- what happens if $|V_1| = |V_2|$ (thru control from outside world).

$$P_1 = P_2 \,, \quad Q_2 = Q_1 - \omega L |I|^2$$

$$S_1 = V_1 I^* \,, \quad S_2 = V_2 I^* \quad \underset{|V_1| = |V_2|}{\Longrightarrow} \quad |S_1| = |S_2|$$

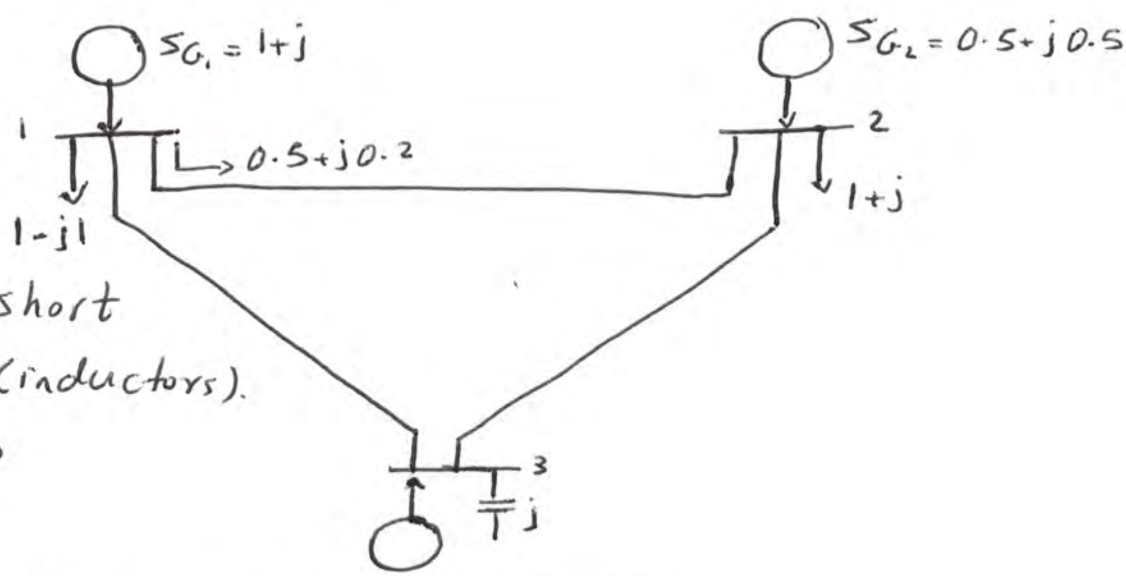
$$\Rightarrow P_1^2 + Q_1^2 = P_2^2 + Q_2^2 \quad \Rightarrow \quad Q_2 = -Q_1$$

$$\Rightarrow \boxed{S_2 = S_1^*}$$

Symbols:

- Generator: ○→
- Bus |
- Load (constant power) ←
- Transmission Line ——

Example:



- Lines are short (inductors).
- Find S_{G_3} ?

