

# Strong Mixed-Integer Formulations for Power System Islanding and Restoration

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**Abstract**—Intentional Controlled Islanding (ICI) and Black Start Allocation (BSA) are two examples of problems in the power systems literature that have been formulated as Mixed Integer Programs (MIPs) and allow reconfiguration of the power system topology. A key consideration in both of these problems is that each island that appears after a reconfiguration must have at least one energized generator. In this paper, we examine three alternative MIP formulations for this restriction, show their equivalence, and prove that two of them are stronger in terms of their linear programming relaxation than the formulation most commonly used in the power systems literature. Since the time to solve MIPs can vary significantly between equivalent formulations, we also present computational experiments on IEEE test systems for the ICI and BSA problems. We observe that a polynomially separable, exponential in size, strong formulation yields the best performance for the BSA problem and exhibits a comparable performance to a linear in size, weak formulation for the ICI problem.

**Index Terms**—Black Start Allocation, Power System Restoration, Controlled Islanding, Power Systems Resilience, Mixed Integer Programming, Graph Partitioning, Network Topology Optimization

## I. INTRODUCTION

Mixed integer programming (MIP) formulations, i.e. optimization models that involve integer as well as real variables, are becoming ubiquitous in power systems applications (unit commitment, power system restoration, capacity expansion planning, optimal islanding). The main reasons for their popularity are that they offer broad modeling capabilities and that specialized commercial MIP solvers have improved significantly over the past years, making MIPs tractable for many practical applications. In these applications, binary variables are used to represent commitment, scheduling, time dependencies, component energization, as well as to approximate nonlinear curves with piecewise linear functions. Two of the problems that have been formulated as MIPs in power systems are Intentional Controlled Islanding (ICI) and Black Start Allocation (BSA).

ICI is a measure employed to prevent cascading power system outage by splitting the grid into smaller, stable and easily controllable islands via switching off lines [1]–[16]. One straightforward approach to model the problem is to represent

the switching decisions with binary variables and formulate an optimization problem [1]–[11]. The optimization objective and constraints embody the requirements that the splitting should satisfy, such as minimum power flow disruption, size and capacity of the resulting islands and isolating or grouping generators in coherent sets.

The BSA for Power System Restoration (PSR) problem [17]–[27] aims to allocate Black Start (BS) resources in the grid in an efficient way, in order to ensure a successful restoration of the power system after an outage. The problem is often modeled as a MIP, with binary decisions representing the BSA and the restoration of generators/lines/buses of the system [17]–[24]. The optimization objective is to maximize the energization of the system components over a time horizon, or to minimize the load shedding for critical loads, whereas the various constraints ensure that the allocation and restoration plans are feasible.

Both of the aforementioned problems, and possibly others, allow the switching of lines of the power system. It is therefore often necessary to include a constraint to ensure that at all times each island has at least one generator to set up the voltage. More abstractly, the constraint ensures that a graph is partitioned into connected subgraphs, each of which contains a special type of node. In the power systems context, this node is one with an energized generator or corresponding to a set of coherent generators. We will refer to this constraint as the *island energization* (IE) constraint. An example of a system state that violates this requirement is depicted in Figure 1. Network commodity flow formulations have been used to explicitly enforce this requirement [1], [2], [6], [8], [10], [11], [18]–[20] and this is currently the most common approach for power systems applications.

A similar requirement appears in other contexts as well. For instance, in [28], the authors propose three formulations to obtain a connected subgraph that includes a set of terminal nodes in a graph, and provide computational results to test the comparative performance and strength of the different formulations. In their problem, they must select a single connected subgraph that includes certain terminal nodes, whereas we can select multiple connected subgraphs (islands) that must contain at least one generator. While projections of the two problems can be shown to be equivalent using the appropriate network transformations, one of their formulations restricts the connected subgraphs to be trees. For similar reasons, our problem also differs from the Steiner Tree problem and multiple generalizations of it (such as the Prize Collecting Steiner Tree Problem [29]), because these problems restrict

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their connected subgraphs to be trees, whereas our islands can include cycles. Furthermore, all the previous formulations utilize only edge and/or node binary variables. On the other hand, we use binary variables for edges, nodes and terminal nodes, since all of these variables have a physical meaning for power systems applications (buses, branches, generators). As a result, the feasible region we describe in our problem includes the feasible regions for most other problems discussed above.

The motivation for studying equivalent reformulations of the same set of constraints is that, despite the continuous improvements of MIP solvers, the solution times highly depend on the problem formulation employed. As a result, equivalent reformulations of the same requirement can lead to very different solver performances. For a minimization problem, the lower bounds found by the solvers of a MIP are based on solving successive continuous relaxations of the problem, i.e. optimization problems that relax the integrality requirement of some variables. Therefore, equivalent reformulations of the problem with tighter continuous relaxations can lead to better lower bounds and hence a smaller B&B tree. Unfortunately, tighter formulations usually come at the expense of more variables and/or constraints. As a result, there is a computational trade-off between the use of different formulations that has to be resolved based on theoretical and computational results for every particular problem.

The main contributions of this work are the following:

- 1) We present three different formulations of the IE constraint using binary variables for generators, buses and branches: a single-commodity flow formulation  $F_1$  and a multi-commodity flow formulation  $F_2$  based on existing ideas in power systems, and a new, exponential in size, cut-set formulation  $F_3$ . We show that they are all equivalent, that the Linear Programming (LP) relaxation of  $F_2$  is stronger than that of  $F_1$ , and that the LP relaxations of  $F_2$  and  $F_3$  are equally strong. We propose a polynomial time separation algorithm to identify a violated constraint of the LP relaxation of the exponential in size, strong formulation  $F_3$ .
- 2) We present a new formulation for the variant of the optimal ICI problem considered in [1]. Our formulation has fewer variables and constraints and exhibits a better computational performance for the instances examined.
- 3) We demonstrate through computational experiments that: (i) For both problems (ICI and BSA), the size of  $F_2$  makes it impractical to use in realistic applications, despite its strength. (ii) For the optimal BSA problem, formulation  $F_3$  is significantly better (at least one order of magnitude faster) than  $F_1$  in the instances considered. (ii) For the optimal ICI problem,  $F_3$  performs no better than  $F_1$  because of the computational overhead of separating the constraints of  $F_3$ .

