- Normal operating condition for high voltage line:

$$|V_1| \simeq |V_2|, \quad \measuredangle Z \simeq 90^\circ, \quad \theta_{12} = \text{small } (<10^\circ)$$

$\Rightarrow$ Decoupling between control of active and reactive powers:

- P_{12} : couples strongly with θ_{12}
- Q_{12} : couples strongly with $|V_1| - |V_2|$

$$\Delta P_{12}\Big|_{1,1,0} = \frac{|V_1||V_2|}{X} \Delta\theta_{12}$$

$$\Delta Q_{12}\Big|_{1,1,0} = \frac{|V_1|}{X}\left(\Delta|V_1| - \Delta|V_2|\right)$$

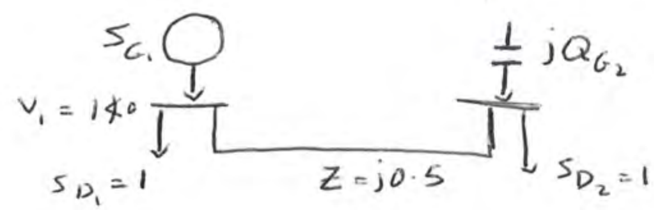
<u>Example:</u>

- Balanced 3ϕ transmission line with $Z = 1\measuredangle 85^\circ$, $\theta_{12} = 10^\circ$
- Find S_{12} and $-S_{21}$ for
  1- $|V_1| = |V_2| = 1$
  2- $|V_1| = 1.1$, $|V_2| = 0.9$

1- $S_{12} = 0.1743$, $\quad -S_{21} = 0.1717 - j0.0303$

2- $S_{12} = 0.1917 + j0.2192$, $\quad -S_{12} = 0.1856 + j0.1493$

<u>Example:</u>



1- pick Q_{G_2} so that $|V_2| = 1$.

2- what happens if $Q_{G_2} = 0$?

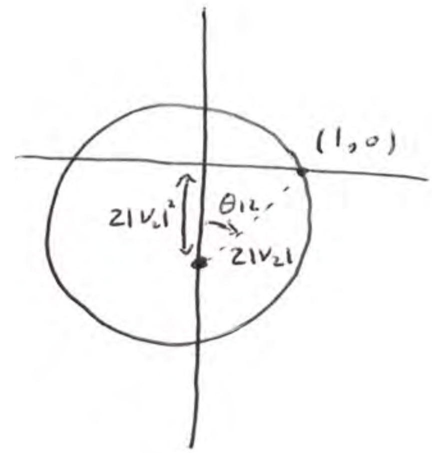
1) $P_{12} = -P_{21} = 1$

$$P_{12} = \frac{|V_1||V_2|}{X} \sin\theta_{12} \quad \Rightarrow \quad \theta_{12} = 30^\circ$$

$$Q_{G_2} = Q_{21} = \frac{|V_2|^2}{X} - \frac{|V_1||V_2|}{X} \cos\theta_{12} = 0.268$$

$Q_{G_2} > 0 \quad \Rightarrow \quad \underline{\underline{C}}$ can be found.

2) $Q_{G_2} = 0 \quad \Rightarrow \quad S_{21} = -1$

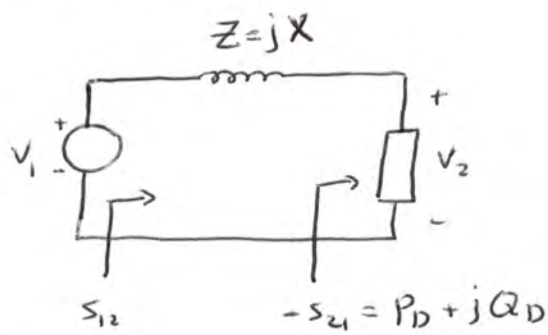


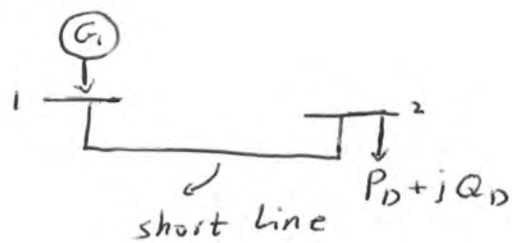
$$\Rightarrow (2|V_2|^2)^2 + 1^2 = (2|V_2|)^2$$

$$\Rightarrow V_2 = 0.707 \angle -45^\circ$$

This is voltage drop.

