Nonlinear and Discrete Optimization—Homework Set 6

1. Find all regular points of the set $\{x \in \mathbf{R}^4 \mid 2(x_1+x_2+x_3)^3+3(x_1+x_2+x_3)^2 = 1,\ x_1^2+x_2^2+x_3^2+x_4 = 1\}$.
2. Find all local minima, global minima, local maxima and global maxima of the function $x_1x_2 + x_2x_3 + x_3x_1$ over the sphere $x_1^2 + x_2^2 + x_3^2 = 1$.
3. Maximize the function $(x_1 - 1)^2 + (x_2 - 2)^2 + (x_3 - 3)^2$ over the sphere $x_1^2 + x_2^2 + x_3^2 = 1$.
4. Using the first-order and second-order conditions, solve the optimization problem:

$$\begin{aligned} \min_{x\in\mathbb{R}^3} \quad & x_1 + x_2^2 + x_2x_3 + 4x_3^2 \\ \text{s.t.} \quad & \frac{1}{2}(x_1^2 + x_2^2 + x_3^2) = 1 \end{aligned}$$

5. Solve the following optimization problem:

$$\begin{aligned} \min_{x\in\mathbb{R}^2} \quad & \frac{50}{x_1} + \frac{20}{x_2} + x_1x_2 \\ \text{s.t.} \quad & x_1 \geq 1 \\ & x_2 \geq 1 \end{aligned}$$

6. Solve the following optimization problem:

$$\begin{aligned} \min_{x\in\mathbb{R}^2} \quad & e^{-x_1} + e^{-2x_2} \\ \text{s.t.} \quad & x_1 + x_2 \leq 1 \\ & x_1 \geq 0 \\ & x_2 \geq 0 \end{aligned}$$