The practical importance of the work can be summarized as follows. In an optimization problem for which the IE requirement is valid (as is the case for the ICI and BSA problems examined in the paper), an improvement in the computational time of the problem can be achieved by adding or substituting the existing formulation with the strong formulation  $F_3$  (or

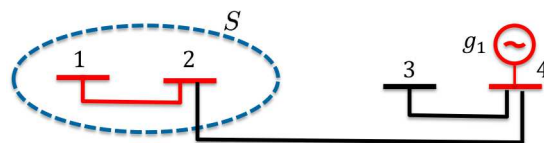


Fig. 1: A small power system with four buses, three branches and one generator. Red color indicates an energized component, whereas black color indicates de-energized components. Note that nodes 1 and 2 and branch (1, 2) form an energized island without any generator, hence this is an infeasible topology that violates the IE constraint.

the concise formulation  $F_1$ ).

The rest of this paper is organized as follows: section II describes the requirement that the IE constraint imposes; section III sets up the notation for the paper; section IV gives the equivalent reformulations; section V proves all the theoretical results of the paper; section VI presents the computational experiments; and section VII draws conclusions based on the results. The full formulations of the two power systems problems considered are provided in the Appendix.

## II. MOTIVATION

The power system restoration (PSR) problem, which is solved for a given allocation of BS units in the system, aims to gradually restore a power system to an operational state after a complete or partial outage. The optimal ICI problem deals with temporarily reconfiguring the grid, by switching lines on and off, as a measure to improve the system security. Both problems involve a series of stepwise actions (usually switchings of lines and generators), while the system moves through a number of different states. Each state can be captured through the status of every bus, line or generator (on or off, i.e. energized or de-energized), as well as through other system characteristics (power flows, generation, etc).

Every step of the process in both problems should respect the island energization (IE) constraint. This requirement may be obvious to the experts that actually perform the switching operations to reconfigure the grid, but when an optimization model is employed to determine a switching plan, the IE constraint has to be imposed. If the ac power flow equations are utilized to model the power system in the optimization model, the IE constraint is implicitly imposed. However, in order to achieve tractability or obtain optimality guarantees, the power flow equations are often relaxed or substituted with approximations and relaxations in the power systems literature which can violate the IE constraint. In such cases, the IE constraint must be explicitly imposed, as in [1], [2], [6], [8], [10], [11], [18]–[20].

The IE constraint formulations we examine in this paper are valid for distribution systems as well. However, for distribution systems one can exploit the graph structure even more, due to the mostly radial nature of the system. As a result, for these systems, there already exist specialized constraints that impose the connectivity requirement, such as the ones used in [30]–[33]. These formulations are specific for distribution systems and do not work for general networks. Our formulations are tailored for general networks, aiming to allow cycles, which are common in transmission systems.

### III. NOTATION

Let  $(N, E)$  be the directed graph derived from the physical graph of the power system, where buses correspond to nodes (set  $N$ ) and branches to directed edges (set  $E$ ). The direction of the edges is defined by arbitrarily picking a ‘‘From Node’’ and ‘‘To Node’’ for every system branch, as is common in power systems literature. Let  $G$  be the set of all generators and  $G(i)$  the set of generators that are connected to bus  $i \in N$ . Let  $u_i$  denote the energization state of bus  $i \in N$  (where  $u_i = 1$  indicates an energized bus, whereas  $u_i = 0$  indicates a de-energized bus),  $u_{ij}$  denote the energization state of branch  $(ij) \in E$  ( $u_{ij} = 1$  indicates an energized branch) and  $u_g$  the energization state of generator  $g \in G$  ( $u_g = 1$  indicates an energized generator). The energization state of the system is completely described by the binary vector  $\mathbf{u} \in \mathbb{B}^{|N| \times |E| \times |G|}$ . We also use auxiliary variables  $\mathbf{f}$ , corresponding to energization network flows (not power flows), in the definition of the single- and multi-commodity flow formulations. Finally, if  $S$  is a subset of the nodes  $S \subseteq N$ , the undirected cut-set  $\delta(S)$  is defined as the set that contains all the edges in  $E$  with one node in  $S$  and one node not in  $S$ , regardless of the direction of the edge.

### IV. FORMULATIONS

In this section, we present three different formulations to impose the IE constraint. Constraints (1) and (2) are included in all the formulations.

- 1) If a generator  $g \in G$  connected to node  $i \in N$  is energized, then the node is considered energized.

$$u_g \leq u_i, g \in G(i), i \in N \quad (1)$$

- 2) If a line is energized, both the nodes at its endpoints are considered energized.

$$u_{ij} \leq u_i, u_{ij} \leq u_j, (ij) \in E \quad (2)$$

#### A. Single-Commodity Flow Formulation

The first formulation is given by:

$$F_1 = \{\mathbf{u} \in \mathbb{B}^{|N| \times |E| \times |G|} : \exists f_g \forall g \in G, f_{ij} \forall (ij) \in E : (1), (2), (4a) - (4c)\} \quad (3)$$

where:

$$0 \leq f_g \leq u_g, g \in G \quad (4a)$$

$$-u_{ij} \leq f_{ij} \leq u_{ij}, (ij) \in E \quad (4b)$$

$$\sum_{j:(ji) \in E} f_{ji} - \sum_{j:(ij) \in E} f_{ij} + \sum_{g \in G(i)} f_g = \frac{1}{|N|} u_i, i \in N \quad (4c)$$