<u>Voltage collapse:</u>





$$P_D = P_{12} = \frac{|V_1||V_2|}{X} \sin\theta_{12}$$

$$Q_D = -Q_{21} = -\frac{|V_2|^2}{X} + \frac{|V_2||V_1|}{X} \cos\theta_{12}$$

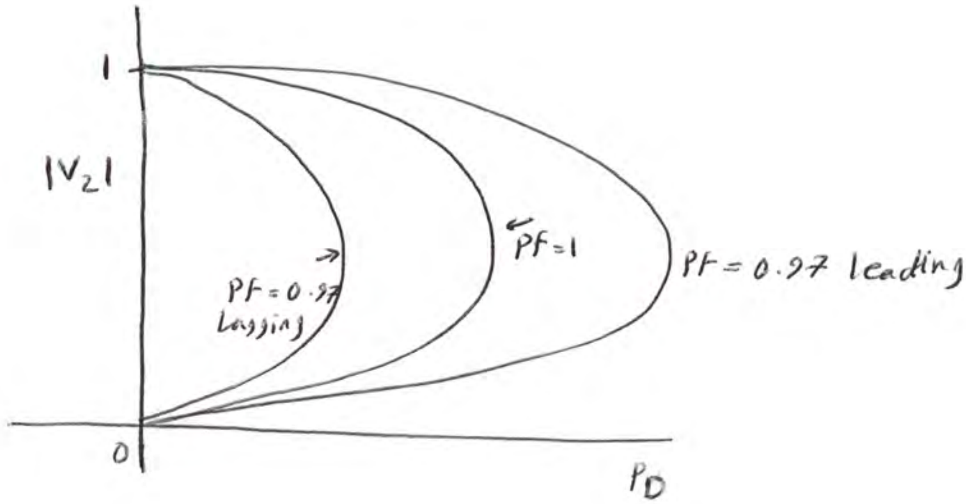
Eliminate θ_{12}: $\quad S_D = |V_2||I_2| e^{j\angle\phi} = P_D(1 + j\beta)$

where $\beta = \tan\phi$, $\phi = \angle S_D = \angle V_2 - \angle I_2$

, $\cos\phi$ = power factor

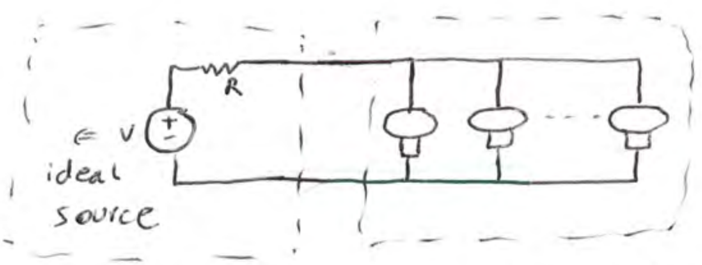
$$\left(\beta P_D + \frac{|V_2|^2}{X}\right)^2 = \left(\frac{|V_2||V_1|}{X}\right)^2 - P_D^2$$

- Assume $|V_1| = 1$ (per unit)



- Look at $|V_2|$ v.s. P_D: There is a point at which "voltage collapse" occurs.

- Interpretation: we want to increase a room brightness.



