Nonlinear and Discrete Optimization—Homework 7

1. Consider the optimization problem:

$$\begin{aligned} \max_{x\in\mathbb{R}^2} \quad & -8x_1^2 - 10x_2^2 + 12x_1x_2 - 50x_1 + 80x_2 \\ \text{s.t.} \quad & x_1 + x_2 \le 1 \\ & 8x_1^2 + x_2^2 \le 2 \\ & x_1 \ge 0 \\ & x_2 \ge 0 \end{aligned}$$

   - Verify that Slaters condition holds.
   - Obtain the KKT conditions for this problem.
   - Prove that $(0, 1)$ is a global solution of this problem.
   - Assume that the above problem is perturbed as

$$\begin{aligned} \max_{x\in\mathbb{R}^2} \quad & -8x_1^2 - 10x_2^2 + 12x_1x_2 - 50x_1 + 80x_2 \\ \text{s.t.} \quad & x_1 + x_2 \le 1 - \varepsilon \\ & 8x_1^2 + x_2^2 \le 2 + 2\varepsilon \\ & x_1 \ge 2\varepsilon \\ & x_2 \ge -5\varepsilon \end{aligned}$$

     for an infinitesimal number ε. This makes the solution $(0, 1)$ perturb a bit. How much will the corresponding optimal objective value change?

2. Find the dual of the optimization problem

$$\begin{aligned} \min_{x\in\mathbb{R}^3} \quad & x_1^2 + x_2^2 + x_3^2 + x_1x_3 + x_1x_2 \\ \text{s.t.} \quad & x_1^2 - x_2^2 + x_1x_2 + x_3^2 + 2x_2x_3 \le 6 \\ & x_1^2 + x_2^2 + x_3^2 = 1 \end{aligned}$$

3. Find the dual of the optimization problem

$$\begin{aligned} \min_{x\in\mathbb{R}^3} \quad & x_1^2 + x_2^2 + e^{x_3} \\ \text{s.t.} \quad & x_1 + x_2 + x_3 \le 1 \\ & x_3 \ge 2 \end{aligned}$$

4. Consider the problem

$$\begin{aligned} \min_{x\in\mathbb{R}^n} \quad & e^{-x_1} + e^{-x_2} + \cdots + e^{-x_n} \\ \text{s.t.} \quad & -1 \le x_1 + x_2 + \cdots + x_n \le 1 \end{aligned}$$

   - Is this problem convex?
   - Calculate its dual.