A set of auxiliary network flow variables  $f_g, g \in G$  and  $f_{ij}, (ij) \in E$ , are employed. An energized node (i.e.  $u_i = 1$ ) will act as a sink of  $\frac{1}{|N|}$  amount of network flow, captured in the right hand side of (4c). Network flow can only be generated from energized generators, due to (4a). Finally, it can only flow through energized lines due to (4b). This ensures that there will be a path from any energized node to an energized generator that uses only energized lines (this is the path that the network flow follows to move from the energized generator

to the energized node). For example, the topology of Figure 1 is infeasible, since node 1 would act as a source of  $1/4$  amount of network flow, but network flow can only be generated at node 4 by the energized generator  $g_1$  and cannot pass through the de-energized line  $(2, 4)$ . Note that the size of the sinks is  $\frac{1}{|N|}$ , so that the topology where a single generator (that can generate up to 1 unit of network flow) energizes all the nodes of a (connected) power system belongs in  $F_1$ . The idea behind the single-commodity flow formulation is the one most commonly used in power systems applications [1], [2], [6], [8], [10], [11], [18]–[20]. The formulation has  $|G| + |E|$  flow variables and  $|N| + |E| + |G|$  constraints.

#### B. Multi-Commodity Flow Formulation

An alternative formulation approach, following the same logic, would be to consider a different type of flow corresponding to the energization of each node. This would lead to the following formulation

$$F_2 = \{\mathbf{u} \in \mathbb{B}^{|N| \times |E| \times |G|} : \exists f_g^k \forall k \in N \forall g \in G, f_{ij}^k \forall k \in N \forall (ij) \in E : (1), (2), (6a) - (6d)\} \quad (5)$$

where:

$$0 \leq f_g^k \leq u_g, k \in N, g \in G \quad (6a)$$

$$-u_{ij} \leq f_{ij}^k \leq u_{ij}, k \in N, (ij) \in E \quad (6b)$$

$$\sum_{j:(ji) \in E} f_{ji}^k - \sum_{j:(ij) \in E} f_{ij}^k + \sum_{g \in G(i)} f_g^k = u_i, k \in N, i \in N : i = k \quad (6c)$$

$$\sum_{j:(ji) \in E} f_{ji}^k - \sum_{j:(ij) \in E} f_{ij}^k + \sum_{g \in G(i)} f_g^k = 0, k \in N, i \in N : i \neq k \quad (6d)$$

The idea behind this formulation is that each node is treated separately and is associated with its own type of network flow and constraints. If node  $k \in N$  is energized, then one or more of the energized generators will need to generate the type  $k$  network flow, that needs to pass through energized lines. The only sink for that type of network flow is node  $k$ , which means that the network flow of type  $k$  is preserved at every other node  $i \neq k$ . In this case, the normalization of  $\frac{1}{|N|}$  is not necessary in (6c), since a single energized generator can generate all  $|N|$  types of network flows to energize all the nodes. This formulation has  $|N| \cdot (|G| + |E|)$  flow variables and  $|N| \cdot (|N| + |E| + |G|)$  constraints.

#### C. Cut-Set Formulation

The third formulation only employs the binary variables. More specifically,

$$F_3 = \{\mathbf{u} \in \mathbb{B}^{|N| \times |E| \times |G|} : (1), (2), (8)\} \quad (7)$$

where:

$$\sum_{(ij) \in \delta(S)} u_{ij} + \sum_{i \in S} \sum_{g \in G(i)} u_g \geq u_n, n \in S, S \subseteq N \quad (8)$$

The idea behind this formulation is that, given any subset  $S$  of the nodes, if any node in that subset is energized (i.e. if  $u_n$  in the right hand side of (8) is equal to 1 for some  $n \in S$ ), then an energized generator must be providing the energizing



flow. Therefore, either one generator within the set  $S$  should be energized (i.e.  $\sum_{i \in S} \sum_{g \in G(i)} u_g \geq 1$  in the left hand side of (8)), so that the energizing flow comes from that generator, or at least one edge in the cut-set should be energized (i.e.  $\sum_{(ij) \in \delta(S)} u_{ij} \geq 1$  in the left hand side of (8)), so that the energizing flow comes from a generator outside the set  $S$ . For example, the topology of Figure 1 is infeasible, since if we pick the set  $S = \{1, 2\}$  and the node  $n = 1$  with  $u_1 = 1$ , the left hand side of (8) is zero (since there are no generator nodes in  $S$  and the only edge in the cut-set has  $u_{24} = 0$ ), while the right hand side is one. Finally, for every node  $n \in N$ , there are  $2^{|N|-1}$  subsets  $S$  of  $N$  that contain it, so the formulation has a total of  $|N| \cdot 2^{|N|-1}$  constraints, which is exponential in the size of  $|N|$ .

## V. THEORETICAL RESULTS

### A. Formulation Equivalence

We proceed to show that the three formulations are equivalent, i.e. that they represent the exact same binary space.

**Proposition 1.** *Formulations  $F_1$ ,  $F_2$  and  $F_3$  are equivalent.*

*Proof.* We show that  $F_1 \subseteq F_3$ ,  $F_3 \subseteq F_2$ , and  $F_2 \subseteq F_1$ .