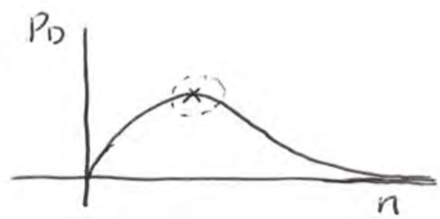
<u>DC case</u>

claim: Adding bulbs increases the brightness up to some point, then the room gets darker!

proof: model light bulb as r

$$P_D = \frac{V_D^2}{r} \times n \ , \quad r_e = \frac{r}{n} \quad \Rightarrow \quad V_D = \frac{r_e}{R + r_e} V = \frac{r}{Rn + r} V$$

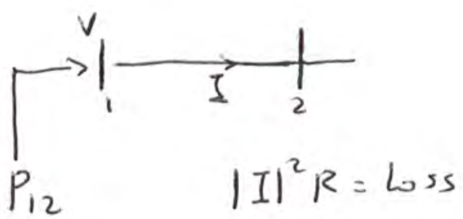
$$\Rightarrow P_D = \frac{V^2 r n}{(Rn + r)^2} \quad \Rightarrow$$



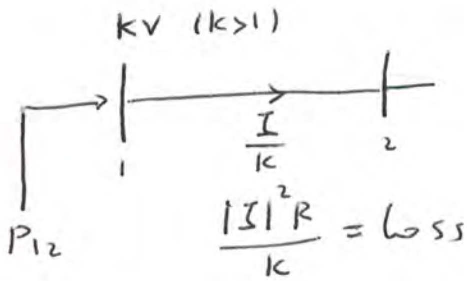
- Generation is typically at line voltages 11 - 30 kV
- Transmission over long distances requires high voltages, say 138, 230, 345, 765 kV.
- Distribution takes place at low voltages, say 2400 or 41600 V.
- Industrial load needs 440 V.
- Commercial or residential use needs 240/120 V.

Advantages of going to high voltage:

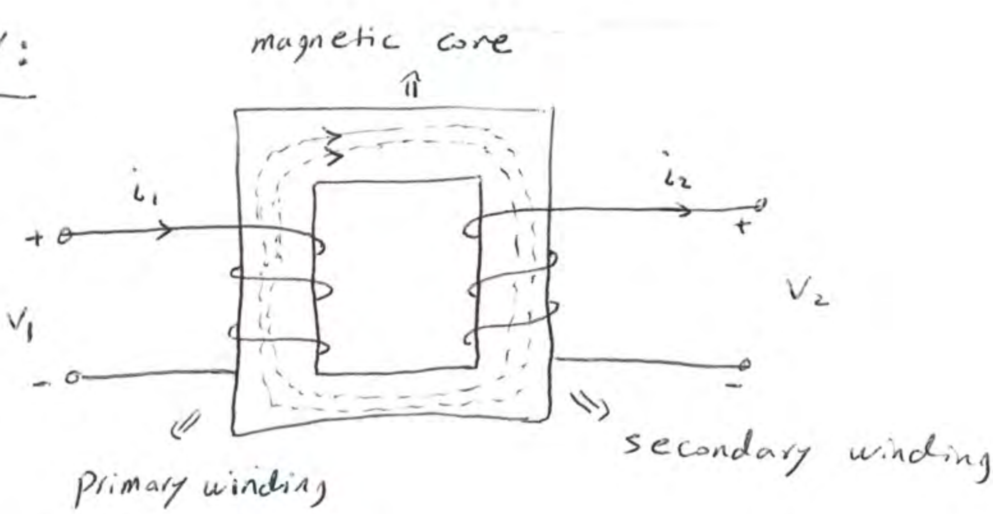
- $P_{12} = \dfrac{|V_1||V_2|}{X} \sin\theta_{12}$ $\Rightarrow$ transmission capacity increases
- Loss reduction:

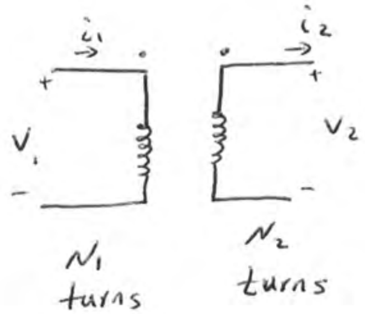


,



Transformer:



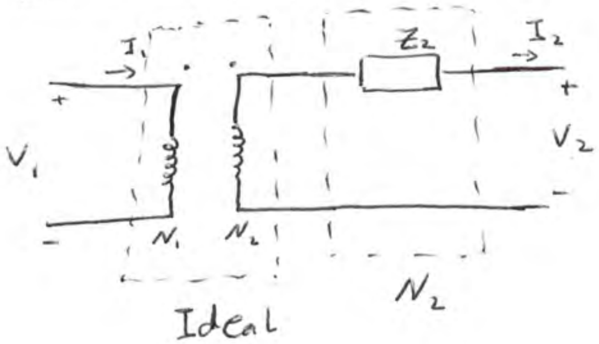


Ideal Trans :

