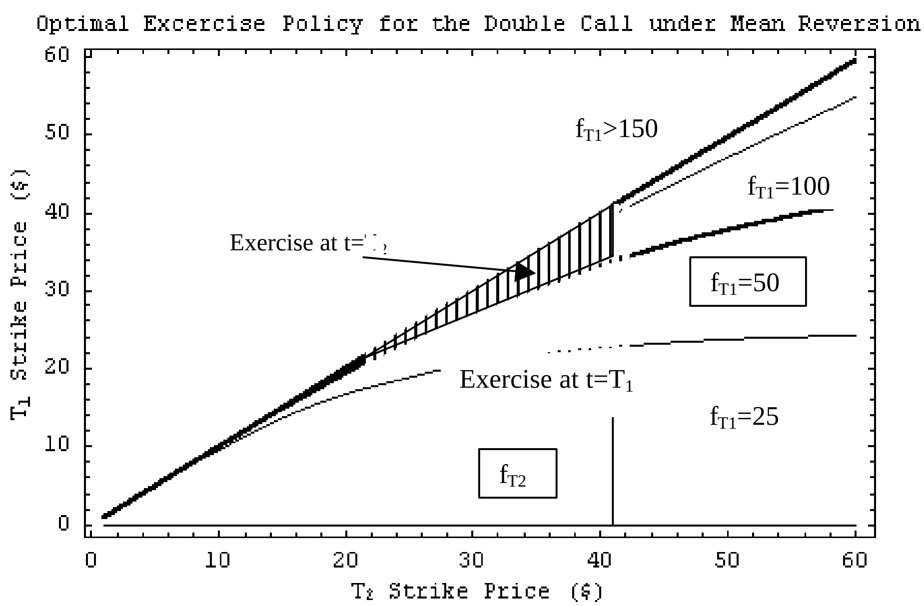

Figure 7. Optimal Exercise Policy for the Double Call Option under an Affine Diffusion (Mean Reverting)

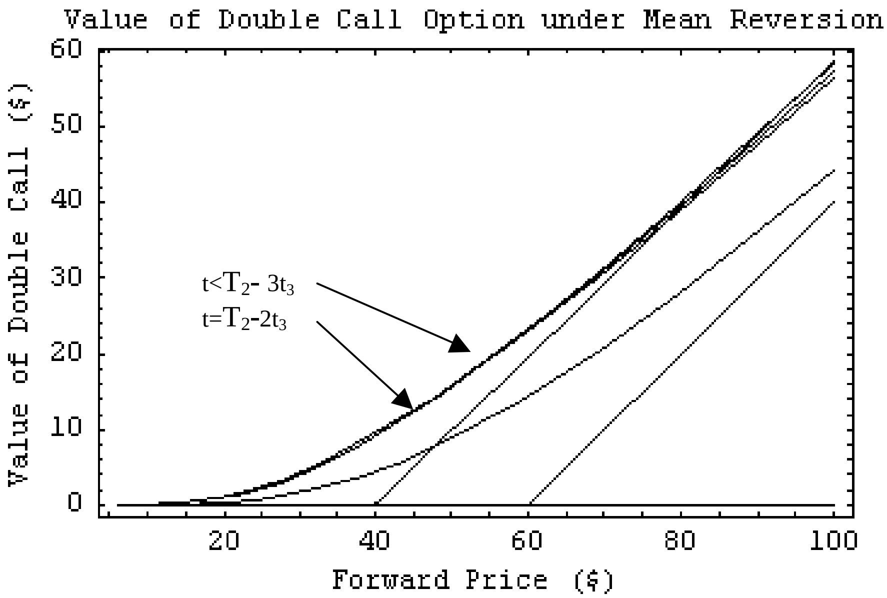

Figure 8. Value of Double Call Option under an Affine Diffusion (Mean Reverting) (t_3 = T_2- T_1).

4 Affine Jump-Diffusion (AJD)

This section extends the models in section three to include jump behavior in the price process. Non-storability of electricity causes steep changes in prices with changing supply and demand conditions. Deng (1999) uses AJDs to model electricity prices. Deng presents several specifications, one having regime switching and one with another factor to capture stochastic volatility. Apart from flexibility that this class of processes offers, one advantage is that one can use transform analysis to arrive at almost-closed form solutions of the option price. Including stochastic volatility or regime switching behavior means that call option prices will depend on two random factors. While this does not create difficulties in pricing simple call options, analytical pricing of the double-call becomes very difficult. The main difficulty comes in specifying the set over which the double-call will be excercised at time T_1. We therefore concentrate on the specification having only one factor and leave the others for future research. We assume that the log spot price follows an affine jump-diffusion under the risk neutral measure.

$$dX_t = (k_0 + k_1 X_t)dt + s\,dB_t^Q + dZ_t \qquad (11)$$

where X_t = lnS_t, s is the volatility of the spot price (which will be taken to be constant), B_t is standard brownian motion under the risk-neutral measure, and Z is a pure jump process with arrival intensity λ and jump-size transform ϕ(c, t). The drift and variance terms have an affine form (we assume constant volatility).

Both previous cases can be seen as special cases of an AJD. The GBM formulation uses κ_0 = r-1/2s^2, κ_1=0. The mean-reverting affine diffusion uses $\kappa_0 = \kappa\theta$, $\kappa_1 = -\kappa$. Both specifications use constant volatility, s, and do not model jump behavior. For this section we continue to use the affine diffusion parameters and add jump behavior to the model. We assume two independent jump processes with intensities, λ_1 and λ_2 and jump-size tranform:

$$f_j(c,t) = \frac{1}{1 - m_j c} \tag{12}$$

4.1 Transforms and Forward Prices

Define the transform of the distribution at T_2 as:

$$\Psi(v,t,T_2,X_t) = E_t^Q[e^{-r(T_2-t)}\exp\{v X_{T_2}\} \big| f_t]$$

where E_t^Q denotes expectation under the risk-neutral measure Q.