**Part 1.**  $\mathbf{u} \in F_1 \implies \mathbf{u} \in F_3$

Assume for contradiction that  $\mathbf{u} \in F_1$  but  $\mathbf{u} \notin F_3$ . Based on (8), that means  $\exists S_0 \subseteq N, \exists n_0 \in S_0 : \sum_{(ij) \in \delta(S_0)} u_{ij} + \sum_{i \in S_0} \sum_{g \in G(i)} u_g < u_{n_0}$ . Since the right hand side is binary, and the left hand side is integer, the only way for strict inequality to hold is if  $\sum_{(ij) \in \delta(S_0)} u_{ij} + \sum_{i \in S_0} \sum_{g \in G(i)} u_g = 0$  and  $u_{n_0} = 1$ . Since the first equality is a sum of non-negative terms equal to zero, each one of them has to equal zero, so we obtain that:

$$u_{ij} = 0, (ij) \in \delta(S_0) \quad (9a)$$

$$u_g = 0, g \in G(i), i \in S_0 \quad (9b)$$

$$u_{n_0} = 1 \quad (9c)$$

Now, since  $\mathbf{u} \in F_1$ , by summing over equations (4c) for  $i \in S_0$ , we obtain:

$$\sum_{i \in S_0} \left( \sum_{j: (ji) \in E} f_{ji} - \sum_{j: (ij) \in E} f_{ij} \right) + \sum_{i \in S_0} \sum_{g \in G(i)} f_g = \frac{1}{|N|} \sum_{i \in S_0} u_i$$

The left hand side (LHS) can be simplified by observing that the sum of the flows inside the set  $S_0$  will cancel each other, while the flows on branches that have only one node in  $S_0$  (i.e. belong in  $\delta(S_0)$ ), are all zero, due to (4b) and (9a).

$$\begin{aligned} \text{LHS} &= \sum_{i \in S_0} \left( \sum_{j: (ji) \in E} f_{ji} - \sum_{j: (ij) \in E} f_{ij} \right) + \sum_{i \in S_0} \sum_{g \in G(i)} f_g \\ &= \sum_{i \in S_0} \sum_{g \in G(i)} f_g \leq \sum_{i \in S_0} \sum_{g \in G(i)} u_g = 0 \end{aligned}$$

where the last line uses (4a) and (9b). On the other hand, the right hand side (RHS) yields:

$$\text{RHS} \geq \frac{1}{|N|} u_{n_0} = \frac{1}{|N|}$$

where we used that  $n_0 \in S_0$  and the rest of the binary variables in the summation are non negative, together with (9c). Based

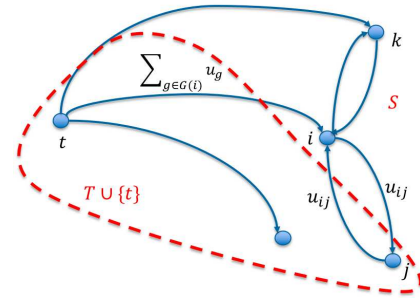


Fig. 2: Graph used in the proofs of Proposition 1, Proposition 3, and Proposition 4.

on the inequalities for the LHS and RHS, we obtain  $0 \geq \frac{1}{|N|}$ , which is a contradiction.

**Part 2.**  $\mathbf{u} \in F_3 \implies \mathbf{u} \in F_2$

Based on  $\mathbf{u}$ , construct a directed graph with nodes  $N \cup \{t\}$ , where  $t$  is a dummy node, and edges: for each node  $i \in N$  that has at least one generator (i.e.  $G(i) \neq \emptyset$ ), add a directed edge from  $t$  to  $i$  with capacity  $\sum_{g \in G(i)} u_g$ , and for each directed edge  $(ij) \in E$ , add two directed edges, one from  $i$  to  $j$  and one from  $j$  to  $i$ , both with capacity  $u_{ij}$ . Now pick a node  $k \in N$ . For  $T \subseteq N$  and  $S = N \setminus T$ , the capacity of any  $t$ - $k$  cut is given by (see Figure 2):

$$C(T \cup \{t\}, S) = \sum_{(ij) \in \delta(S)} u_{ij} + \sum_{i \in S} \sum_{g \in G(i)} u_g \quad (10)$$

which is greater than or equal to  $u_k$ , since  $k \in S$  and  $\mathbf{u} \in F_3$ . Therefore, the min-cut has capacity  $v \geq u_k$ , which means the max-flow has capacity  $v$ . We can scale all the flows of the max-flow by the positive quantity  $\frac{u_k}{v}$ , which is less than one, to obtain flows that retain feasibility and inject  $u_k$  amount of network flow at node  $k$ . Define  $f_g^k$  based on the flow on the edge  $t$ - $i$ , where  $g \in G(i)$  (if more than one generators are connected to bus  $i$ , assign to each  $f_g^k$  flow proportional to the capacity  $u_g$ ). For every edge  $(ij) \in E$ , assign  $f_{ij}^k$  equal to the difference of the flows on the arcs in the graph we created, which is guaranteed to be in  $[-u_{ij}, u_{ij}]$  due to the feasibility of the max-flows. We can repeat this process for every node  $k \in N$ , and hence generate feasible flows for formulation  $F_2$ , which shows that  $\mathbf{u} \in F_2$ .

**Part 3.**  $\mathbf{u} \in F_2 \implies \mathbf{u} \in F_1$

Since  $\mathbf{u} \in F_2$ , based on the multi-commodity flows of formulation  $F_2$ , let:

$$f_g = \frac{1}{|N|} \sum_{k \in N} f_g^k, g \in G$$

$$f_{ij} = \frac{1}{|N|} \sum_{k \in N} f_{ij}^k, (ij) \in E$$

By summing over all  $k \in N$  the constraints of formulation  $F_2$ , one can observe that the flows defined above satisfy the constraints of formulation  $F_1$ .  $\square$

## B. Formulation Strength

The LP relaxation of  $F_1$  is defined as:

$$\text{LP}(F_1) = \{ \mathbf{u} \in [0, 1]^{|N| \times |E| \times |G|} : \exists f_g \forall g \in G, \\ f_{ij} \forall (ij) \in E : (1), (2), (4a) - (4c) \} \quad (11)$$

We have similar definitions for the relaxations of  $F_2$  and  $F_3$ . When comparing two MIP formulations  $F_i$  and  $F_j$ , we say that  $F_i$  is (strictly) stronger than  $F_j$  if the LP relaxation of  $F_i$  is a (strict) subset of the LP relaxation of  $F_j$ . A stronger formulation yields better bounds in the execution of B&B (the B&B algorithm is based on solving successive relaxations of the problem with additional constraints). Therefore, pruning, branching and identifying integer points will be more efficient for the stronger formulation and the resulting B&B tree is expected to be smaller.

