Convex Optimization–Homework 4

1. Write a code to get the incidence matrix of an arbitrary graph as an input and solve the SDP relaxation of its associated max cut problem (suggestion: use CVX at cvxr.com/cvx/).

2. Consider the optimization
   \[ \min_{x_1, x_2} e^{x_1 + 3x_2 - 0.1} + e^{x_1 - 3x_2 - 0.1} + e^{-x_1 - 0.1} \]
   Write a code to solve this optimization using the gradient method with the backtracking parameters \( \alpha = 0.1 \) and \( \beta = 0.6 \). Draw \( f(x^{(k)}) \) versus \( k \) for \( k = 0, 1, 2, \ldots, 50 \) on a log-linear plot.

3. Consider the optimization
   \[ \min_{x \in \mathbb{R}^n} - \sum_{i=1}^{n} \log(1 - x_i^2) - \sum_{i=1}^{n} \log(1 - a_i^T x) \]
   where \( n = 5000 \) and \( a_i \) are randomly generated vectors. Solve this optimization using Newton’s method with the backtracking line search (\( \alpha = 0.01 \) and \( \beta = 0.5 \)). Draw \( f(x^{(k)}) \) versus \( k \) for \( k = 0, 1, \ldots, 30 \) on a log-linear plot.

4. Consider the optimization
   \[ \min_{x_1, x_2, x_3, y} \quad f_1(x_1, y) + f_2(x_2, y) + f_3(x_3, y) \]
   \[ \text{s.t.} \quad g_1(x_1, y) + g_2(x_2, y) + g_3(x_3, y) \leq 0, \]
   where \( f_i \)'s and \( g_i \)'s are convex. Assume that \( x_i \) is the private variable of user \( i = 1, 2, 3 \) and \( y \) is a public variable. Propose a distributed algorithm to solve the above problem.