As this is the discounted payoff of a single random variable, Ψ e^{-rt} will be a martingale under the risk-neutral measure (some regularity conditions are required (see Duffie et al, 1998)). Following Duffie et al (1998), it is conjectured that the transform will have the form:

$$\Psi(v,t,T_2,X_t) = \exp[a'((T_2-t),v) + b'((T_2-t),v) X_t] \tag{13}$$

As special cases of this process are Gaussian, the exponential affine form is a natural choice. It can be shown, by applying Ito's lemma, that the transform takes this form for the one timepoint case, where α' and β' solve:

$$\begin{aligned} &\frac{d}{dt} b'(t,v) + B(b'(t,v),t) = 0 \qquad b'(0,v) = v \\ &\frac{d}{dt} a'(t,v) + A(b'(t,v),t) = 0 \qquad a'(0,v) = 0 \end{aligned} \tag{14}$$

where, for complex c,

$$\begin{aligned} &B(c,t) = -kc \\ &A(c,t) = kqc + \tfrac{1}{2} s\ c^2 - r + \sum l_j (f_j(c,t) - 1) \end{aligned} \tag{15}$$

We can integrate the differential equations to get:

$$b'(t,v) = n\, e^{-kq\,t}$$

$$a'(t,v) = q\,n(1 - e^{-kq\,t}) + \tfrac{s^2 n^2}{4k}(1 - e^{-2kt}) - rt - \sum_j \tfrac{l}{k} \ln[\tfrac{m_j - 1}{m_j e^{-kt} - 1}] \tag{16}$$

The forward price is given by:

$$f_t = e^{r(T_2-t)} \Psi(1,t,T_2,X_t)$$

$$= \exp\{q\,(1-e^{-kq\,t}) + \tfrac{s^2}{4k}(1-e^{-2kt}) - rt - \sum_j \tfrac{l}{k}\ln[\tfrac{m_j-1}{m_j e^{-kt}-1}] + e^{-kq\,t}X_t\} \qquad (17)$$

We take parameter values from Deng (1999) (see Table 1 in Appendix) for illustrative purposes. Figure 9 shows forward curves under contango (spot price = \$24.63) and backwardation (spot price = \$120.00). The longterm mean spot price is \$30.00.

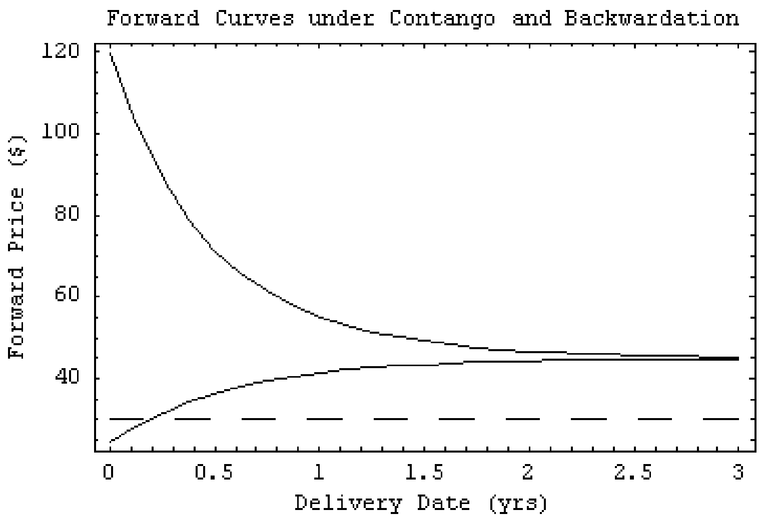

Figure 9. Forward Curves under Contango and Backwardation (Affine Jump-Diffusion).

The forward price will converge to a higher quantity than the longterm mean, depending on the parameters for the mean-reverting and jump parts of the model. Observe that the increased volatility from the jumps contributes a significant amount in forward prices and that they converge well above the long-term mean.

4.2 Option Pricing

We calculate option prices under this model by expressing the option price in terms of simpler securities and price these directly using the transform calculated in (13) and (16). Let $G_{a,b}(y)$ denote the price of a security that pays $e^{aX}{}_T$ at time T in the event $b.X_T \le y$. Thus, one can express the call option price expiring at T_2, in terms of these simpler securities:

$$C_t(k_{T_2}\,|\,f_t) = G_{1,-1}(-\ln k_{T_2}) - k_{T_2} G_{0,-1}(-\ln k_{T_2}) \qquad (18)$$

To determine the price of the simpler securities we express them in terms of the transform of the uncertainty in log spot price at T_2, and then use inversion formulae to derive almost-closed form solutions of the call option price (see Figure 10 for the value of the simple call option under an affine jump-diffusion).

Proposition 4 The option value in a "callable forward with early notification" after time T_1 is the price of a simple call option on the forward. The discount to the forward price in a "callable forward" is e^{rt_2} times the option price given by:

$$C(k_{T_2}\big|f_t) = e^{-rt_2} f_t\left(\frac{1}{2} - \frac{1}{p}\int_0^\infty \frac{\mathrm{Im}[\Psi(1-iv,t,T_2,X_t)e^{rt_2+iv\ln k_{T_2}}]}{vf_t}dv\right) -$$

$$e^{-rt_2} k_{T_2}\left(\frac{1}{2} - \frac{1}{p}\int_0^\infty \frac{\mathrm{Im}[\Psi(-iv,t,T_2,X_t)e^{rt_2+iv\ln k_{T_2}}]}{v}dv\right) \qquad (19a)$$

where, f_t is the forward price at the time of contracting; k_{T_2} is the strike price; Ψ is the transform of the spot price distribution at T_2.

The premium on the forward price in a "putable forward" is e^{rt_2} times the price of a simple put option given by:

$$P(k_{T_2}\big|f_t) = e^{-rt_2} k_{T_2}\left(\frac{1}{2} - \frac{1}{p}\int_0^\infty \frac{\mathrm{Im}[\Psi(iv,t,T_2,X_t)e^{rt_2-iv\ln k_{T_2}}]}{v}dv\right) -$$

$$e^{-rt_2} f_t\left(\frac{1}{2} - \frac{1}{p}\int_0^\infty \frac{\mathrm{Im}[\Psi(1+iv,t,T_2,X_t)e^{rt_2-iv\ln k_{T_2}}]}{vf_t}dv\right) \qquad (19b)$$

Proof: see Appendix.

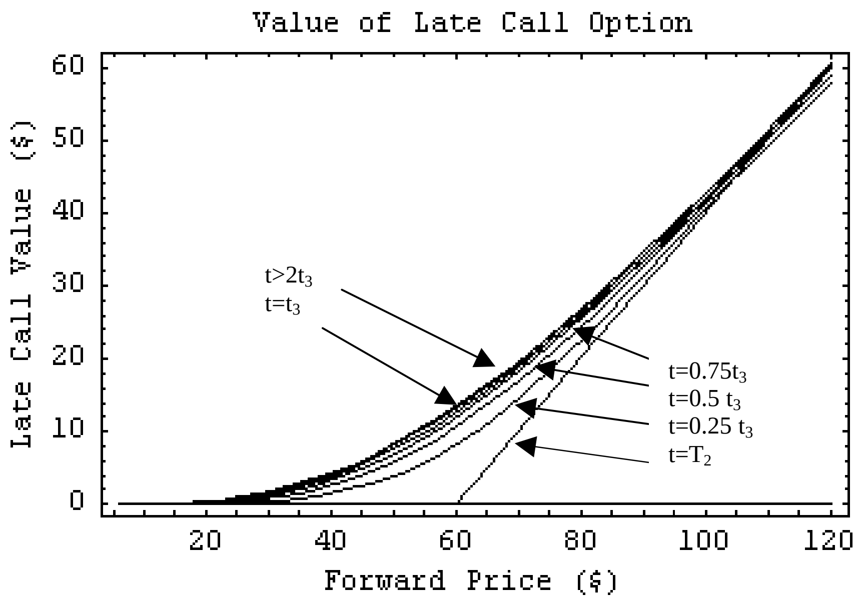

Figure 10. Value of late Call option under an Affine Jump-Diffusion.