The following propositions give a result for the relative strength of the different formulations.

**Proposition 2.** *The LP relaxation of  $F_2$  is strictly stronger than the LP relaxation of  $F_1$ .*

*Proof.* We first need to show that  $\mathbf{u} \in \text{LP}(F_2) \implies \mathbf{u} \in \text{LP}(F_1)$ . To see that, note that the proof employed in Part 3 of Proposition 1 did not use the integrality of the variables in  $\mathbf{u}$ . Therefore, the same proof can be used to show the inclusion in this case.

To see the strictness of the inclusion, consider a graph with two nodes  $N = \{1, 2\}$ , one line  $E = \{(1, 2)\}$ , and one generator  $G = \{g_1\}$  connected to node 1. Consider the point  $(u_1, u_2, u_{12}, u_{g_1}) = (1, 1, 1/2, 1)$ . Picking  $f_{g_1} = 1, f_{12} = 1/2$ , we can see that  $(1, 1, 1/2, 1) \in \text{LP}(F_1)$ . However, the point does not belong in  $\text{LP}(F_2)$ , since for  $k = 2$  the line capacity of  $1/2$  prevents the 1 unit of type 2 flow to pass from the generator to the sink in node 2.  $\square$

**Proposition 3.** *The LP relaxation of  $F_2$  is the same as the LP relaxation of  $F_3$ .*

*Proof.* To see that  $\mathbf{u} \in \text{LP}(F_3) \implies \mathbf{u} \in \text{LP}(F_2)$ , notice that the proof in Part 2 of Proposition 1 did not use the integrality of the variables  $\mathbf{u}$ . Therefore the same proof can be employed to show this result as well.

To see that  $\mathbf{u} \in \text{LP}(F_2) \implies \mathbf{u} \in \text{LP}(F_3)$ , for each  $k \in N$ , consider the max-flow problem from  $t$  to  $k$  in the graph of Figure 2. Due to the constraints in  $F_2$ , a feasible flow of at least  $u_k$  exists. Therefore, the maximum flow is at least  $u_k$ , which means that the minimum cut is at least  $u_k$ . That implies that any other cut, whose capacity has the form (10), will be greater than or equal to the minimum cut, so greater than or equal to  $u_k$ . Since this holds for all  $k \in N$ , the constraints of  $\text{LP}(F_3)$  are satisfied.  $\square$

## C. Constraint Separation

The constraints in formulation  $F_3$  are exponentially many. Even though we cannot include all of them in the model that is passed to commercial optimization software, we can actually efficiently identify a violated constraint of  $\text{LP}(F_3)$ , based on the following proposition. Therefore, we can use solver callbacks to dynamically add the constraints at any point the solver reaches (fractional or not), only if they are violated.

**Proposition 4.** *Given a point  $\mathbf{u} \in [0, 1]^{|N| \times |E| \times |G|}$ , we can identify a violated constraint from  $\text{LP}(F_3)$  or verify that none exists (separation problem), in polynomial time.*

*Proof.* Given a point  $\mathbf{u} \in [0, 1]^{|N| \times |E| \times |G|}$ , for every  $k \in N$ , construct the graph from Figure 2. Then find the minimum  $t - k$  cut in this graph and compare the value to  $u_k$ . Given the fact that the capacity of a cut in that graph has the form (10), there are two cases for the capacity of the minimum cut  $C_{\min}^k$ :

- 1) If  $C_{\min}^k < u_k$ , for some  $k \in N$ , then node  $k$  and the minimum cut set  $S_{\min}$  yield a violated constraint in  $\text{LP}(F_3)$ .
- 2) If  $C_{\min}^k \geq u_k$ , for all  $k \in N$ , then all constraints in  $\text{LP}(F_3)$  are satisfied.

Since the min-cut problems can be solved in polynomial time, and we only need to solve at most  $|N|$  of them, the separation problem can be solved in polynomial time. Note that, if the point is integral, a graph traversal to identify the islands and check if there exists one without a generator, is enough to identify if all constraints are satisfied in linear time (if we find an island with no generator, we can then generate a violated constraint with  $S$  corresponding to that island).  $\square$

## VI. SIMULATION RESULTS

All optimization problems were formulated using Gurobi with Python. Each simulation was executed at a single node of the Lawrence Livermore National Laboratory quartz server (128 GB RAM, 2.1 GHz, 18 CPUs). For formulation  $F_3$  all constraints with  $|S| = 1$  were *a priori* included in the formulation, and the rest were lazily added using incumbent callbacks, i.e. each candidate incumbent found by the solver is checked for feasibility and a lazy constraint is added if the candidate is infeasible. A 20,000s time limit was set to the solver and a 1% optimality gap termination was considered (i.e. a solution guaranteed to be within 1% of the optimal is denoted as optimal) for all simulations. Some instances caused the B&B tree to run out of memory due to the size of the problem. For these instances (indicated with \*), we present results using settings that limit the solver's memory use by restricting the number of threads to 4 and using the file system as temporary storage.

### A. Optimal Intentional Controlled Islanding

The most common practice behind ICI is that the generators of the grid can be divided into groups of coherent generators, based on their relative angle response to a disturbance. By isolating unstable generators or grouping together only generators that are coherent to each other, a cascaded outage may be avoided. In [1], an optimal ICI model was devised to propose switching actions. The goal was to create a partition of the grid into islands of coherent generators with minimal power-flow disruption.