$$S_{13} = S_{G_1} - (1-j) - (0.5+j0.2) = -0.5 + j1.8$$

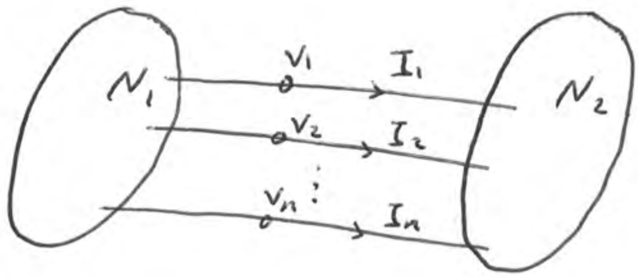
$$S_{31} = -S_{13}^* = 0.5 + j1.8$$

$$S_{23} = S_{G_2} - (1+j) - (-S_{12}^*) = -j\,0.7$$

$$S_{32} = -S_{23}^* = -j\,0.7$$

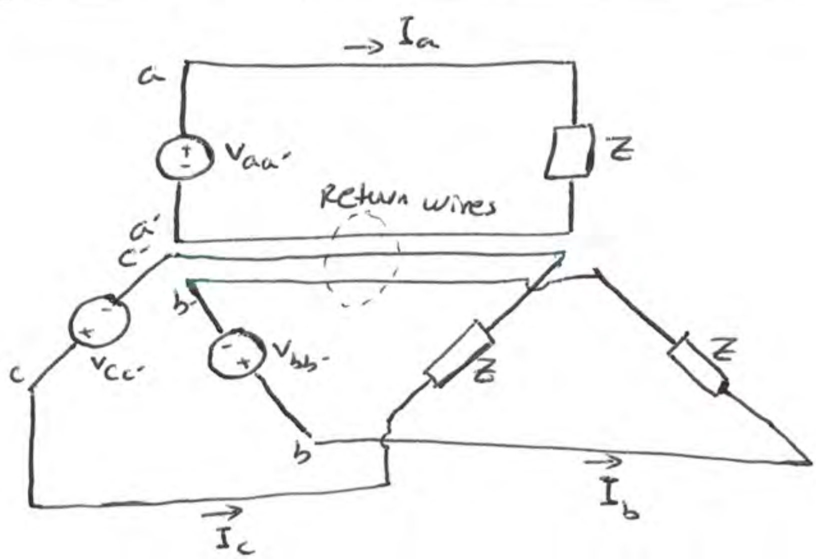
$$\Rightarrow \quad S_{G_3} = S_{32} + S_{12} - j = 0.5 + j\,0.1$$

- Power Transfer:



power transferred from N_1 to N_2 $= S = \sum V_i I_i^*$

Balanced Three-phase Network:



- Connect a, b, c
- Connect a', b', c'

Balanced 3ϕ: - $|V_{aa'}| = |V_{bb'}| = |V_{cc'}|$

- Angle difference is ± 120 (for every two phases)

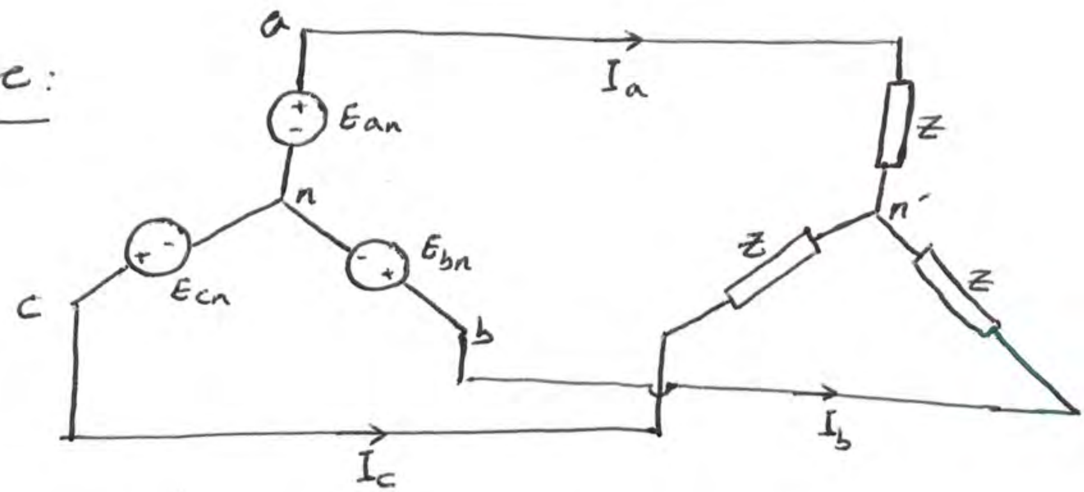
Then: $I_a + I_b + I_c = 0 \Rightarrow$ No current on return wire

Remove the return wires:

- Saving in wiring
- Less loss ($Loss = |I|^2 r$)

Balanced 3ϕ = balanced sources + balanced Loads (Z)

Example:



