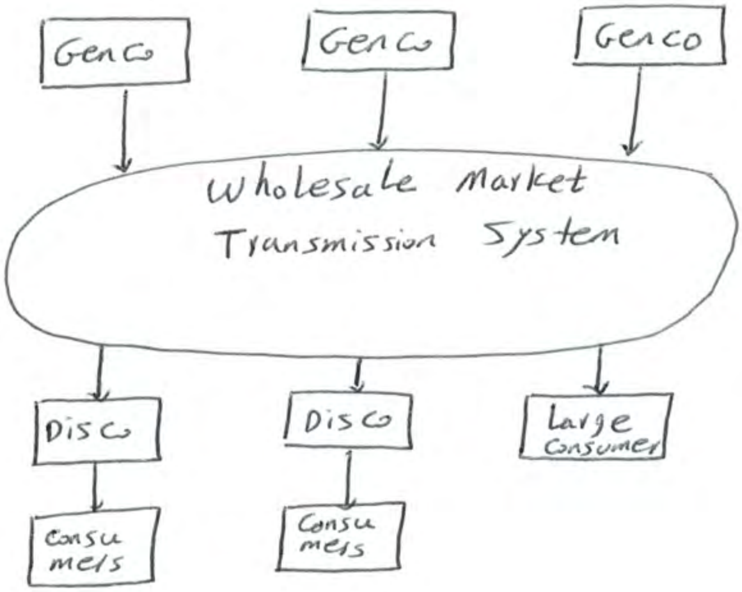
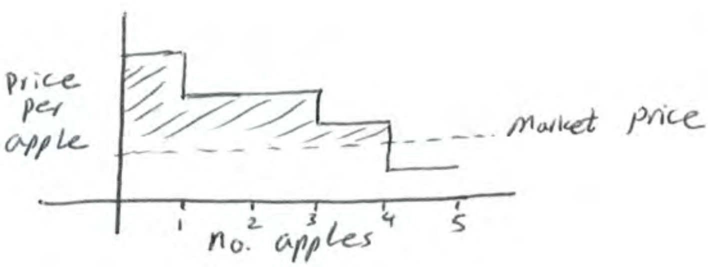
Electricity Market:



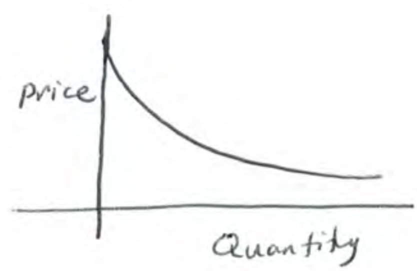
Economics:



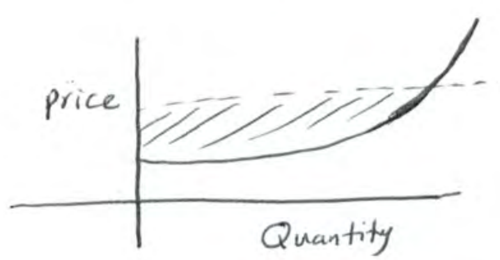
shaded area = Consumer's surplus

= "extra value" that you get by buying multiple apple at the same price

- demand function $q = D(\pi)$
- inverse demand function $\pi = D^{-1}(q)$



- supply function $q = S(\pi)$
- inverse supply function $\pi = S^{-1}(q)$



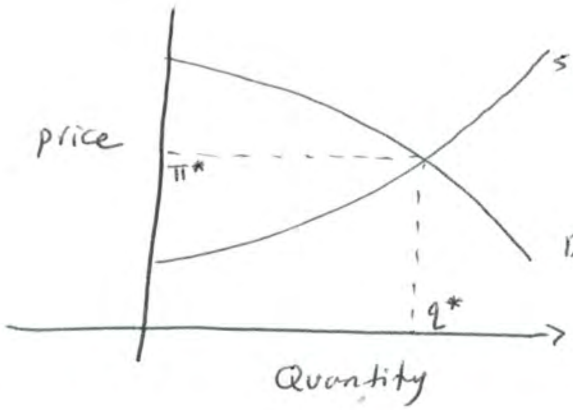
shaded area = producer's profit

- perfectly competitive market: No one can affect the price, or all market participants take the price as given.