It may seem surprising that for a given forward price, the simple call option value under an AJD is lower than under an AD. This is because a given forward price under the two models does not correspond to equal spot prices at the time of contracting. The corresponding spot price under an AD would be significantly higher and hence would yield a higher option price.

4.3 Callable Forward with Early Notification

As before, one can determine a unique $\bar{k}$ at time T_1, as the forward price at which : $\bar{k} - k_{T_1} = e^{rt_3} C_{T_1}(k_{T_2} \big| f_{T_1} = \bar{k})$. The optimal exercise policy will therefore remain the same. One can now proceed in a similar manner pricing the double-call option before time T_1 by breaking up the payoff and using transform analysis. For $t < T_1$

$$\hat{C}_t(k_{T_1}, k_{T_2} \big| f_t) = C'_{t,T_1}(\bar{k} \big| f_t) + e^{-rt_1} C_{T_1}(k_{T_2} \big| f_{T_1} = \bar{k}) - P^{co}_{t,T_1}(e^{-rt_3}(\bar{k} - k_{T_1}) \big| f_t) \qquad (20)$$

where,

The first term is the value of a special call for time T_1 and strike price $\bar{k}$.

The second term is the discounted sure value of the later call option at price $f_t = \bar{k}$

The third term is the price of a compound put option that allows the holder to sell a call option for time T_2 at strike price $k_1 = e^{-rt_3}(\bar{k} - k_{T_1})$. The payoff of this put at time T_1 is :

$[k_1 - C_{T_1}(k_{T_2} \big| f_{T_1})]^+$.

In section 3 we had derived the process followed by the forward price and we could price the intermediate call option, C', directly using this process. Here, we begin by determining the transform of the forward price uncertainty at T_1. We can write this as:

$$\Phi(g, t, T_1, X_t) = E_t^Q[e^{-r(T_1-t)} \exp\{g\, Y_{T_1}\} \big| f_t] \qquad (22)$$

where, Y_{T_1} is log forward price at T_1. Using the definition of f_{T_1} from (17), we have:

$$gY_{T_1} = g \ln f_{T_1} = ga'((T_2 - T_1),1) + gb'((T_2 - T_1),1) X_{T_1}$$

$$\Phi(g, t, T_1, X_t) = E_t^Q[e^{-r(T_1-t)} \exp\{ga'((T_2 - T_1),1) + gb'((T_2 - T_1),1) X_{T_1}\} \big| f_t] \qquad (23)$$

The forward price transform at T_1 can now be written as:

$$\begin{aligned}\Phi(g, t, T_1, X_t) = \exp[&ga'((T_2 - T_1),1) + a'((T_1 - t), gb'((T_2 - T_1),1)) \\ &+ b'((T_1 - t), gb'((T_2 - T_1),1))\, X_t]\end{aligned} \qquad (24)$$

One can now proceed as before and price the call option using the methodology in Proposition 4. For the compound put option in the double-call, we need to work with a joint transform of the forward price at T_1 and T_2. To see this, we write the put option as the expected value of the discounted payoff under the risk-neutral measure[3]:

$$
\begin{aligned}
P_t^{co} &= E_t^Q[e^{-rt_1}\{k_1 - C_{T_1}(k_{T_2} \mid f_{T_1})\}^+] \\
&= E_t^Q[e^{-rt_1}k_1 \Uparrow_{k_1 \geq C_{T_1}(k_{T_2}\mid f_{T_1})}] - E_t^Q[e^{-rt_1}E_{T_1}^Q[e^{-rt_3} f_{T_2} \Uparrow_{\ln f_{T_2} \geq \ln k_{T_2}}] \Uparrow_{k_1 \geq C_{T_1}(k_{T_2}\mid f_{T_1})}] \\
&\quad + E_t^Q[e^{-rt_1}E_{T_1}^Q[e^{-rt_3}k_{T_2} \Uparrow_{\ln f_{T_2} \geq \ln k_{T_2}}] \Uparrow_{k_1 \geq C_{T_1}(k_{T_2}\mid f_{T_1})}]
\end{aligned} \tag{25}
$$

now, $k_1 \geq C_{T_1}(k_{T_2} \mid f_{T_1}) \equiv \ln\ f_{T_1} \leq \ln \bar{k}$ (see Figure 2).

Using the law of iterated expectations we can write this as:

$$
\begin{aligned}
&= k_1 E_t^Q[e^{-rt_1} \Uparrow_{\ln f_{T_1} \leq \ln \bar{k}}] - E_t^Q[e^{-rt_2}\exp\{\ln\ f_{T_2}\} \Uparrow_{\ln f_{T_1} \leq \ln \bar{k}} \Uparrow_{\ln f_{T_2} \geq \ln k_{T_2}}] \\
&\quad + k_{T_2} E_t^Q[e^{-rt_2} \Uparrow_{\ln f_{T_1} \leq \ln \bar{k}} \Uparrow_{\ln f_{T_2} \geq \ln k_{T_2}}]
\end{aligned} \tag{26}
$$

