Power Systems Analysis - Homework 3

1. Given symmetric matrices $A_0, A_1, \ldots, A_n \in \mathbb{R}^{m\times m}$, define:

$$M \triangleq \{(x_1, \ldots, x_n) \in \mathbb{R}^n | A_0 + x_1 A_1 + \cdots + x_n A_n \succeq 0\}.$$

   Prove that M is a convex set ($\succeq$ is the positive semidefinite sign).

2. Prove that the following functions are all convex:

   (a) $ax + b$

   (b) e^{ax}

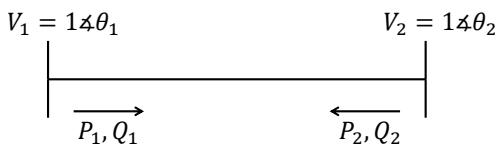
   (c) x^a on the domain $\{x \in \mathbb{R} | x > 0\}$ where $\alpha > 1$ or $\alpha < 0$.

   (d) $x \log(x)$ on the domain $\{x \in \mathbb{R} | x > 0\}$

   (e) $-\log(x)$ on the domain $\{x \in \mathbb{R} | x > 0\}$.

3. Consider two functions $g : \mathbb{R} \longrightarrow \mathbb{R}$ and $h : \mathbb{R} \longrightarrow \mathbb{R}$. Define $f(x) \triangleq h(g(x))$. Prove that f is convex if g and h are convex and h is increasing.

4. Consider the figure



   (a) Write P_1, Q_1, P_2, Q_2 as functions of the parameter $\theta = \theta_1 - \theta_2$.

   (b) Show that there exist a matrix $A \in \mathbb{R}^{2\times 2}$ and a vector $b \in \mathbb{R}^2$ such that

$$\begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} = A \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} + b$$

   where A and b are constant and do not depend on θ.

   (c) Define M as:

$$M = \{(P_1, P_2) | P_1 \leq P_1^{\max}, P_2 \leq P_2^{\max}, Q_1 \leq Q_1^{\max}, Q_2 \leq Q_2^{\max}\}$$

   How does the feasible set M look like?