- Equilibrium price or market clearing price π^*:

$$D(\pi^*) = S(\pi^*)$$

- Equilibrium quantity: $D^{-1}(q^*) = S^{-1}(q^*)$

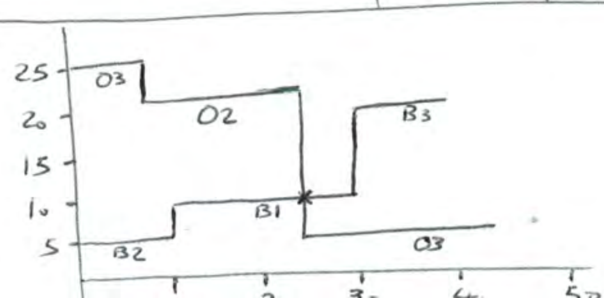


An economic situation is <u>pareto efficient</u> if the benefit of no parties can be increased without reducing the benefit of some one else.

$$q^* = \text{pareto optimal}$$

- Electrical energy is treated as a commodity.
- Electricity market is different from other markets because of lack of storage

	Identifier	Amount (MW)	price ($/MWh)
Bids to sell energy	B1	20	10
	B2	10	5
	B3	10	20
Offers to buy energy	O1	20	5
	O2	15	20
	O3	10	25



⟸ supply and demand curves (This is one way to clear the market)

- Instead of price v.s. quantity. Look at price v.s. cost:

<u>producer i</u>: $\underbrace{\max \Omega_i}_{\text{profit}} = \max (\underbrace{\Pi}_{\text{given price}} \cdot P_i - \underbrace{C_i(P_i)}_{\text{cost}})$

$$\Rightarrow \quad \Pi = \underbrace{C_i'(P_i)}_{} \rightarrow \text{marginal cost}$$

marginal cost = change in total cost when the quantity produced changes by one unit.

$C_i(P_i)$: Quadratic, piecewise linear, etc.

- what if $c'(P_i)\big|_{P_i^{max}} < \Pi \Rightarrow$ generate P_i^{max}

$c'(P_i)\big|_{P_i^{min}} > \Pi \Rightarrow$ generate P_i^{min}

<u>perfect Competition</u>: N producers with no network.

$$\left.\begin{aligned} &\min \sum_{i=1}^{N} C_i(P_i) \\ &\sum_{i=1}^{N} P_i = l \end{aligned}\right\} \text{economic dispatch} \longrightarrow l(P_1,\dots,P_N,\lambda) = \sum_i C_i(P_i) + \lambda\left(l - \sum_i P_i\right)$$

$$\Rightarrow \frac{dc_1}{dP_1} = \frac{dc_2}{dP_2} = \dots = \frac{dc_N}{dP_N} = \lambda$$

$\Rightarrow$ Lagrange multiplier λ (shadow price) = competitive price.

<u>Imperfect competition</u>: Each producer can potentially influence the market price thru their actions.

X_i = action of producer $\underline{i}$, X_{-i} = action of other producers.

$\Omega_i = \Pi P_i - C_i(P_i)$, Π depends on the actions.

$\Rightarrow \quad \Omega_i = \Omega_i(X_i, X_{-i})$

- Assume all producers behave in a rational way to maximize their profits.

$\Rightarrow$ find X_i^* such that $\Omega_i(X_i^*, X_{-i}^*) \geq \Omega_i(X_i, X_{-i}^*)$

- $(X_1^*, X_2^*, \dots, X_N^*)$ is called a Nash equilibrium.

(to find this point, we need to fix the actions of all but one producers). $\longrightarrow$ Non-cooperative game in game theory.