To evaluate these expectations we define the joint transform of the uncertainty at T_1 and T_2:

$$\Phi'(u,v,t,T_1,T_2,X_t) = E[e^{-r(T_2-t)}\exp\{u\,Y_{T_1} + v\,Y_{T_2}\}] \tag{27}$$

where $Y_{T_1} = \ln f_{T_1}$ and $Y_{T_2} = \ln f_{T_2} = X_{T_2}$, as the forward price will converge to the spot price at delivery. We conjecture the same form as in the single timepoint case (this holds for the first two models where this distribution is a bivariate normal):

$$\Phi'(u,v,t,T_1,T_2,X_t) = \exp[a(t_2,u,v) + b(t_2,u,v)\,X_t] \tag{28}$$

Again, as this can be seen as the discounted payoff of a random variable, $e^{-rt}\Phi'_t$ will be a martingale under the risk-neutral measure. Applying Ito's lemma (see Protter (1990) for the complex version) we see that, as before, α and β have to follow (14) and (15). To determine boundary conditions consider Φ_{T_1}:

$$
\begin{aligned}
\Phi'_{T_1} &= E_{T_1}^Q[e^{-r(T_2-T_1)}\exp\{uY_{T_1} + vY_{T_2}\}] \\
&= \exp\{uY_{T_1}\}E_{T_1}^Q[e^{-r(T_2-T_1)}\exp\{vY_{T_2}\}] \text{ (as } Y_{T_1} \text{ is known at time } T_1\text{)}
\end{aligned} \tag{29}
$$

One can recognize the second term as the transform of the forward price for the one time point case and substitute from (24). Thus one can write Φ'_{T_1} as:

$$
\begin{aligned}
\Phi'(u,v,T_1,T_1,T_2,X_{T_1}) = \exp\{&urt_3 + ua'(t_3,1) + a'(t_3,v) \\
&+ [ub'(t_3,1) + b'(t_3,v)]\,X_{T_1}\}
\end{aligned} \tag{30}
$$

One can now solve for Φ'_t by solving (14) and (15) for the boundary conditions:

[3] In what follows we use following notation for the indicator function: $\Uparrow_{x \geq y} = 1 \text{if } x \geq y,\ = 0\ o.w.$

$$a(t_3,u,v) = urt_3 + ua'(t_3,1) + a'(t_3,v)$$
$$b(t_3,u,v) = u + b'(t_3,v) \tag{31}$$

Rather than extending the inversion formula used for the simple call option, we use an alternative approach to evaluate the expectations in (26). The compound call can be expressed as:

$$P^{co}_{t,T_1}(k_1 | f_t) \quad = k_1 G^{t,T_1}_{0,-1}(\ln \bar{k}) - e^{-rt_2} f_t \Pi_1(t,T_1,T_2) + e^{-rt_2} k_{T_2} \Pi_2(t,T_1,T_2) \tag{32}$$

where,

k_1 is the strike price for the compound call, P^{co}; $G^{t,T1}$ is as defined in section 4.2; Π_1 and Π_2 are defned as:

$$\Pi_1(t,T_1,T_2) \ = \frac{E^Q_t[e^{-rt_2} \exp\{\ln f_{T_2}\} \Uparrow_{\ln f_{T_1} \le \ln \bar{k}} \Uparrow_{\ln f_{T_2} \ge \ln k_{T_2}}]}{E^Q_t[e^{-rt_2} \exp\{\ln f_{T_2}\}]}$$
$$\Pi_1(t,T_1,T_2) = \frac{E^Q_t[e^{-rt_2} \Uparrow_{\ln f_{T_1} \le \ln \bar{k}} \Uparrow_{\ln f_{T_2} \ge \ln k_{T_2}}]}{E^Q_t[e^{-rt_2}]} \tag{33}$$

It can be easily confirmed that Π_1 and $\Pi_2 \in [0, 1]$ and thus they can be determined after calculating their characteristic functions and inverting according to the method developed in Shepard (1991) (see Bakshi and Madan (1998) for an application to option pricing). To evaluate Π_1 and Π_2 observe that if we treat these as distribution functions, we can express their characteristic functions as:

$$\Theta_1(f,j,t,T_1,T_2;Y_{T_1},Y_{T_2}) = \frac{\Phi'(if,1+ij,t,T_1,T_2,X_t)}{\Phi'(0,1,t,T_1,T_2,X_t)}$$
$$\Theta_2(f,j,t,T_1,T_2;Y_{T_1},Y_{T_2}) = \frac{\Phi'(if,ij,t,T_1,T_2,X_t)}{\Phi'(0,0,t,T_1,T_2,X_t)} \tag{34}$$

This leads to Proposition 5. Figure 11 shows plots of the double call option under an affine jump-diffusion.

Proposition 5 The discount in the "callable forward with early notification" is equal to e^{rt_2} times the price of the double-call option at the time t:

$$\hat{C}_t(k_{T_1},k_{T_2} | f_t) =$$

$$\left\{ e^{-rt_2} f_t(\frac{1}{2} - \frac{1}{p}\int_0^\infty \frac{\mathrm{Im}[\Phi(1-iv,t,T_1,X_t)e^{rt_1+iv\ln\bar{k}}]}{vf_t}dv) - e^{-rt_2}\bar{k}(\frac{1}{2} - \frac{1}{p}\int_0^\infty \frac{\mathrm{Im}[\Phi(-iv,t,T_1,X_t)e^{rt_1+iv\ln\bar{k}}]}{v}dv) \right\}$$

$$+\left\{ e^{-rt_2}\bar{k}\left(\frac{1}{2}-\frac{1}{p}\int_0^{\infty}\frac{\mathrm{Im}[\Psi(1-iv,t,T_2,X_t)e^{rt_3+iv\ln k_{T_2}}]}{vf_t}dv\right)-e^{-rt_2}k_{T_2}\left(\frac{1}{2}-\frac{1}{p}\int_0^{\infty}\frac{\mathrm{Im}[\Psi(-iv,t,T_2,X_t)e^{rt_3+iv\ln k_{T_2}}]}{v}dv\right)\right\}$$

$$-\Big\{ e^{-rt_1}k_1\left(\frac{1}{2}-\frac{1}{p}\int_0^{\infty}\frac{\mathrm{Im}[\Phi(-iv,t,T_1,X_t)e^{rt_1-iv\ln\bar{k}}]}{v}dv\right)-e^{-rt_2}f_t\Big[\left(\frac{1}{2}-\frac{1}{p}\int_0^{\infty}\frac{\mathrm{Re}[\Theta_1(0,f,t,T_1,T_2,Y_{T_1})e^{-if\ln\bar{k}}]}{f}df\right)$$

$$-\Big(-\frac{1}{4}+\frac{1}{2}\left(\frac{1}{2}-\frac{1}{p}\int_0^{\infty}\frac{\mathrm{Re}[\Theta_1(f,0,t,T_1,T_2,Y_{T_1})e^{-if\ln\bar{k}}]}{if}df\right)+\frac{1}{2}\left(\frac{1}{2}-\frac{1}{p}\int_0^{\infty}\frac{\mathrm{Re}[\Theta_1(0,j,t,T_1,T_2,Y_{T_2})e^{-if\ln k_{T_2}}]}{ij}dj\right)$$