Calculate $V_{nn'}$?
(neutral-neutral voltage)

$$I_a = \frac{E_{an} + V_{nn'}}{Z}$$

$$I_b = \frac{E_{bn} + V_{nn'}}{Z}, \quad I_c = \frac{E_{cn} + V_{nn'}}{Z}$$

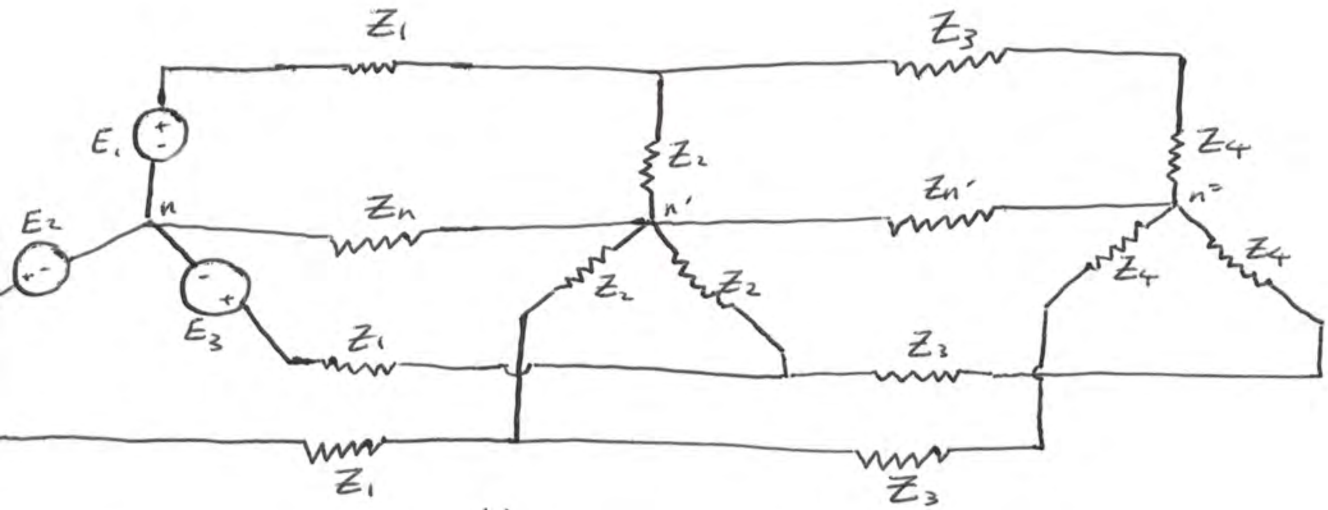
$I_a + I_b + I_c = 0$, $E_{an} + E_{bn} + E_c = 0$ (why ?)

$$\Rightarrow \frac{3 V_{nn'}}{Z} = 0 \quad \Rightarrow V_{nn'} = 0$$

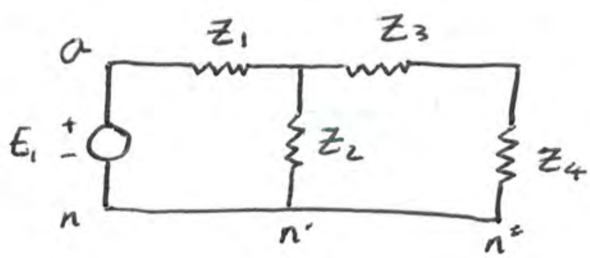
- what if $\underline{\underline{n}}$ is connected to $\underline{\underline{n'}}$ via an impedance Z_n ?

$$I_{nn'} = \frac{V_{nn'}}{Z_n} \quad \Rightarrow \left(\frac{3}{Z} + \frac{1}{Z_n} \right) V_{nn'} = 0$$

$\Rightarrow$ If $\frac{3}{Z} + \frac{1}{Z_n} \neq 0$, then $V_{nn'} = 0$

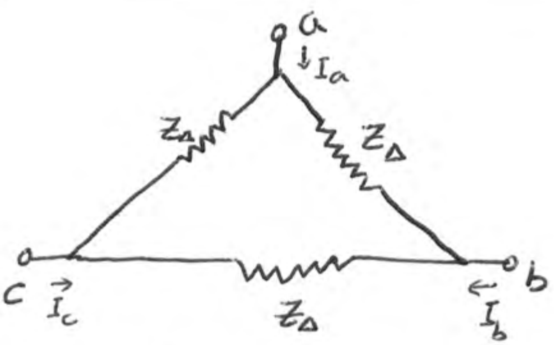


$\Downarrow$



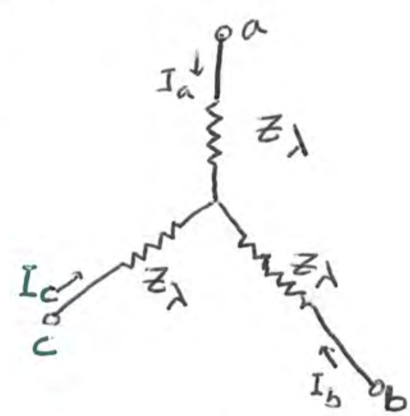
per phase analysis

Delta-wye Load Transformation:



Δ-Y Trans.