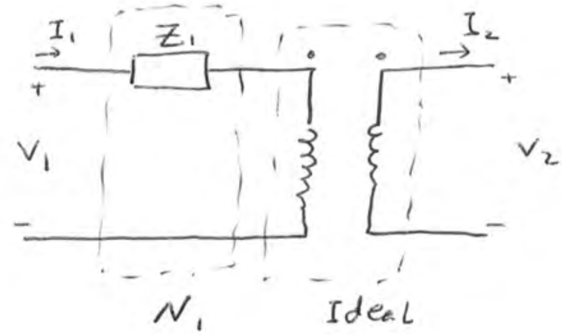
$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = n \quad , \quad \frac{i_2}{i_1} = \frac{1}{n} = a$$

$$\Rightarrow \quad i_1 V_1 = i_2 V_2 \quad \text{(lossless device)}$$

Equivalence:



$\simeq$

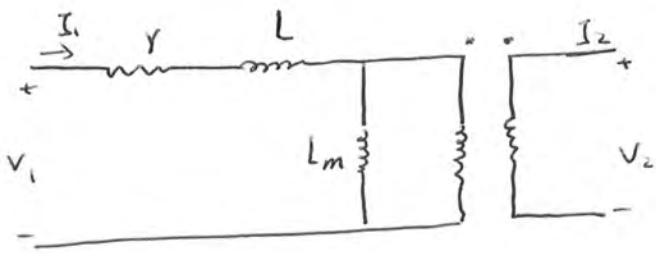


$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & \frac{1}{a} \end{bmatrix} \begin{bmatrix} 1 & Z_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \quad , \quad \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1 & Z_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & 0 \\ 0 & \frac{1}{a} \end{bmatrix}$$

$$\Rightarrow \quad Z_1 = a^2 Z_2$$

- True even for moving a shunt (not series) impedanc

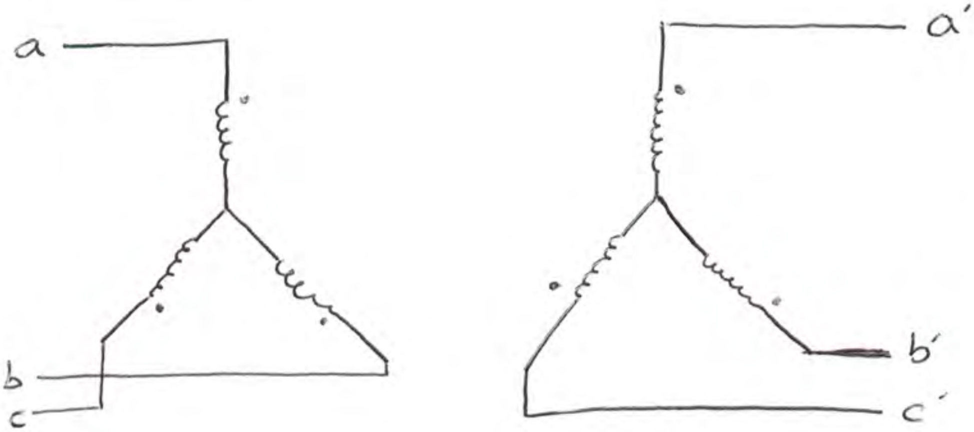
More realistic model:



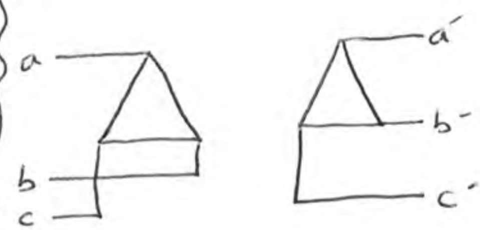
Most of the time, L_m is ignored.

- Various ways to have 3-ϕ transformers.