$$-\left(\frac{1}{2p^2}\int_0^{\infty}\int_0^{\infty}\left(\frac{\mathrm{Re}[\Theta_1(f,j,t,T_1,T_2,Y_{T_1},Y_{T_2})e^{-if\ln\bar{k}-ij\ln k_{T_2}}]}{fj}-\frac{\mathrm{Re}[\Theta_1(f,-j,t,T_1,T_2,Y_{T_1},Y_{T_2})e^{-if\ln\bar{k}+ij\ln k_{T_2}}]}{fj}\right)dfdj\right)\Big)\Big]$$

$$+e^{-rt_2}k_{T_2}\Big[\left(\frac{1}{2}-\frac{1}{p}\int_0^{\infty}\frac{\mathrm{Re}[\Theta_2(0,f,t,T_1,T_2,Y_{T_1})e^{-if\ln\bar{k}}]}{f}df\right)-$$

$$\Big(-\frac{1}{4}+\frac{1}{2}\left(\frac{1}{2}-\frac{1}{p}\int_0^{\infty}\frac{\mathrm{Re}[\Theta_2(f,0,t,T_1,T_2,Y_{T_1})e^{-if\ln\bar{k}}]}{if}df\right)+\frac{1}{2}\left(\frac{1}{2}-\frac{1}{p}\int_0^{\infty}\frac{\mathrm{Re}[\Theta_2(0,j,t,T_1,T_2,Y_{T_2})e^{-if\ln k_{T_2}}]}{ij}dj\right)$$

$$-\left(\frac{1}{2p^2}\int_0^{\infty}\int_0^{\infty}\left(\frac{\mathrm{Re}[\Theta_2(f,j,t,T_1,T_2,Y_{T_1},Y_{T_2})e^{-if\ln\bar{k}-ij\ln k_{T_2}}]}{fj}-\frac{\mathrm{Re}[\Theta_2(f,-j,t,T_1,T_2,Y_{T_1},Y_{T_2})e^{-if\ln\bar{k}+ij\ln k_{T_2}}]}{fj}\right)dfdj\right)\Big)\Big]\Big\}$$

where, Ψ is as defined in (13); Φ as defined in (24) and Θ_1 and Θ_2 as defined in (34).

Proof: see Appendix.

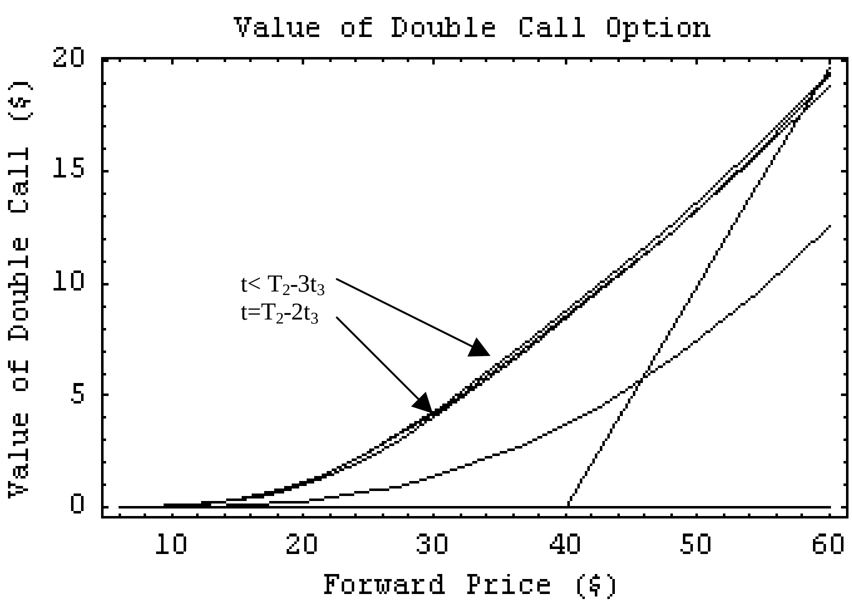

Figure 11. Value of the Double Call option under an affine jump-diffusion.

As seen in the mean reverting case the double-call option does not have much time-value for long periods before expiration i.e., its value approaches a steady state with respect to time to expiration.

5 Concluding Remarks and Further Research

In a competitive electricity market, financial instruments and derivatives based on underlying commodity contracts will play an important role as means for risk management. Such instruments can also emulate traditional contracts between customers, utilities and independent power producers aimed at improving the efficiency of resource utilization.

This paper studies the pricing problem of three efficiency motivated instruments in the electric power industry. Using increasingly complex price processes, the instruments were priced using forward contracts and option-like derivatives. The contract prices were first calculated under the canonical geometric brownian motion model. This was then extended to a mean-reverting spot price diffusion process by showing that option prices have the same form under this model with a different variance term. Further extension to include jump behavior was done with the use of transform analysis and almost closed form formulae were obtained under this model.

It was observed that mean-reversion causes option prices to reach a steady state value with respect to time to expiration, unlike the GBM case for which option prices approach a constant growth rate. This convergence is due to the fact that under the mean reversion assumption the distribution of the underlying at time of expiration converges to a steady state distribution. It was also observed that volatility from jump behavior can contribute significantly to forward and option prices.

Further research is needed to extend the present model to include another factor, such as regime switching behavior or a factor that models stochastic volatility in spot prices. An immediate problem in extending the transform analysis technique to these models is that option prices depend on two random variables. While this does not pose a problem for the "callable forward" and the "putable forward" (see Deng (1999) for models with multifactor specifications), pricing the "callable forward with early notification" becomes analytically intractable. In this case, determining the optimal exercise policy will require that both electricity price and the volatility level (or the regime) will need to be observed at time T_1. This implies that there will not be a unique electricity price at which the later call option price is equal to the payoff from killing it at T_1. There will instead be a family {(forward price, volatility)} or {(forward price, regime)}as there will be one such price for each level of volatility or regime. The problem of pricing the double-call option before time T_1 cannot be broken into a simple call option and a compound put option as before (see Figure 12). One would need to directly evaluate the expectation using Monte Carlo simulation or numerically solve the partial differential equation associated with the double-call option.