The formulation provided in [1], which we denote with  $F_4$ , utilizes a mixed integer program to identify the optimal islanding. The authors of [1] deal with the intractability

| Optimal ICI | B&B Nodes | Gap [%] | Upper Bound | Lower Bound | Time [s] |
|-------------|-----------|---------|-------------|-------------|----------|
| IEEE-118    |           |         |             |             |          |
| $K = 4$     |           |         |             |             |          |
| $F_1$       | 1         | Optimal | 3.6846      | 3.6846      | 0.16     |
| $F_3$       | 328       | Optimal | 3.6846      | 3.6846      | 1.03     |
| $F_4$       | 1         | Optimal | 3.6846      | 3.6846      | 0.94     |
| Polish      |           |         |             |             |          |
| $K = 3$     |           |         |             |             |          |
| $F_1$       | 3         | Optimal | 17.7717     | 17.6123     | 5        |
| $F_3$       | 1         | Optimal | 17.7717     | 17.7710     | 29       |
| $F_4$       | 1         | Optimal | 17.7717     | 17.7717     | 509      |
| $K = 4$     |           |         |             |             |          |
| $F_1$       | 13        | Optimal | 27.3901     | 27.3901     | 12       |
| $F_3$       | 1         | Optimal | 27.4225     | 27.2373     | 36       |
| $F_4$       | 2635      | Optimal | 27.3901     | 27.3901     | 3098     |
| $K = 5$     |           |         |             |             |          |
| $F_1$       | 32        | Optimal | 44.9159     | 44.9159     | 13       |
| $F_3$       | 1         | Optimal | 44.9508     | 44.8052     | 34       |
| $F_4^*$     | 5138      | 1.46    | 44.9159     | 44.2587     | 20000    |

TABLE I: Optimal ICI for the IEEE-118 and Polish systems, splitting the system into  $K$  islands.

of formulation  $F_4$  by constructing a heuristic based on LP relaxations. In this paper, we present equivalent formulations of  $F_4$  using the IE constraint formulations  $F_1$ ,  $F_2$  and  $F_3$ ; see Appendix A for the full formulations.

In order to perform a computational comparison between the different formulations of the problem, we considered two test systems: the IEEE-118 bus system (118 buses, 186 branches, 54 generators) and an instance of the Polish system (3374 buses, 4161 branches, 596 generators). Due to the size of the resulting model and the memory limitations, formulation ( $F_2$ ) was impractical and was not implemented.

The results for the ICI problem are presented in Table I. The IEEE-118 system solves to optimality relatively quickly for all three formulations considered. For the Polish system, we observe that  $F_4$  seems to be two orders of magnitude slower than  $F_1$ . The exponential formulation  $F_3$  performs slightly worse than  $F_1$  (with, in fact, more than 80% of its simulation time spent in separating the violated constraints). In this case the trade-off between a tighter formulation and separation of an exponential number of constraints is inconclusive.

### B. Optimal Black Start Allocation

The optimal BSA problem aims to allocate the black start resources in the grid in a way that will ensure an efficient restoration of the system in the case of a blackout. The formulation used in this paper is a simplified adaptation of the one explained in [19]. The main simplification employed is that, instead of detailed nodal power balance equations and active and reactive power flow approximations, we only use one aggregate constraint for the active power balance and one for the reactive power capability at every time step of the underlying restoration problem (given the BSA). This is a typical assumption for this problem [17], [23], made mostly to ensure its tractability. The complete formulation used in this paper can be found in Appendix B.

We use three test systems: the IEEE-39, the IEEE-118, and the IEEE-300 power systems. The results of the simulations are shown in Table II. The IEEE-39 bus problem solves to optimality relatively quickly for all formulations. Formulation  $F_2$  is slower due to its size (every linear program solved at

| Optimal BSA | B&B Nodes | Gap [%] | Upper Bound | Lower Bound | Time [s] |
|-------------|-----------|---------|-------------|-------------|----------|
| IEEE-39     |           |         |             |             |          |
| $F_1$       | 1335      | Optimal | 1550        | 1536        | 2        |
| $F_2$       | 640       | Optimal | 1536        | 1533        | 22       |
| $F_3$       | 108       | Optimal | 1545        | 1530        | 1        |
| IEEE-118    |           |         |             |             |          |
| $F_1$       | 474738    | 4.81    | 5946        | 5673        | 20000    |
| $F_2$       | 3177      | 1.30    | 5730        | 5656        | 20000    |
| $F_3$       | 4624      | Optimal | 5721        | 5670        | 99       |
| IEEE-300    |           |         |             |             |          |
| $F_1$       | 203933    | 2.86    | 12873       | 12515       | 20000    |
| $F_2^*$     | 1028      | 2.15    | 12840       | 12570       | 20000    |
| $F_3$       | 5172      | Optimal | 12668       | 12593       | 108      |

TABLE II: Optimal BSA results for the IEEE-39, IEEE-118 and IEEE-300 systems.

intermediate steps takes more time), however the number of explored B&B nodes for the solution is smaller (due to the strength of the formulation).

In the larger systems considered, the performance of the formulations is very different. For the IEEE-118 system,  $F_1$  and  $F_2$  are unable to solve the problem to optimality within the time limit of 20,000s, whereas  $F_3$  easily achieves a solution with the desired gap within 99 seconds.  $F_2$  performs better than  $F_1$ , however due to its size it is unable to explore enough nodes in the B&B tree to reduce the optimality gap to the desired levels. Finally, for the IEEE-300 system  $F_3$  is able to yield an optimal solution in around 100 seconds.  $F_1$  only achieves a 2.86% solution and  $F_2$  only a 2.15% solution in more than 100-fold the time it took for  $F_3$  to optimally solve the problem.