- Bertrand : compete on the price : $X_i = \Pi_i$
- Cournot : compete on the output : $X_i = P_i$

Example for cournot:

$\Pi = 100 - D$ \$/mwh (inverse demand curve)

Two firms:

$C_A = 35 \cdot P_A$ \$/h

$C_B = 45 \cdot P_B$ \$/h

$$\begin{cases} \Omega_A(P_A, P_B) = \Pi(D) \cdot P_A - C_A(P_A) \\ \Omega_B(P_A, P_B) = \Pi(D) \cdot P_B - C_B(P_B) \end{cases}$$

(P_A, P_B unknown $\Rightarrow$ D unknown)

$D = P_A + P_B$, max profit $\Rightarrow \dfrac{\partial \Omega_A}{\partial P_A} = \dfrac{\partial \Omega_B}{\partial P_B} = 0$

$$\Rightarrow \quad \Pi(D) - \frac{dC_A(P_A)}{dP_A} + P_A \frac{d\Pi(D)}{dD} \times \frac{dD}{dP_A} = 0$$

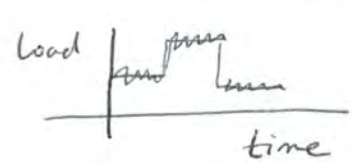
$$\Rightarrow \quad \begin{cases} P_A = \frac{1}{2}(65 - P_B) \\ P_B = \frac{1}{2}(55 - P_A) \end{cases}$$

- Each generator has a true cost function (say quadratic)
- Independent system operator (ISO) has a demand function $D(\pi)$.
- ISO asks each supplier to submit a supply function $P_i(\pi)$.
- ISO expects that $P_i(\pi) = C_i(\pi)$, which is not the case.
- ISO requires $P_i(\pi)$ to satisfy some properties (piece-wise linear with $\underline{\underline{6}}$ segments).
- Question: How does a supplier choose $P_i(\pi)$ to maximize its profit?

$$D(\pi) = \sum_i P_i(\pi) \quad , \quad \Omega_i = \pi P_i - C_i(P_i)$$
$$= \pi\left(D(\pi) - \sum_{-i} P_{-i}(\pi)\right) - C_i\left(D(\pi) - \sum_{-i} P_{-i}(\pi)\right)$$

Take derivative w.r.t. π to find the optimal coefficients for $P_1(\pi), \dots, P_N(\pi)$.

- Ancillary services: Balancing $\Rightarrow$ supply = demand
- Balancing is a must for power grids.
  - regulation service: handle small, rapid fluctuations.
  - Load-following service: handle slower fluctuations (predictable)



  - reserve service:
    handle large and unpredictable power deficits.

- Need ancillary service to ensure security of the grid.

$\Rightarrow$ Market for ancillary service

$$P_i \longrightarrow (P_i, \underset{\text{Reserve}}{R_i}) \quad , \quad P_i \le P_i^{max} \longrightarrow P_i + R_i \le max$$

$$C_i(P_i) \longrightarrow C_i^1(P_i) + C_i^2(R_i)$$

Mathematical formulation of competitive market:

- Lossless, no generating limits, no network:

$$D = \sum_{i=1}^{n} P_i \quad , \quad \min \sum_i C_i(P_i) \quad \Rightarrow \quad \lambda = C_i'(P_i)$$

lag. multiplier = price

- Lossy, no generating limits, infinite-capacity network:

$$L = \sum_{i=1}^{n} P_i - D = f(P_2, \dots, P_n) \qquad (\text{Bus } \underline{1} : \text{slack bus})$$

Loss

$$\Rightarrow \quad \text{Lagrangian} = \sum_i C_i(P_i) - \lambda\left(\sum_i P_i - D - f(P_2, \dots, P_n)\right)$$

$$\Rightarrow \quad C_1'(P_1) = \lambda \quad , \quad C_i'(P_i) - \lambda\left(1 - \frac{\partial f}{\partial P_i}\right) = 0 \qquad i = 2, \dots, n$$

$$\Rightarrow \quad \lambda = \underbrace{\frac{1}{1 - \frac{\partial f}{\partial P_i}}}_{L_i} C_i'(P_i) \quad , \quad L_1 = 1$$

$$\frac{\partial f}{\partial P_i} \overset{?}{>} 0$$ $\leftarrow$ because more generation expects to bring more losses.

Nodal price: the cost of suppling an additional megawatt of load at the node under consideration by the cheapest possible means.