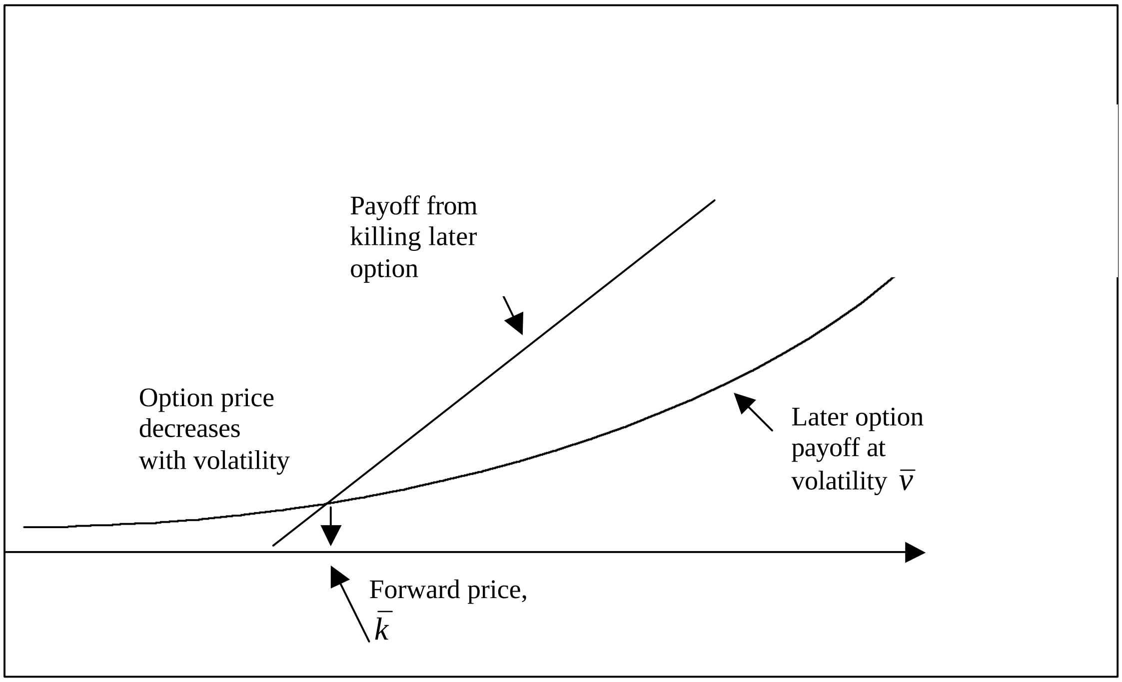

Figure 12. Option payoff under an affine jump-diffusion with stochastic volatility.

APPENDIX

Table 1. Parameter values for examples in the paper (from Deng (1999)).

Parameter	Value
κ	1.70
θ	3.40
σ	0.74
λ_1	6.08
μ_1	0.19
λ_2	7.00
μ_2	-0.11

We use $t_3 = T_2 - T_1 = 0.5$ years in all our examples.

Proof of Proposition 1: We need to evaluate the expression:

$$C_t(k_{T_2} \mid f_t) = E^Q[e^{-r(T_2-t)}(f_{T_2} - k_{T_2})^+ \mid f_t] \tag{A1}$$

where E^Q denotes the expectation under the risk-neutral measure. This can be evaluated as:

$$\begin{aligned} C_t(k_{T_2} \mid f_t) &= \int_{-\infty}^{\infty} e^{-r(T_2-t)}\{(f_t \exp\{-\tfrac{1}{2}s^2(T_2-t)+y\} - k_{T_2})^+ \tfrac{1}{2ps\sqrt{(T_2-t)}} \exp(-\tfrac{y^2}{2s^2(T_2-t)})\}\,dy \\ &= \int_{y^*}^{\infty} e^{-r(T_2-t)} f_t \exp\{-\tfrac{1}{2}s^2(T_2-t)+y\} \tfrac{1}{2ps\sqrt{(T_2-t)}} \exp(-\tfrac{y^2}{2s^2(T_2-t)})\,dy \\ &\quad - \int_{y^*}^{\infty} e^{-r(T_2-t)} k_{T_2} \tfrac{1}{2ps\sqrt{(T_2-t)}} \exp(-\tfrac{y^2}{2s^2(T_2-t)})\,dy \end{aligned} \tag{A2}$$

$$\textit{where } y^* = \ln(k_{T_2}/f_t) + \tfrac{1}{2}s^2(T_2-t)$$

using the expression for the forward price in (5). This can be simplified to Black's formula. The put option price can be easily calculated using put-call parity:

$$P_t(k_{T_2} \mid f_t) = C_t(k_{T_2} \mid f_t) + e^{rt_2} k_{T_2} - e^{rt_2} f_t \tag{A3}$$

$$P_t(k_{T_2} \mid f_t) = e^{-rt_2}[f_t N(-d_1) - k_{T_2} N(-d_2)] \tag{A4}$$

Proof of Proposition 2: We need to show that the option price formulae for the simpler derivatives are as shown. We have:

$$\hat{C}_t(k_{T_1}, k_{T_2} \big| f_t) = C^{'}_{t,T_1}(\bar{k} \big| f_t) + e^{-r(t-T_1)} C_{T_1}(k_{T_2} \big| f_{T_1} = \bar{k}) - P^{co}_{t,T_1}(e^{-rt_3}(\bar{k} - k_{T_1}) \big| f_t) \quad \text{(A5)}$$

where,

The first term is the value of a special call option expiring at time T_1 with strike price $\bar{k}$;

The second term is the discounted sure value of the later call option at forward price $\bar{k}$;

The third term is the price of a compound put option that allows the holder to sell a call option for time T_2 at strike price e^{-rt_3} ($\bar{k}$ - k_{T_1}). The payoff of this put at time T_1 is $[e^{-rt_3}(\bar{k} - k_T) - C_{t,T}(k_0 \big| f_T)]^+$.

At t = T_1, the option payoff is:

$$\max[e^{-rt_3}(f_{T_1} - k_{T_1}), C_{T_1}(k_{T_2} \big| f_{T_1})] \quad \text{(A6)}$$

Therefore the option will be excercised when the two terms are equal. As the option price is increasing in forward prices, one can determine a unique effective strike price, $\bar{k}$, for the call option at T_1 . $\bar{k}$ solves the implicit equation:

$$\bar{k} - k_{T_1} = e^{rt_3} C_{T_1}(k_{T_2} \big| f_{T_1} = \bar{k}) \quad \text{(A7)}$$

The first term can be written as:

$$C^{'}_{t,T_1}(\bar{k} \big| f_t) = E^Q[e^{-rt_1} e^{-rt_3}(f_{T_2} - k_{t_2})^+ \big| f_t] \quad \text{(A8)}$$

One can factor out e^{-rt_3} and that leaves us with the price of a simple call option which is given in Proposition 1. The second term is also a simple call option price from Proposition 1.

