Nonlinear and Discrete Optimization—Homework 4

1. (2 points) Consider the optimization problem

$$\min_{x_1,x_2,x_3} \quad e^{x_1-1} + e^{-x_1+1} + e^{x_2-2} + e^{-x_2+2} + e^{x_3-3} + e^{-x_3+3} + (x_1 - 7x_2 + 3x_3)^6$$

Write a code in CVX to find the global minimum x^*. Then, write a code that implements the Gradient method with your choice of the backtracking parameters. Draw $|f(x^{(k)}) - f(x^*)|$ verses k for $k = 0, 1, 2, \ldots, 50$ on a log-linear plot. Show the trajectory of the points $x^{(0)}, x^{(1)}, ..., x^{(50)}$ in the 3-dimensional (x_1, x_2, x_3) plane.

2. (2 point) Redo the previous problem with Newton's method.

3. (6 points) Consider the optimization problem

$$\min_{x\in\mathbb{R}^n} \quad -\sum_{i=1}^{n} \log(4 - x_i^2) - \sum_{i=1}^{n} \log(2 + a_i^T x)$$

where $n = 1250$ and $a_i \in \mathbb{R}^n$ are randomly generated vectors. Write a code in CVX to find the global minimum x^*. Then, write a code that implements the Newton's method with the backtracking line search. Draw $|f(x^{(k)}) - f(x^*)|$ versus k for $k = 0, 1, \ldots, 100$ on a log-linear plot.