$\Longleftrightarrow$



$$I_a = \frac{V_{ab}}{Z_\Delta} + \frac{V_{ac}}{Z_\Delta} = \frac{V_{ab}+V_{ac}}{Z_\Delta}$$

$$V_{ab} = Z_\lambda (I_a - I_b)$$

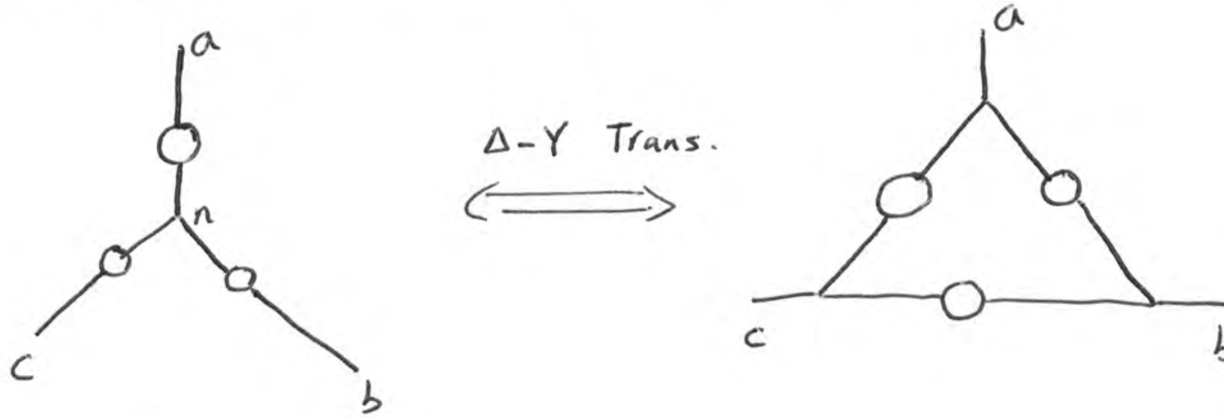
$$V_{ac} = Z_\lambda (I_a - I_c)$$

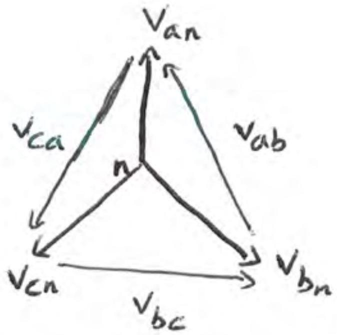
$$\Rightarrow V_{ab} + V_{ac} = Z_\lambda (I_a + 2 I_a)$$

$\Downarrow$

$$Z_\Delta = 3 Z_\lambda$$

Delta-wye source Transformation:





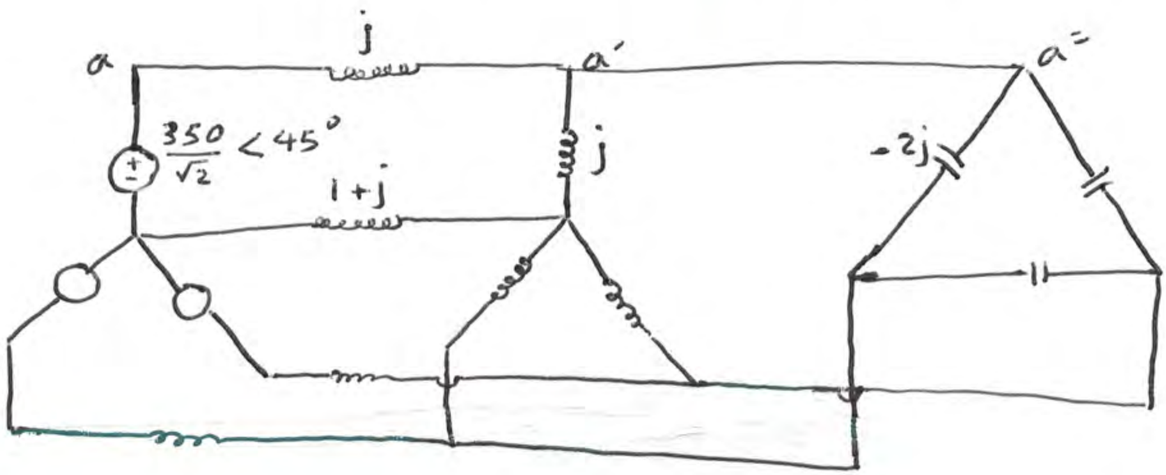
$\Rightarrow \quad V_{ab} = \sqrt{3}\, V_{an}\, e^{j\pi/6}$