The third term can be written as:

$$\begin{aligned} P^{co}_{t,T_1}(k_1 \big| f_t) &= E^Q[e^{-rt_1}(k_1 - C_{T_1}(k_{T_2} \big| f_{T_1})^+ \big| f_t] \\ &= E^Q_t[e^{-rt_1} k_1 \Uparrow_{k_1 \ge C_{T_1}(k_{T_2} | f_{T_1})}] - E^Q_t[e^{-rt_1} e^{-rt_3} f_T N(d_1) \Uparrow_{k_1 \ge C_{T_1}(k_{T_2} | f_{T_1})}] \\ &\quad + E^Q_t[e^{-rt_1} e^{-rt_3} k_{T_2} N(d_2) \Uparrow_{k_1 \ge C_{T_1}(k_{T_2} | f_{T_1})}] \end{aligned} \quad \text{(A9)}$$

where $k_1 = e^{-rt_3}(\bar{k} - k_{T_1})$ and we have used the definition of the call option price from Proposition 1. It can be noted that $k_1 \ge C_{T_1}(k_{T_2} \big| f_{T_1}) \equiv \ln f_{T_1} \le \ln \bar{k}$. Also, since given f_t the log futures price at time T_1 is a $N(\ln f_t - \frac{1}{2} s^2 t_1, s^2 t_1)$ random variable, all the expectations in the above formula can be expressed in terms of normal or bivariate normal distributions.

Consider the 2nd term:

$$E_t^Q[e^{-rt_1}e^{-rt_3}\, f_{T_1} N(d_1) \Uparrow_{k_1 \geq C_{T_1}(k_{T_2} | f_{T_1})}]$$

$$= e^{-rt_2}\int_{-\infty}^{\ln \bar{k}} N(d_1)\frac{1}{\sqrt{2p}\, s\ \sqrt{t_1}} e^x \exp\{-\tfrac{1}{2}(\frac{x-\ln\ f_t\ +\tfrac{1}{2}s\ ^2 t_1}{s\ \sqrt{t_1}})\}dx \qquad \text{(A10)}$$

where we have used the fact that the log futures price is normally distributed at T_1.

Making a change of variables:

$$y = \frac{x - \ln\ f_t\ + \tfrac{1}{2} s\ ^2 t_1}{s\ \sqrt{t_1}}$$

we get,

$$= e^{-rt_2}\ f_t \int_{-\infty}^{-a_1} N(d_1)\, j\,(y)dy$$

where $j(y)$ is the standard normal density function. and

$$a_1 = \frac{\ln(\, {}^{f_t}\!/\!_{k}) + \tfrac{1}{2}\ s\ ^2 t_1}{s\ \sqrt{t_1}} \qquad \text{(A11)}$$

Now,

$$d_1 = \frac{\ln(\, {}^{f_{T_1}}\!/\!_{k_{T_2}}) + \tfrac{1}{2}\ s\ ^2 t_3}{s\ \sqrt{t_3}} \qquad \text{(A12)}$$

substituting for ln f_{T2}, we have:

$$d_1 = \frac{s\ \sqrt{t_1} y + \ln\ f_{T_1}\ + \tfrac{1}{2}\ s\ ^2 t_1 - \ln k_{T_2}\ + \tfrac{1}{2}\ s\ ^2 t_3}{s\ \sqrt{t_3}}$$

$$= \frac{b_1 - r\ y}{\sqrt{1 - r\ ^2}} \qquad \text{(A3)}$$

where,

$$b_1 = \frac{\ln(\, {}^{f_t}\!/\!_{k_{T_2}}) + \tfrac{1}{2}\ s\ ^2 t_2}{s\ \sqrt{t_2}}\ andr\ = -\sqrt{\frac{t_1}{t_2}}$$

Therefore the 2nd term:

$$= \ e^{-rt_2}\ f_t N_2(-a_1, b_1; -\sqrt{\tfrac{t_1}{t_2}})$$

where $N_2(a, b;\ r)$ is the bivariate Normal distribution with correlation r. One can do similar substitutions for the other terms to get:

$$P_{t.T_1}^{co}(k_1 | f_t) = e^{-rt_2} k_{T_2} N_2(-a_2, b_2; -\sqrt{\tfrac{t_1}{t_2}}) - e^{-rt_2}\ f_t N_2(-a_1, b_1; -\sqrt{\tfrac{t_1}{t_2}}) + e^{-rt_1} k_1 N(-a_2) \quad \text{(A13)}$$

where a_1 and b_1 are as above and

$$a_2 = a_1 - s\sqrt{t_1} \text{ and } b_2 = b_1 - s\sqrt{t_2} \; . \tag{A14}$$

Proof of Proposition 3: We begin by specifying the price process for the log spot price under the risk-neutral measure.

$$dX_t = k\,(q - X_t) + s\; dB_t^Q \tag{A15}$$

where $X_T = \ln S_t$.

Lemma 1: Given X_t, X_T will be Gaussian and its mean and variance are given by:

$$E_t^Q[X_T] = q + (X_t - q)\exp\{-k\,(T-t)\} = m_t^T \tag{A16}$$

$$Var_t^Q[X_T] = \tfrac{s^2}{2k}[1 - \exp\{-2k\,(T-t)\}] = J^{2\,T}_{\;\;t} \tag{A17}$$

Proof: We integrate the stochastic differential equation for X using e^{ks} as an integrating factor (see Oksendal (1995)):

$$\int_t^T e^{ks} dX_s = \int_t^T e^{ks} k\, q ds - \int_t^T e^{ks} k X_s ds + \int_t^T e^{ks} s\; dB_s^Q \tag{A18}$$

We can eliminate the terms containing X by applying Itô's lemma to $Y_s = e^{\kappa s} X_s$. The remaining terms can be easily integrated to arrive at the the spot price at T:

$$X_T = q + (X_t - q)\exp\{-k\,(T-t)\} + \int_t^T s\, e^{-k\,(T-s)} dB_s^Q \tag{A19}$$

As the Itô integral is a Gaussian random variable, X_T will be Gaussian with mean (the Itô integral has zero mean):

$$E_t^Q[X_T] = q + (X_t - q)\exp\{-k\,(T-t)\} = m_t^T \tag{A20}$$

Also, using Itô isometry (see Oksendal (1995)):

$$Var[X_T] = E[(\int_t^T s\, e^{-k\,(T-s)} dB_s^Q)^2] = \int_t^T s^{\;2} e^{-2k\,(T-s)} ds \tag{A21}$$

which can be integrated to give.

$$Var_t^Q[X_T] = \tfrac{s^2}{2k}[1 - \exp\{-2k\,(T-t)\}] = J^{2\,T}_{\;\;t} \; . \tag{A22}$$

As before, the log forward price will be normal under the risk-neutral measure (spot price will be log-normal).