## VII. CONCLUSIONS

The main message of this work is that, while it might be easy to formulate a power systems problem as a MIP, choosing the right way to formulate the problem can make a significant difference in the solution times. Among others, some issues to consider when selecting the right formulation are the tightness of the formulation (i.e. how tight the relaxation of the feasible region is around the integral points) and the size of the formulation (number of constraints and variables). There is usually a trade-off between the size and the tightness of the formulation, that can be resolved in practice for each problem. Also, even if a problem is intractable in practice with one formulation, a reformulation could yield an acceptable computational performance. We presented empirical proof of all the aforementioned points by examining two problems in power systems.

For the ICI problem, even though formulations  $F_4$  and  $F_1$  are equivalent and are both employing single-commodity flow to enforce connectivity, their computational performance was very different, since  $F_1$  is more compact than  $F_4$  (fewer variables and constraints). The problem formulated using  $F_4$  solved orders of magnitude slower compared to  $F_1$ . The size of  $F_2$  made it completely intractable. Finally, even though  $F_3$  is imposing tighter constraints in general, the exponential size of the formulation (that forced the implementation to employ lazy constraints added through callbacks) made the computational



performance of the formulation slightly worse than the weaker  $F_1$ .

The situation for the optimal BSA problem was different. Formulation  $F_2$  was still impractical, due to its size. However,  $F_3$  significantly outperformed  $F_1$ , even though it employed an exponential number of constraints. The reasons for that are twofold: firstly, formulation  $F_3$  employs stronger constraints, as shown in this paper. Secondly, the constraints of  $F_3$  other than those with  $|S| = 1$  are rarely violated, so candidates found by solver heuristics early on the tree are often feasible and optimal. The intuition behind this is that for the restoration problem we expect that the direction of the problem is such that edges get energized (rather than de-energized). If lines are mostly getting energized around energized generators, the situation where islands without an energized generator will show up are actually rare, so lazily generating the cuts for  $F_3$  is faster than including the entire formulation  $F_1$  in the optimization solver.

## APPENDIX A OPTIMAL ISLANDING FORMULATION

The optimal ICI formulation presented here is an equivalent reformulation of the problem formulated in [1], based on a previous work by the same authors in [34]. The goal is to find a minimum cost partitioning of the grid to islands given a partitioning of the generators. More specifically, the generators are divided into  $|K|$  subsets of coherent generators  $G_h, h \in K$ , i.e. of generators that will be in the same island after the reconfiguration of the grid and in different islands from the generators of the other subsets. We note that our single-commodity flow formulation approach is similar in nature to the model used in [1], with the exception that we make use of the fact that the generators in each coherent group are forced to belong to the corresponding partition, so one of them can be used as the source of the network flow.

Let  $i \in N$  denote a bus of the system,  $(ij) \in E$  a branch, and  $g \in G$  a generator. We denote the generators connected to node  $i$  with  $G(i)$ . Also, for each set  $G_h$ , denote one of the generators (assumed to be the isochronous one, even though the specific choice is not important) with  $G_h^{(0)} \in G_h$ . We denote the node that generator  $g$  is connected by  $n(g)$ . Let the binary variables  $u_i^h/u_{ij}^h/u_g^h$  denote (if equal to 1) that node  $i$ /branch  $(ij)$ /generator  $g$  belongs to partition  $h \in K$ . Let the binary variable  $z_{ij}$  denote that branch  $(ij)$  is switched on (i.e. it belongs to some partition). Let the variables  $f_{ij}^h, f_g^h$  denote the network flows that will ensure the connectivity of partition  $h \in K$ . Finally, there is a cost  $d_{ij}$  associated with switching off branch  $(ij)$  and a minimum size requirement  $M$  of every bus set in the partition.

$$\begin{aligned} & \text{minimize} && \sum_{(ij) \in E} \frac{1}{2} d_{ij} (1 - z_{ij}) \\ & \text{s.t.} && \\ & z_{ij} = && \sum_{h \in K} u_{ij}^h, (ij) \in E \end{aligned} \quad (12a)$$

$$\sum_{h \in K} u_i^h = 1, i \in N \quad (12b)$$

$$\sum_{i \in N} u_i^h \geq M, h \in K \quad (12c)$$

$$u_{ij}^h \leq u_i^h, u_{ij}^h \leq u_j^h, (ij) \in E, h \in K \quad (12d)$$

$$u_g^h \leq u_i^h, g \in G(i) \quad (12e)$$

$$0 \leq f_g^h \leq u_g^h, g \in G, h \in K, \quad (12f)$$

$$-u_{ij}^h \leq f_{ij}^h \leq u_{ij}^h, (ij) \in E, h \in K, \quad (12g)$$

$$\sum_{j:(ji) \in E} f_{ji}^h - \sum_{j:(ij) \in E} f_{ij}^h + \sum_{g \in G(i)} f_g^h = \frac{1}{N} u_i^h, \quad i \in N, h \in K \quad (12h)$$

$$u_g^h = \begin{cases} 1, & \text{if } g = G_h^{(0)}, h \in K \\ 0, & \text{otherwise} \end{cases} \quad (12i)$$

$$u_{n(g)}^h = 1, g \in G_h, h \in K \quad (12j)$$

The objective of the problem minimizes the weighted cost of switching off edges. Constraint (12a) imposes that a bus is switched on if it belongs in one of the partitions. Constraint (12b) ensures that each bus is assigned to exactly one partition, constraint (12c) ensures that each partition has at least  $M$  buses, constraints (12d) require that if a branch belongs to a partition, both its endpoints belong to it, constraint (12e) ensures that if a generator belongs to partition  $h$ , the node it is connected to will belong to the same partition. Constraints (12f)-(12h), together with (12d) and (12e), are the IE constraints of formulation  $F_1$  imposed for every partition  $h$ . These constraints can be equivalently substituted with the constraints of  $F_2$  or  $F_3$ , as we have shown in this paper. Finally, (12i) allows only the generator  $G_h^{(0)}$  to generate the network flow that ensures the connectivity of each partition  $h$  and (12j) forces each node of the coherent generators to belong to the corresponding partition.

