Nonlinear and Discrete Optimization—Homework 5

1. Prove that the following set is convex:

$$\{x \in \mathbb{R}^3 \mid -2x_1 + 6x_3 \leq 5x_2 + 4 \leq 3x_3 + 2x_1 + 10\}$$

2. Prove that the following set is convex:

$$\left\{ x \in \mathbb{R}^2 \;\middle|\; \begin{bmatrix} 6x_1 - 5x_2 + 2 & -x_1 + x_2 \\ -x_1 + x_2 & 4x_2 + 5 \end{bmatrix} \succeq 0 \right\}$$

3. Find all constant coefficients a_0, a_1, a_2, a_3 for which the univariate function $f(x) = \frac{1}{12}x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$ is convex.

4. Prove that the following function is convex over the set of positive vectors:

$$f(x_1, x_2, x_3) = -\sqrt{x_1x_2} + 2x_1^2 + 2x_2^2 + 3x_3^2 - 2x_1x_2 - 2x_2x_3$$

5. Prove that every local solution of the following problem is a global solution as well:

$$\begin{aligned} \min_{x \in \mathbb{R}^3} \quad & e^{x_1 - x_2} + (x_1 + 5x_2 - x_3)^6 + (6x_1 - x_2 + x_3)^8 \\ \text{s.t.} \quad & x_1 + 2x_2 + 3x_3 = 5 \\ & x_1^4 + x_2^4 + e^{-10x_3} \leq 10 \\ & x_1 \geq 0 \\ & x_2 \geq 0 \\ & x_3 \geq 0 \end{aligned}$$