- IEOR 160: Nonlinear and Discrete optimization

- Note: IEOR 162 covers linear programs in more details

- 2 recitations on Fridays.
- Next Friday: sample problems + Homework
- weekly homework: usually out on Wed. afternoon
  due on Thu. afternoon (5pm)
- 10% will be deducted for each day of late submission (can't hand in homework after the TAs have solved them in class)
- office hours: Mon. 10-11, Wed. 11-12 in 4121 Etch
  (please don't send emails about homework)
- Grading policy:

15%	15%	15%	55%
Homework	midterm 1 (Oct 7)	midterm 2 (Nov 4)	Final exam (Dec 17)

- Textbook: Introduction to Mathematical Programming, volume one.

Syllabus:
- Convex sets and functions
- modeling and formulation
- local optimality
- optimality conditions
- convex optimization
- Duality
- Numerical algorithm
- Integer programming
- Branch and bound
- cutting plane
- convex relaxation
- Conic optimization
- Applications

Optimization: Find <u>optimal decisions</u>
to <u>maximize/minimize an objective</u>
subject to <u>constraints/conditions</u>

Example: Fly from SF to NY:

minimize : fuel/costs/delays
maximize : reliability
Constraints : safety/route/physics

- Canonical form: $\min_{x \in \mathbb{R}^n} f_0(x)$

  s.t. $f_i(x) \leq 0 \quad i = 1, \dots, m$

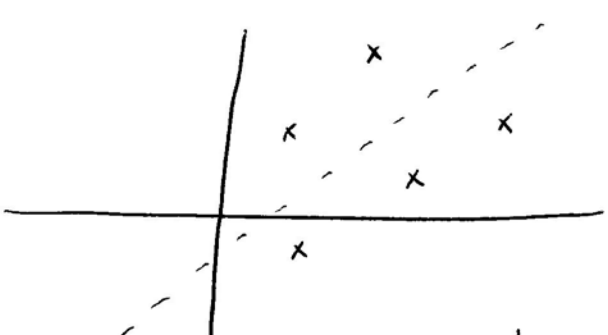
- Example: $x = [x_1, x_2]$, $\min_{x} x_1^2 + x_2^2$ s.t. $\underbrace{x_1 - x_2 = 5}$

  $x_1 - x_2 - 5 \leq 0$

  $-(x_1 - x_2 - 5) \leq 0$

- special case of optimization: Feasible strategy
- set objective to zero to find such a feasible point

- Complexity: minutes $\longrightarrow$ hours $\longrightarrow$ days $\longrightarrow \dots$
- How to measure complexity ?
- Easy problem: fit a set of points to a line.



$\Longrightarrow$ This is an easy task because it has an explicit solution (Formula)

- This doesn't work for almost all optimization problems. $\Longrightarrow$ No formula!

- strategy: Use an iterative algorithm:

Guess for solution $\xrightarrow{\text{improvement}}$ Guess 2 $\xrightarrow{\text{improve}}$ Guess 3 $\longrightarrow \dots$

$\dots \longrightarrow$ Converge to a solution.

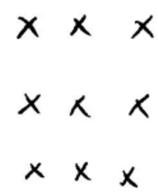
- Can talk about finite v.s. infinite convergence
- Convergence rate ?
- Complexity: $x_1^2=1,\ x_2^2=1, \ldots, x_n^2=1$

  $\Rightarrow$ Feasible points: 2^n of those

- 2^{500} is more than number of atoms in observable universe.
- Complexity: a function of n: $n, n^2, 2^n, \ldots$

Feasible set:

- $x \in D$ $\quad (x_i^2=1,\ i=1,\ldots,n)$



Infinite set:

- $x \in C$ $\quad (x_1+x_2+\cdots+x_n=1)$

Counterintuitive: Continuous optimization is usually easier than discrete optimization

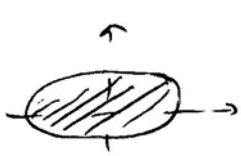
---

Easy problems v.s. hard problems

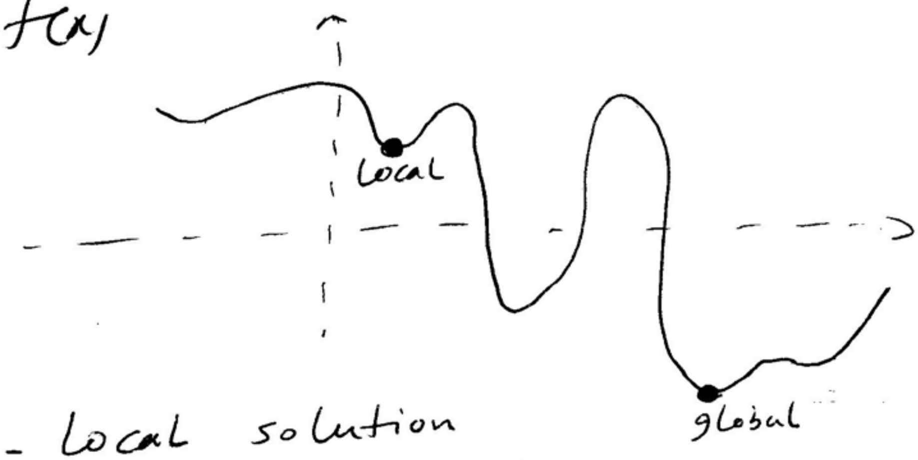
- objective

- set

Convex v.s. nonconvex



- Example: $\min_{x \in \mathbb{R}} f(x)$



* Important concepts:
- local solution v.s. global solution
- optimality conditions
- Duality: define prices