## APPENDIX B OPTIMAL BLACK START ALLOCATION FORMULATION

The optimal BSA problem aims to allocate black start units across the grid so that an efficient system restoration is ensured after a blackout. The formulation used in this paper is a simplified adaptation of the formulation in [19]. Let  $t \in T$  denote the set of time instances, starting from  $1 \in T$ ,  $g \in G$  the generators,  $i \in N$  the buses, and  $(ij) \in E$  the branches. We denote the generators connected to node  $i$  with  $G(i)$ . The binary variables  $u_g^t, u_i^t, u_{ij}^t$  denote the energization (when set to 1) of generator  $g$ /bus  $i$ /branch  $(ij)$  at time  $t$ , and  $u_g^0, u_i^0$  denote the initial state of the generator  $g$  and bus  $i$  (here assumed zero for all components, i.e. we examine a total blackout). The variables  $f_g^t, f_{ij}^t$  denote the network flows of the  $F_1$  formulation,  $p_g^t$  is the active generation of generator  $g$ , and  $p_{SH_i}$  is the load shed at bus  $i$ . Finally, the parameters of the problem are the cost  $C_{BS_g}$  of allocating generator  $g$  to be a BS unit, the total allocation budget  $B$ , the generator capability  $\bar{P}_g$ , cranking time  $T_{CR_g}$ , cranking power  $P_{CR_g}$ , ramping rate  $K_{R_g}$ , and minimum reactive power capability

$Q_g$ , the bus load  $P_{D_i}$ , angles  $\phi_{D_i}$  and shunt reactance  $Q_{SH_i}$  and the branch susceptance  $B_{SH_{ij}}$ .

$$\text{maximize} \quad \sum_{i \in N, t \in T} u_i^t + \sum_{(ij) \in E, t \in T} u_{ij}^t + \sum_{g \in G, t \in T} u_g^t$$

s.t.

$$\sum_{g \in G} C_{BS_g} u_{BS_g} \leq B \quad (13a)$$

$$u_g^0 = 0, \quad g \in G \quad (13b)$$

$$u_i^0 = 0, \quad i \in N \quad (13c)$$

$$u_{ij}^t \leq u_i^{t-1} + u_j^{t-1}, \quad (ij) \in E, \quad t \in T \quad (13d)$$

$$u_i^t \geq u_i^{t-1}, \quad i \in N, \quad t \in T \quad (13e)$$

$$u_g^t \geq u_{BS_g}, \quad (ij) \in E, \quad t \in T \quad (13f)$$

$$u_g^t \leq u_i^t, \quad i \in N, \quad g \in G(i), \quad t \in T \quad (13g)$$

$$u_{ij}^t \leq u_i^t, u_{ij}^t \leq u_j^t, \quad (ij) \in E, \quad t \in T \quad (13h)$$

$$0 \leq f_g^t \leq u_g^t, \quad g \in G, \quad t \in T, \quad (13i)$$

$$-u_{ij}^t \leq f_{ij}^t \leq u_{ij}^t, \quad (ij) \in E, \quad t \in T, \quad (13j)$$

$$\sum_{j:(ji) \in E} f_{ji}^t - \sum_{j:(ij) \in E} f_{ij}^t + \sum_{g \in G(i)} f_g^t = \frac{1}{N} u_i^t, \quad i \in N, \quad t \in T \quad (13k)$$

$$0 \leq p_g^\tau \leq \bar{P}_g u_g^t, \quad g \in G, \quad \tau \in \{t, t+1, \dots, t+T_{CR_g}+1\}, \quad t \in T \cup \{0\} \quad (13l)$$

$$p_g^t - p_g^{t-1} \leq K_{R_g}, \quad g \in G, \quad t \in T \quad (13m)$$

$$p_g^{t-1} - p_g^t \leq K_{R_g}, \quad g \in G, \quad t \in T \quad (13n)$$

$$\sum_{g \in G} (p_g^t + P_{CR_g} (u_{BS_g} - u_g^t)) = \sum_{i \in N} (P_{D_i} - p_{SH_i}^t), \quad t \in T \quad (13o)$$

$$(1 - u_i^t) P_{D_i} \leq p_{SH_i}^t \leq P_{D_i}, \quad i \in N, \quad t \in T \quad (13p)$$

$$\sum_{i \in N} \sum_{g \in G(i)} \frac{Q_g}{u_g} u_g^{\max\{0, t-T_{CR_g}-1\}} + \sum_{(ij) \in E} B_{SH_{ij}} u_{ij}^t +$$

$$\sum_{i \in N} Q_{SH_i} u_i^t - \sum_{i \in N} (P_{D_i} - p_{SH_i}^t) \tan(\phi_{D_i}) \leq 0, \quad t \in T \quad (13q)$$

The objective of the problem maximizes the energization of the system components, (13a) imposes the allocation budget, (13b) and (13c) initialize the outage, (13d) allows the energization of a bus only if at least one of its endpoints was energized at the previous time step, (13e) forces the nodes to stay energized after their initial energization, (13f) forces a black start generator to get energized, (13g) forces the energization of a bus if one of its generators is energized, (13h) impose that if an edge is energized both of its endpoints should be energized. Constraints (13g)-(13k) are the constraints of formulation  $F_1$  for illustration (which, as proven, can be replaced equivalently with  $F_2$  or  $F_3$ ), repeated for every time step  $t$ . Constraints (13l)-(13n) define the generator startup capability curve. Finally, constraints (13o)-(13q) impose aggregate active and reactive power capacity constraints. The underlying assumptions and justification behind this formulation of the problem can be found in [19].

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