V_{ab} = line-line voltage
= line voltage

V_{an} = line-neutral voltage
= phase voltage

$\Rightarrow$ |line voltage| = $\sqrt{3}$ |phase voltage|

Example:



per phase

$\Longrightarrow$

3ϕ power:

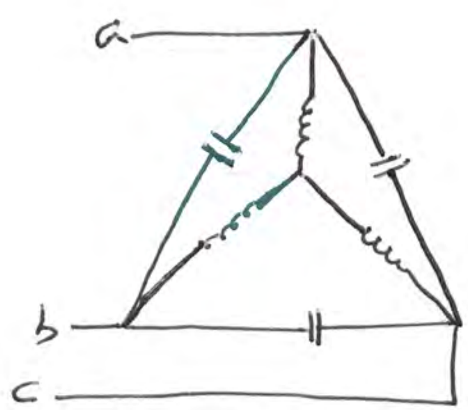
$$S_{3\phi} = V_a I_a^* + V_b I_b^* + V_c I_c^*$$
$$= V_a I_a^* + V_a e^{-j2\pi/3} I_a^* e^{j2\pi/3} + V_a e^{j2\pi/3} I_a^* e^{-2\pi/3}$$
$$= 3 V_a I_a^* = 3S$$

$$P_{3\phi}(t) = V_a(t)\, i_a(t) + V_b(t)\, i_b(t) + V_c(t)\, i_c(t)$$
$$= |V||I| \Big(\cos\phi + \cos(2wt + \measuredangle V + \measuredangle I)$$
$$+ \cos\phi + \cos(2wt + \measuredangle V + \measuredangle I - \tfrac{4\pi}{3})$$
$$+ \cos\phi + \cos(2wt + \measuredangle V + \measuredangle I + \tfrac{4\pi}{3})\Big)$$
$$= 3|V||I|\cos\phi = 3P = \underline{\text{constant}}$$

$\Rightarrow$ Instantaneous power delivered to load is constant.

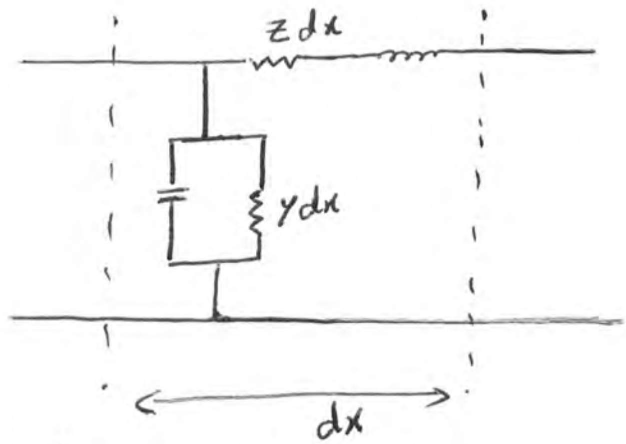
Application in ac motor: constant speed

Question: what to do with this ?