- λ is the price at the Load bus
- $C_i'(P_i)$ is the nodal price at the generator bus $\underline{i}$.
- How to find the penalty factor L_i : (or $\frac{\partial f}{\partial P_i}$)

practical situation $|V_i| \simeq 1$

$$\Rightarrow \quad \frac{\partial P_i}{\partial \theta_j} = |V_i||V_j| \left(G_{ij} \sin(\theta_i - \theta_j) - B_{ij} \cos(\theta_i - \theta_j) \right)$$

$$\left.\begin{aligned} \frac{\partial f}{\partial \theta_j} &= \sum_{i=2}^{n} \frac{\partial f}{\partial P_i} \frac{\partial P_i}{\partial \theta_j} \quad , \quad j = 2, \dots, n \\ , \quad \frac{\partial f}{\partial \theta_j} &= \sum_{i=1}^{n} \frac{\partial P_i}{\partial \theta_j} \quad \left(f(P_1, \dots, P_n) = \sum_i P_i - D \right) \end{aligned}\right\} \Rightarrow \frac{\partial f}{\partial P_i} \text{ obtained}$$

Look at the problem in the context of opf:

$$\left.\begin{aligned} & \sum_{i=1}^{n} C_i(P_i) \\ \text{s.t.} \quad & P_i^{min} \le P_i \le P_i^{max} && \rightarrow \underline{\gamma}_i, \bar{\gamma}_i \\ & P_i = P_{D_i} + \sum_{j \in N(i)} P_{ij} && \rightarrow \lambda_i \\ & P_{ij} = Re\left(V_i (V_i - V_j)^* Y_{ij}^* \right) \\ & |P_{ij}| \le P_{ij}^{max} \\ & V_i^{min} \le |V_i| \le V_i^{max} \end{aligned}\right\} \Rightarrow \lambda_i = C'(P_i) - \underline{\gamma}_i + \bar{\gamma}_i$$

Locational marginal price (LMP)

sensitivity analysis: $\boxed{\lambda_i = \frac{\partial C^*}{\partial P_{D_i}}}$

DC Power Flow Approximation:

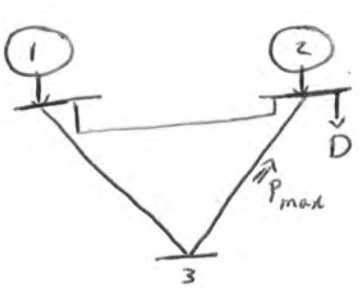
- Resistance of each Line is neglibile compared to the reactance
- The magnitude of bus voltage is equal to its nominal value
- The difference in voltage angles across each Line is so small that: $\cos(\theta_i-\theta_j)\simeq 1$, $\sin(\theta_i-\theta_j)\simeq \theta_i-\theta_j$ (first-order approximation)

$$\Rightarrow \boxed{P_{ij} = Y_{ij}(\theta_i-\theta_j)} \quad \Rightarrow \quad \underline{\underline{P = (Y_{ij})\,\theta}}$$

$\Rightarrow$ The Constraints all become Linear, allowing for finding LMPs.

- DC OPF could be good for market, but not real-time operation: 1- No $|V_i|$'s, 2- No Q_i's, 3- solution is physically meaningless and can't be realized in practice

Can LMP be negative?



Gen 1 : very cheap,
Gen 2 : very expensive,
D : very big

$$\Rightarrow P_{12} = 2P_{13} = P_{max} \Rightarrow P_2 = D - 3P_{max}$$

- what happens if a Load ε is added to bus 3 ?

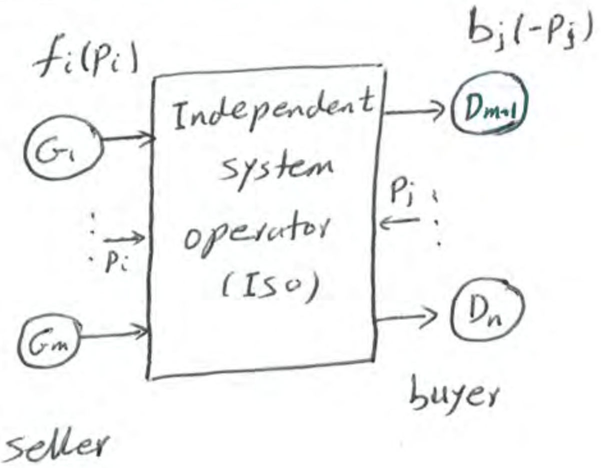
$P_{13}' = P_{max} + \varepsilon$, $P_{12}' = 2P_{13}$ $\Rightarrow$ $P_1' = 3P_{max} + 3\varepsilon$, $P_2' = D - 3P_{max} + 2\varepsilon$

