Nonlinear and Discrete Optimization—Homework 3

1. Consider a function $f(x) : \mathbb{R}^2 \to \mathbb{R}$ with the property that its Gradient and Hessian are equal to $[0\ 0]$ and $\begin{bmatrix} 3 & 4 \\ 4 & 5 \end{bmatrix}$ at some point $\bar{x}$.

   a) Find a direction Δx such that the relation $f(\bar{x} + \varepsilon\Delta x) < f(\bar{x})$ holds for sufficiently small and positive values of the scalar ε.

   b) Find a direction Δx such that the relation $f(\bar{x} + \varepsilon\Delta x) > f(\bar{x})$ holds for sufficiently small and positive values of the scalar ε.

2. Consider a unimodal function $f(x) : \mathbb{R} \to \mathbb{R}$ over the interval $[-3, 3]$. To find an approximate global maximum with the error of at most 10^{-4}, how may iterations of the golden section method are needed?

3. Use the golden section method to estimate the optimal solution of the following optimization problem with the approximation error of at most 1:

$$\max_{x \in \mathbb{R}} \quad x - 2e^x$$
$$\text{s.t.} \quad -3 \leq x \leq 2$$

4. Write a code that implements the golden section method for Problem 3 with the precision of 10^{-5}. Also, write a code in CVX that solves Problem 3 directly, and compare its solution with the solution of your implementation of the golden section method.

5. For each of the following functions, determine whenever it is coercive or not:

   (a) $f(x_1, x_2) = 7x_1^4 + 5x_2^4$

   (b) $f(x_1, x_2) = e^{x_1^2} + e^{x_2^2} - x_1^{200} - x_2^{50}$

   (c) $f(x_1, x_2) = 2x_1^2 - 8x_1x_2 + x_2^2$

   (d) $f(x_1, x_2, x_3) = x_1^3 + x_2^3 + x_3^3$