$$f_{t,T} = E_t^Q[S_T] = E_t^Q[\exp\{X_T\}] \tag{A21}$$

To arrive at an explicit formula, define the (discounted) characteristic of X_T as follows:

$$\Phi(u, t, T, X_T) = E[e^{-r(T-t)} \exp\{u\, X_T\}] \tag{A22}$$

One can use the familiar exponential affine for the characteristic function of X_T ~ N(m,u^2) to get:

$$\Phi(u,t,T,X_T) = \exp\{-r(T-t) + um^T + \tfrac{1}{2}u^2 J^{2^T}\}] \qquad \text{(A23)}$$

The forward price is given by $e^{r(T-t)}\,\Phi(1,t,T,X_T)$:

$$f_t = \exp\{q + (X_t - q)\exp\{-k(T-t)\} + \tfrac{s^2}{4k}[1-\exp\{-2k(T-t)\}] \qquad \text{(A24)}$$

The above implies that forward term structure does not converge to the long-term mean of the spot price but instead to a larger quantity depending on the volatility and rate of mean-reversion.

We can use Itô's lemma to arrive at the process followed by f (this will have zero drift).

$$\begin{aligned} df_t &= \exp\{-k(T-t)\}s\ f\,dB_t^Q \\ d\ln f_t &= -\tfrac{1}{2}s^2\exp\{-2k(T-t)\}dt + \exp\{-k(T-t)\}s\ dB_t^Q \end{aligned} \qquad \text{(A25)}$$

$$\therefore \ln f_T = \ln f_t - \tfrac{1}{2}\tfrac{s^2}{2k}[1-\exp\{-2k(T-t)\}] + \int_t^T s\,e^{-k(T-s)}dB_s^Q \qquad \text{(A26)}$$

Forward prices will also be lognormal under this model with different variance than in the GBM case. Therefore, the same formulae will apply with a different variance term.

Proof of Proposition 4: Define the characteristic of $G_{a,b}(y)$ as:

$$\begin{aligned} \hat{G}_{a,b}(v) &= \int_{\Re} e^{ivy} dG_{a,b}(y) \\ &= \Psi(a+ivb,t,T,f_t) \end{aligned} \qquad \text{(A27)}$$

One can now use inversion methods to determine the call formula explicitly using (see Duffie et al (1998) for an explicit derivation):

$$G_{a,b}(y) = \frac{\Psi(a,t,T,f_t)}{2} - \tfrac{1}{p}\int_0^\infty \frac{\operatorname{Im}[\Psi(a+ivb,t,T,f_t)e^{-ivy}]}{v}dv \qquad \text{(A28)}$$

The call price can now be expressed as:

$$\begin{aligned} C(k_{T_2}\,|\,f_t) = {} & \left(\frac{\Psi(1,t,T,f_t)}{2} - \tfrac{1}{p}\int_0^\infty \frac{\operatorname{Im}[\Psi(1-iv,t,T,f_t)e^{-iv\ln k_{T_2}}]}{v}dv\right) - \\ & \left(\frac{\Psi(0,t,T,f_t)}{2} - \tfrac{1}{p}\int_0^\infty \frac{\operatorname{Im}[\Psi(-iv,t,T,f_t)e^{-iv\ln k_{T_2}}]}{v}dv\right) \end{aligned} \qquad \text{(A29)}$$

To get the call price in terms of the forward price at t, observe that

$$\Psi(1,t,T,f_t) = e^{-rt_3} f_t \ \mathit{and}\ \Psi(0,t,T,f_t) = e^{-rt_3} \qquad \text{(A30)}$$

Substituting these terms in the option pricing formula one can arrive at (19a). We can similarly write the put price as :

$$P_t(k_{T_2}\,|\,f_t) = k_{T_2}G_{0,1}(\ln k_{T_2}) - G_{1,1}(\ln k_{T_2}) \qquad \text{(A31)}$$

One can now proceed and substitute for terms in this equation to get (19b).

: Again, we need to show that the option price formulae for the simpler derivatives

$$C_t(k_{T_1} \quad {}_{T_2} \big| f_t) \quad C_{t\;T_1}(\bar{\ } \big| f_t \quad e^{-rt_1} \quad {}_{T_1}(k_{T_2} \big| f_{T_1} \quad \bar{\ }) \quad P^{co}_{t,T_1} \; e^{\;rt_3}(\bar{\ } \quad k_{T_1}) \big| f_t$$

As before, we can calculate a unique value of the forward price, $\bar{k}$, at which the double call option will be exercised $\bar{k}$ solves the implicit equation:

$$\bar{k} - k_{T_1} = e^{rt_3} C_{T_1}(k_{T_2} \big| f_{T_1} = \bar{k}) \tag{A33}$$

Now, the first two terms are a straightforward application of proposition 4 using the appropriate transform inversion. The third term can also be expressed as (32):

$$P^{co}_{t,T_1}(k_1 \big| f_t) \quad = k_1 G^{t,T_1}_{0,-1}(\ln \bar{k}) - e^{-rt_2} f_t \Pi_1(t, T_1, T_2) + e^{-rt_2} k_{T_2} \Pi_2(t, T_1, T_2) \tag{A34}$$

The first term can be evaluated using an application of proposition 4. To evaluate Π_1 and Π_2 we use the characteristic functions in (34). Now, for a bivariate distribution, F(a , b), of two random variables S and P, we can express *F(S £ a, P ³ b)* as:

F(S £ a, P ³ b) = F(S £ a) - *F(S £ a, P £ b)* (A35)

If the characteristic function, Θ(ϕ, φ; S, P), of F is known we can use the method in Shepard (1991) (see Bakshi and Madan (1998) for an application to option pricing) to invert Θ(ϕ, φ; S, P) to get the desired probabilities given by:

$$F(S \leq a) = \frac{1}{2} - \frac{1}{p}\int_0^{\infty} \mathrm{Re}[\frac{\Theta(f, 0; S)e^{-i f a}}{i f}]df \tag{A36}$$

$$\begin{aligned} &F(S \leq a, P \leq b) = -\frac{1}{4} + \frac{1}{2}F(S \leq a) + \frac{1}{2}F(P \leq b) \\ &-(\frac{1}{2p^2}\int_0^{\infty}\int_0^{\infty}\{\mathrm{Re}\left[\frac{\Theta(f, j; S, P)e^{-ifa-ijb}}{fj}\right] - \mathrm{Re}\left[\frac{\Theta(f, -j)e^{-ifa+ijb}}{fj}\right]\}dfdj) \end{aligned} \tag{A37}$$

The compound option formula can be derived by applying the above results to Π_1 and Π_2.

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