Transmission Lines:

Distributed model:



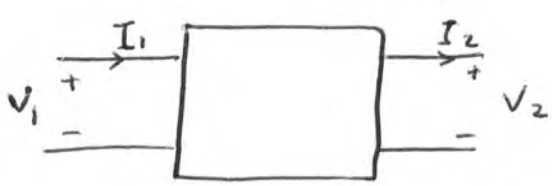
$z = r + j\omega L$ = series impedance per meter

$y = g + j\omega c$ = shunt admittance per meter (to neutral)

$\gamma = \sqrt{yz}$ = propagation constant

$Z_c = \sqrt{z/y}$ = characteristic impedance

m = lengh (meter)



lumped model $\Longrightarrow$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} \cosh \gamma m & Z_c \sinh \gamma m \\ \dfrac{\sinh \gamma m}{Z_c} & \cosh \gamma m \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

Example:

- A 60 Hz 138 kV 3ϕ line is 225 mi long
- Distributed line parameters are:

$r = 0.169\ \Omega/mi$, $L = 2.093\ mH/mi$, $C = 0.014\ \mu F/mi$, $g =$

- It delivers 40 MW at 132 kV with 0.95 power fac lagging
- Find efficiency.

$w = 2\pi f = 2\pi \cdot 60$

$Z = 0.169 + j\,0.789$, $y = j\,5.38 \times 10^{-6}$

$Z_c = \sqrt{Z/y}$, $m\gamma = m\sqrt{Zy}$

$|V_2| = 132 \times 10^3/\sqrt{3} = 76.2$ KV

$P_{Load} = 40/3 = 0.95\,|V_2|\,|I_2|$, $\angle I_2 = -\cos^{-1} 0.95$

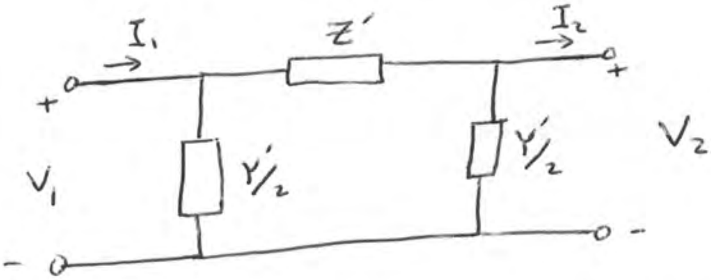
per phase

Given V_2, I_2, we can find V_1, I_1.

$P_{12} = Re\, V_1 I_1^* = 14.45$ MW

$$\Rightarrow \eta = \frac{13.33}{14.45} = 0.92 \Rightarrow \text{Efficiency} = 92\%$$

- lumped circuit Equivalent: (π model)



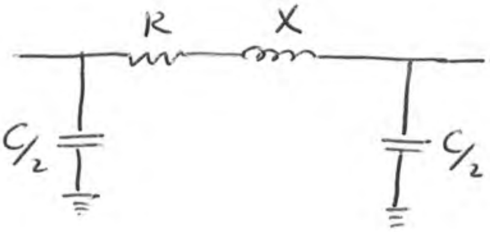
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$V_1 = V_2 + Z'\left(I_2 + \frac{V_2 Y'}{2}\right) = \underbrace{\left(1 + \frac{Z'Y'}{2}\right)}_{A} V_2 + \underbrace{Z'}_{B} I_2$$

$$I_1 = \frac{Y' V_1}{2} + I_2 + V_2 \frac{Y'}{2} = \underbrace{Y'\left(1 + \frac{Z'Y'}{4}\right)}_{C} V_2 + \underbrace{\left(1 + \frac{Z'Y'}{2}\right)}_{D} I_2$$

- π-equivalent model is commonly used.

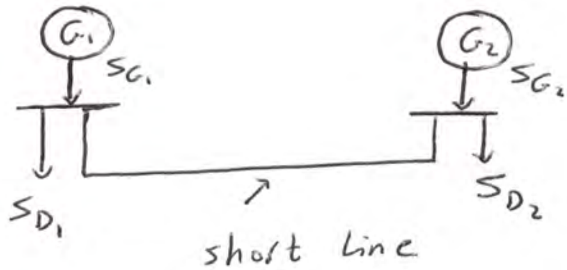
- long line (m > 150 mi) : Use exact π model
- medium line (50 < m < 150) :