Cost for (P_1', P_2') < Cost for (P_1, P_2) $\Rightarrow$ $\boxed{\text{price at bus 3} < 0}$

$\Rightarrow$ Laws of physics combined with network congestion

introduces negative LMPs.

Pool-based electricity market:



- Market participants submit their bids and offers.
- ISO solves an economic dispatch problem (OPF).
- ISO clears the market via LMPs.

Economic dispatch:

$$\max \; -\sum_{i=1}^{m} f_i(P_i) + \sum_{j=m+1}^{n} b_j(-P_i) \longleftarrow$$ social walfare benefit of everyone

s.t. Network constraints

Merchandising surplus (ISO): Money collected - payment made

$\underbrace{\qquad\qquad}_{\text{Due to Loss + Congestion}}$

$$= \sum_{i=1}^{m} \lambda_i P_i - \sum_{j=m+1}^{n} \lambda_j(-P_j)$$

$$= \sum_{i=1}^{n} \lambda_i P_i$$

Nonprofit corporation: This surplus should be redistributed.

Note that: $\displaystyle\sum_{i=1}^{n} \lambda_i P_i = \sum_{i=1}^{n} \lambda_i \Big(\sum_{j\in N(i)} P_{ij}\Big) = \sum_{(i,j)\in E} \underline{P_{ij}(\lambda_i - \lambda_j)}$

Financial Transmission Right: $P_{ij}(\lambda_i - \lambda_j)$

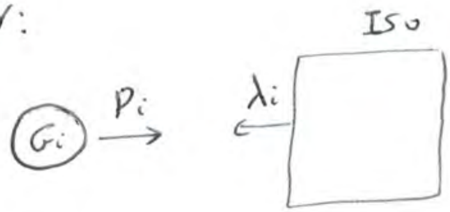
$$\underset{i}{\times} \xrightarrow{\;\overrightarrow{P_{ij}}\;} \underset{j}{\times}$$

FTR holder gets this money. $\longrightarrow$ obtaing FTR: auction

Flowgate right: $P_{ij}\lambda_{ij} + P_{ji}\lambda_{ji}$, produce revenues if congested.

- Decentralized market with perfect competition:

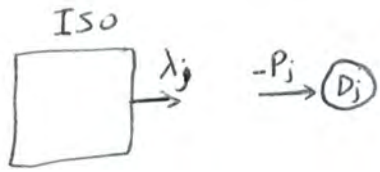
1- Seller:



Given λ_i : $\max_{P_i} \; \lambda_i P_i - f_i(P_i)$

s.t. $P_i^{min} \leq P_i \leq P_i^{max}$

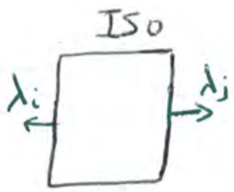
2- Buyer:



Given λ_j : $\max_{-P_j} \; b_j(-P_j) - \lambda_j(-P_j)$

s.t. $D_j^{min} \leq -P_j \leq D_j^{max}$

3- ISO:



$\min_{P} \sum \lambda_i P_i$ ← merchandising surplus (non-profit)

s.t. Network constraints

Assume f's convex, b's concave, Network constraints = convex

- Consider $\lambda^* = (\lambda_1^*, \dots, \lambda_n^*)$: Lagrange multipliers for OPF

- At $\lambda = \lambda^*$, problems (1), (2), (3) have a common solution $P = P^*$ is equal to optimal solution of OPF

⇒ <u>competitive equilibrium is efficient.</u>

- Can have market for reactive power.
- If AC OPF is solved instead of DC OPF, LMPs for active power indirectly account for reactive power.
- Another extension is Dynamic market.
- As wind is becoming important, dynamic market is gaing popularity.