Math Programming II - Homework 3

Problem 1

Consider an $n \times n$ symmetric matrix Q that is sign indefinite (meaning that it is neither positive semidefinite nor negative semidefinite). Let $x^T Q x$ be minimized using the gradient algorithm with the initial point x^0 and a sufficiently small constant step size t. Find all values x^0 that make the algorithm converge to a saddle point of the function $x^T Q x$.

Problem 2

Let $f : R^n \mapsto R$ be a twice continuously differentiable function that satisfies

$$m\|y\|^2 \leq y' \nabla^2 f(x) y \leq M\|y\|^2, \quad \forall x, y \in R^n \tag{1}$$

where m and M are some positive scalars. Let also X be a closed convex set. Show that f has a unique global minimum x^* over X, which satisfies

$$\theta_M(x) \leq f(x) - f(x^*) \leq \theta_m(x), \quad \forall x \in R^n \tag{2}$$

where for all $\delta > 0$, we denote

$$\theta_\delta(x) = -\inf_{y \in X} \left\{ \nabla f(x)'(y - x) + \frac{\delta}{2}\|y - x\|^2 \right\} \tag{3}$$

Problem 3

Consider the conditional gradient method, and assume that the gradient of f satisfies

$$\|\nabla f(x) - \nabla f(y)\| \leq L\|x - y\|, \quad \forall x, y \in X \tag{4}$$

where L is a positive constant. Show that if the stepsize α^k is given by

$$\alpha^k = \min\left\{1, \frac{\nabla f(x^k)'(x^k - \bar{x}^k)}{L\|x^k - \bar{x}^k\|^2}\right\}, \tag{5}$$

then every limit point of $\{x^k\}$ is a stationary point.