Nonlinear and Discrete Optimization—Homework 2

1. Find a second-order approximation of the function $f(x) = e^{x_1+5x_2} + \cos(x_1 - x_2) + 10\sin(x_1)\cos(x_2)$ around the point $(0, 0)$.

2. Find all local minima, local maxima and saddle points of the function $f(x_1, x_2) = x_1^4 - x_1x_2 + x_2^4$.

3. Find all local minima, local maxima and saddle points of the function $f(x_1, x_2, x_3) = x_1x_2 + x_2x_3 + x_3x_1 - x_1^3 - x_2^3 - x_3^3$.

4. A company has n factories. Factory i (for $i = 1, 2, ..., n$) is located at point (x_i, y_i) in the two-dimensional plane $\mathbb{R}^2$. The company wants to locate a warehouse at a point (x, y) that minimizes

$$\sum_{i=1}^{n} (\text{distance from factory } i \text{ to the warehouse})^2$$

   Where should the warehouse be located?

5. Consider the five points $(0, 0), (0, 7), (7, 0), (2, 2), (-4, -4)$ in $\mathbb{R}^2$ and name them (x_i, y_i) for $i = 1, ..., 5$. The objective is to find two coefficients $a, b \in \mathbb{R}$ such that the boundary of the ellipse $ax^2 + by^2 = 1$ is as closely to the above 5 points as possible. To this end, we define the error function:

$$f(a, b) = \sum_{i=1}^{5} (ax_i^2 + by_i^2 - 1)^2$$

   Calculate the optimal values of (a, b) by finding the local minima of the error function $f(a, b)$.