- Y-Y connection:



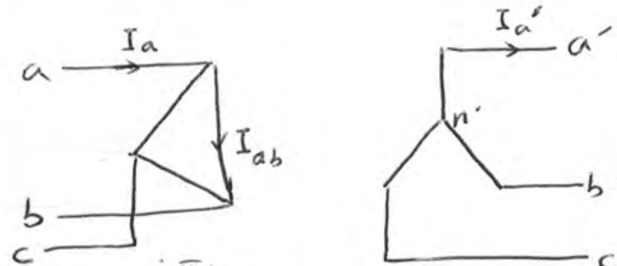
Δ-Δ connection:



- The above settings just change voltage levels.

- How to change phases?

- Consider Δ-Y connection:



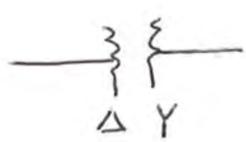
$$V_{a'n'} = n V_{ab} = n(V_{an} - V_{bn}) = \sqrt{3}\, n\, e^{j\pi/6} V_{an}$$

$$\Rightarrow \text{gain } k_1 = \sqrt{3}\, n\, e^{j\pi/6} \quad \text{(for every phase)}$$

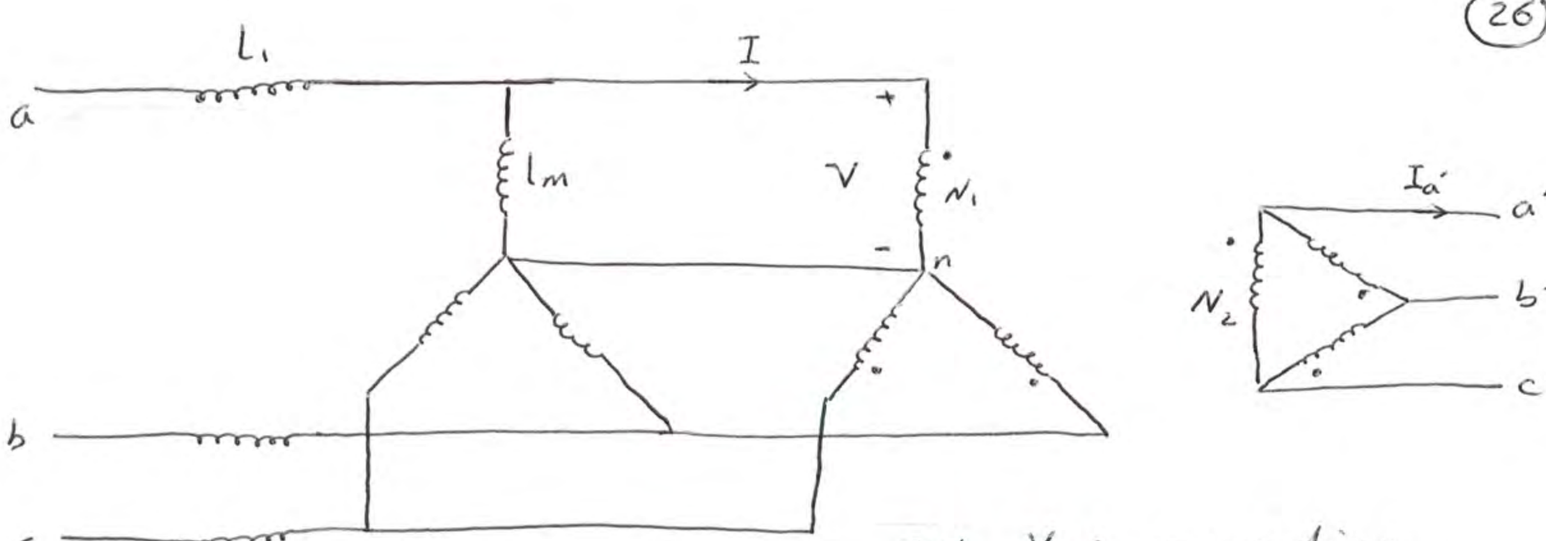
$$I_a = I_{ab} - I_{ca} = n(I_{a'} - I_{c'}) = \sqrt{3}\, n\, e^{-j\pi/6} I_{a'} = k_1^* I_{a'}$$

$$\Rightarrow S' = V_{a'n'} (I_{a'})^* = k_1 V_{an} \left(\frac{I_a}{k_1^*}\right)^* = V_{an} I_a^* = S$$

one-line diagram:

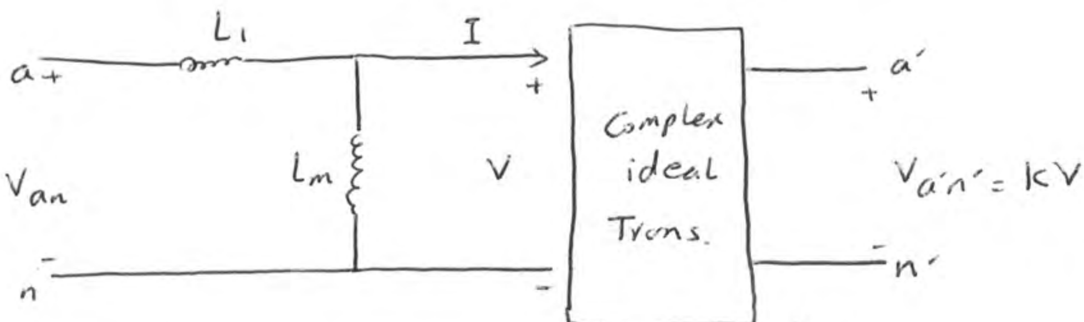


- per phase Analysis:



3ϕ Y-Δ connection

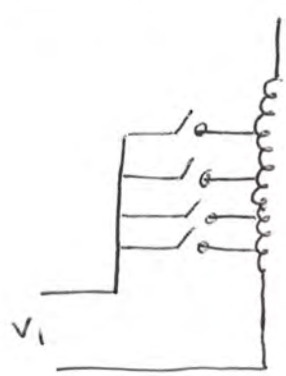
⇓ per phase circuit



$$k = \frac{n}{\sqrt{3}} e^{j\pi/6}$$

- Auto-transformer:

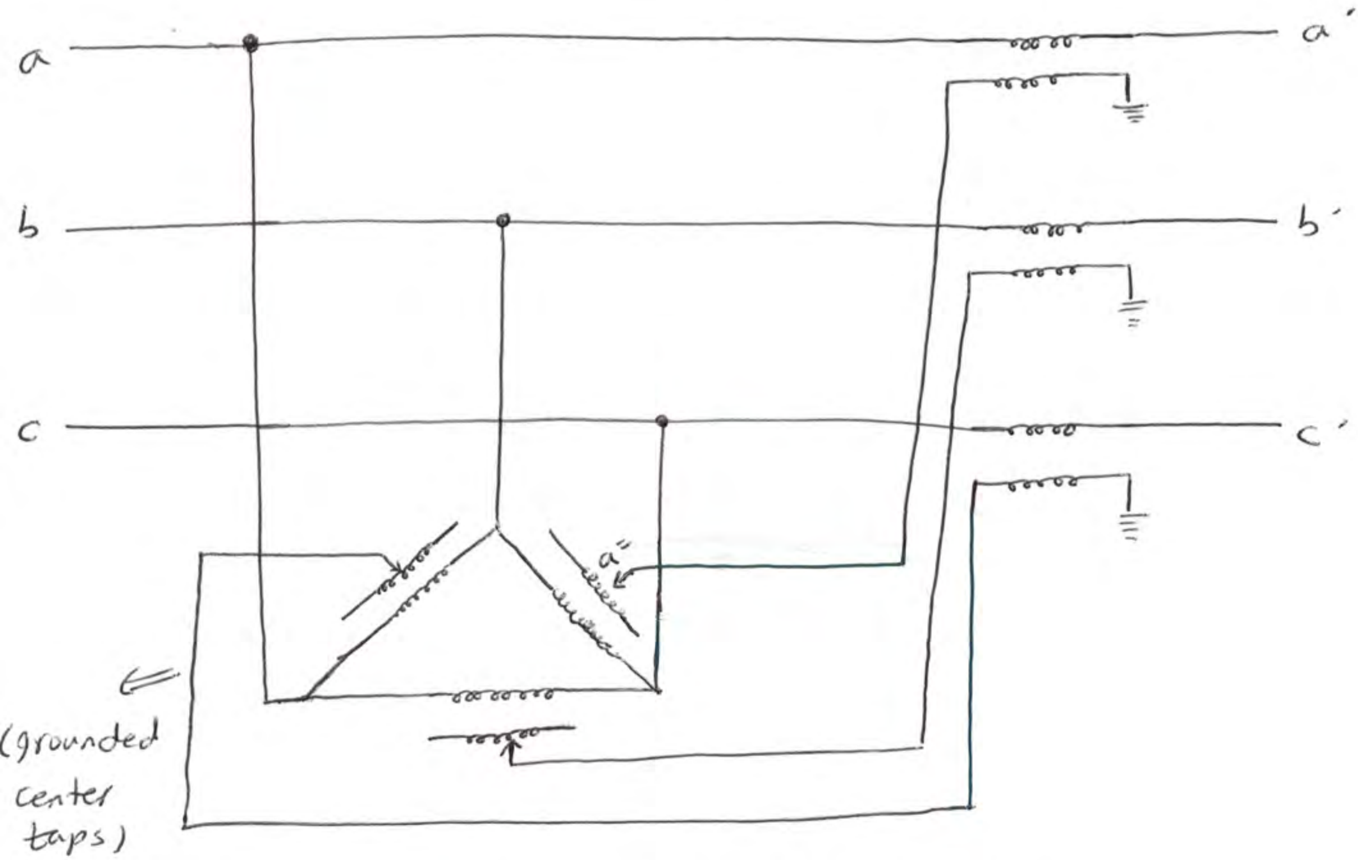
- Tap-changing Transformer:



- How to change phases ?

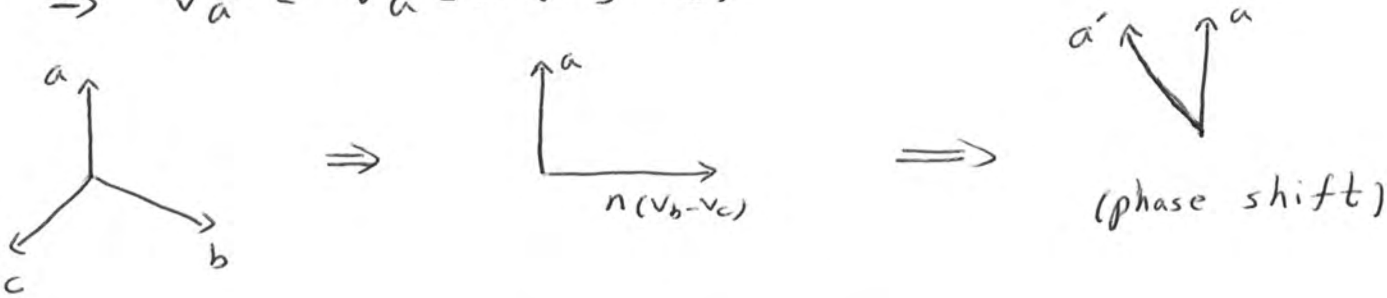
- phase-shifting Transformer:

(phase shifter: Transformer with unity gain)