- Distributed computation:

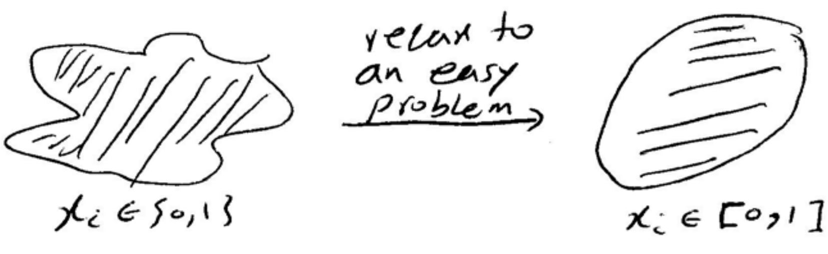
  Look at the iterations $x^{(1)} \to x^{(2)} \to x^{(3)} \cdots$
  - Can we split them among multiple machines?
  - i.e solve a large-scale problem on several machines working in parallel.

- Integer Programming: $\min_{x} x^T Q x$ s.t. $x_i \in \{0,1\}$, $i = 1, \dots, n$

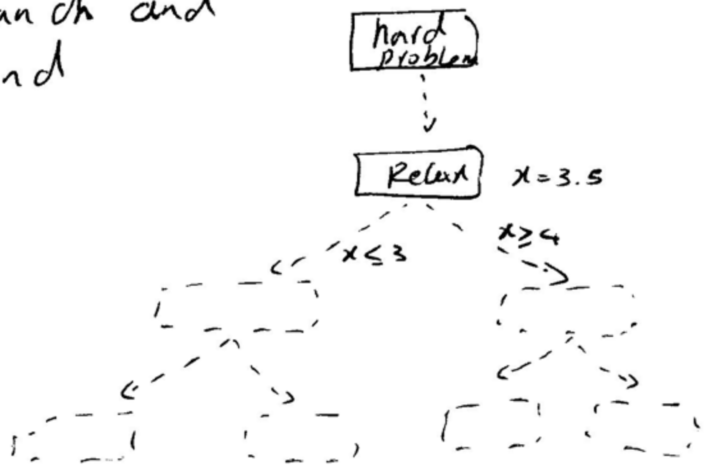
  $\Rightarrow$ 2^n feasible points

  $\Rightarrow$ hard problem.

- Convex relaxations:

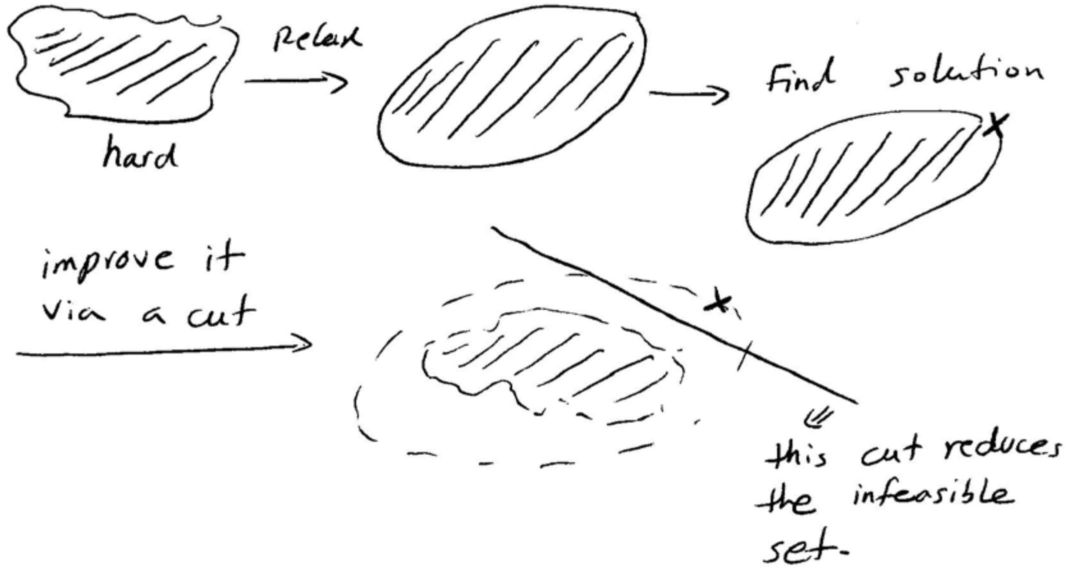


- Branch and bound



$\Longrightarrow$ solve several easy subproblems.

- cutting plane:



- Conic optimization: $LP \subseteq \underbrace{QP \subseteq QCQP \subseteq SOCP \subseteq SDP}_{\text{family of easy problems.}}$

- Applications: OR / OM / Energy / mechanis / Electrical

....