Nonlinear and Discrete Optimization—Homework 1

1. Consider a function $f : \mathbb{R} \to \mathbb{R}$ such that $f'(x) = x^2(x-1)(x-3)$. Find all stationary points of this function and determine their types.

2. Find the globally optimal solution to
$$\begin{aligned} \max \quad & x^3 - x \\ \text{s.t.} \quad & -4 \leq x \leq 2 \end{aligned}$$

3. Find all local solutions to
$$\begin{aligned} \max \quad & x^3 - 3x^2 + 4x - 1 \\ \text{s.t.} \quad & -4 \leq x \leq 4 \end{aligned}$$

4. Show that for all x, we have $e^x \geq x + 1$ (hint: let $f(x) = e^x - x - 1$ and solve the optimization problem $\min\ f(x)$).

5. Find all local minima, local maxima and saddle points of the univariate function $f(x) = 49 \times x^{99} - 99 \times x^{49} + 1$.

6. Given a natural number $n \in \{1, 2, 3, ...\}$, find all local minima, local maxima, saddle points, global minima and global maxima of a univariate function $f(x)$ over the interval $[-10, 10]$ with the property $f'(x) = (x-1)^{3n} + (x-1)^n$.