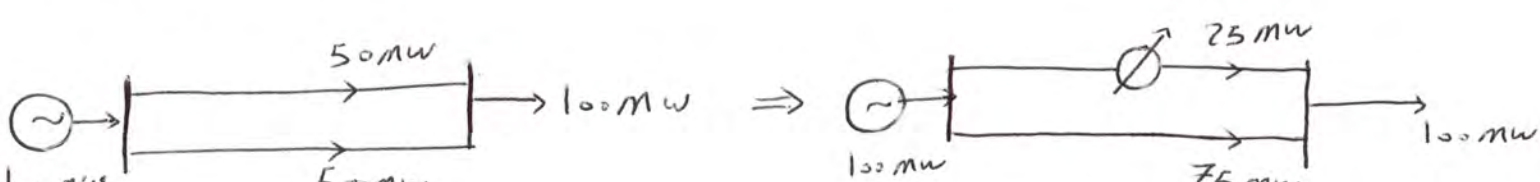


$$V_a - V_{a'} = n_2 V_{a''} = n_2 n_1 (V_b - V_c)$$

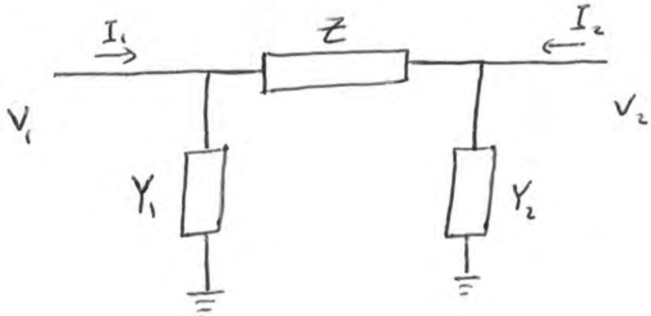
$$\Rightarrow \quad V_{a'} = V_a - n(V_b - V_c)$$



- Application: (Routing power)

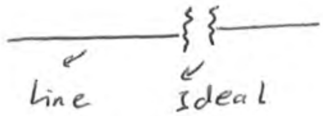


Transformer + Transmission Lines:

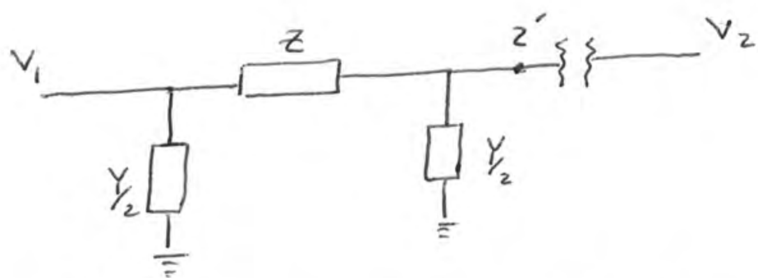


$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_1 + \frac{1}{Z} & -\frac{1}{Z} \\ -\frac{1}{Z} & Y_2 + \frac{1}{Z} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Consider:



$\Longrightarrow$

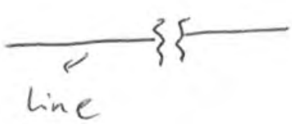


$$\begin{bmatrix} I_1 \\ I_2' \end{bmatrix} = \begin{bmatrix} \frac{Y}{2} + \frac{1}{Z} & -\frac{1}{Z} \\ -\frac{1}{Z} & \frac{Y}{2} + \frac{1}{Z} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2' \end{bmatrix}, \quad V_2' = a V_2, \quad I_2' = \frac{1}{a^*} I_2$$

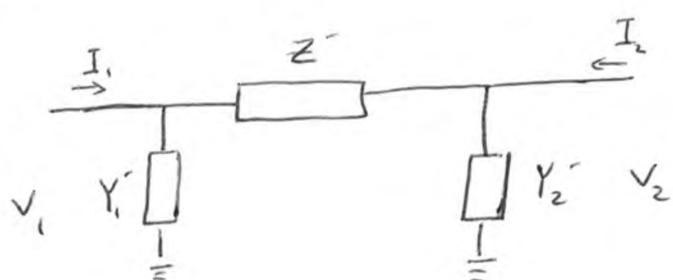
$$\Rightarrow \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{Y}{2} + \frac{1}{Z} & -\frac{a}{Z} \\ -\frac{a^*}{Z} & \left(\frac{Y}{2} + \frac{1}{Z}\right) a a^* \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$B = \frac{Y}{j}$ is called Line charging.

Assume $a = a^*$ (no phase shifter):



$\Longleftrightarrow$



$$Z' = \frac{Z}{a}, \quad Y_1' = \frac{Y}{2} + \frac{1-a}{Z}, \quad Y_2' = \frac{a^2 Y}{2} + \frac{a(a-1)}{Z}$$

- If $a \neq 1$, we are adding a negative shunt resistance