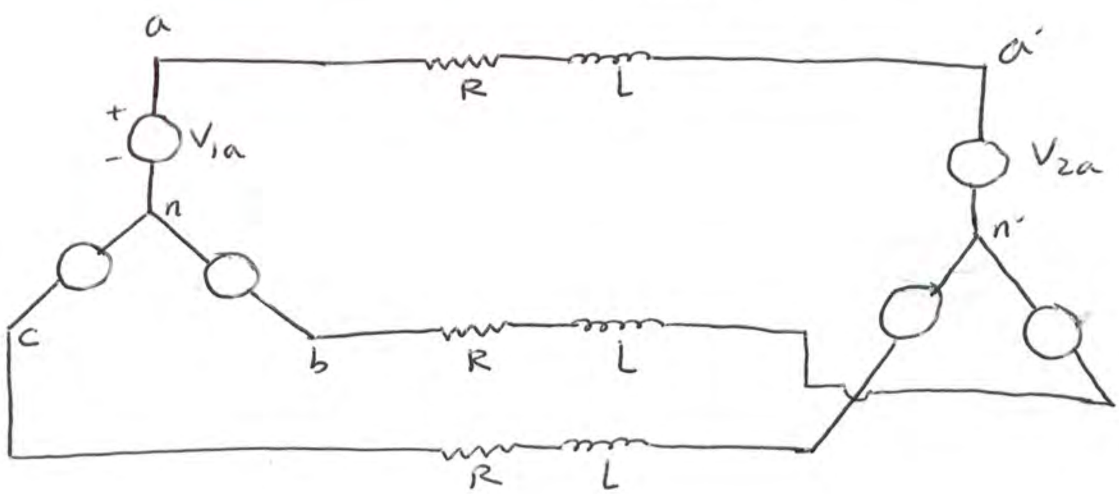


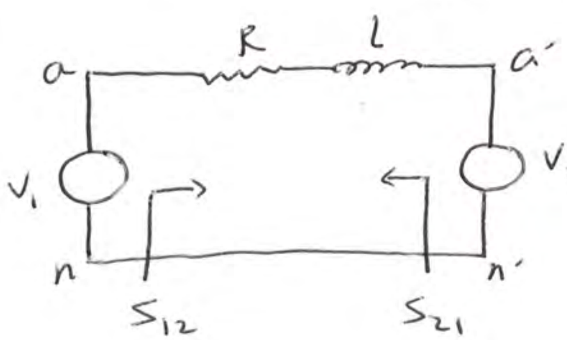
C: total capacitance
R: total resistance
X: total inductance

- short line (m < 50) : Remove $\underline{\underline{C}}$.

Example:







$\theta_{12} = \theta_1 - \theta_2 \;\; (\measuredangle V_1 - \measuredangle V_2)$

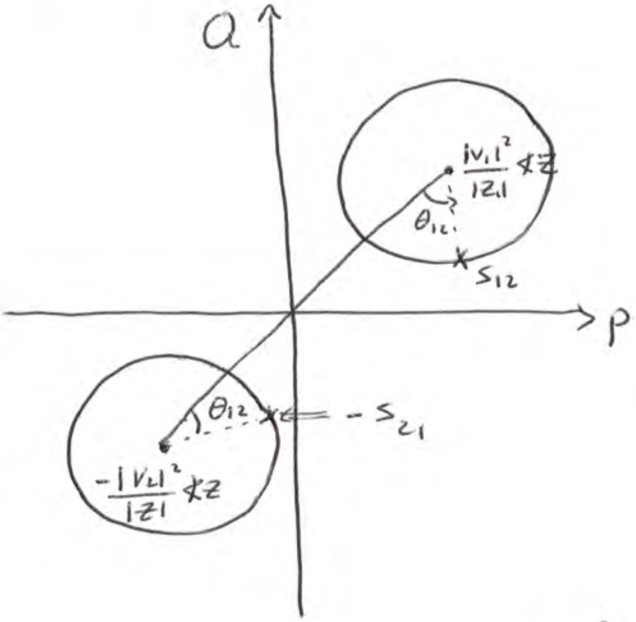
$$\Rightarrow S_{12} = V_1 I_1^* = V_1 \left(\frac{V_1 - V_2}{Z} \right)^*$$

$$= \frac{|V_1|^2}{|Z|} e^{j\measuredangle Z} - \frac{|V_1||V_2|}{|Z|} e^{j(\measuredangle Z + \theta_{12})}$$

and $$S_{21} = \frac{|V_2|^2}{|Z|} e^{j\measuredangle Z} - \frac{|V_2||V_1|}{|Z|} e^{j(\measuredangle Z - \theta_{12})}$$

- what's $S_{12} + S_{21}$? Transmission loss

- In the P-Q plane, draw S_{12} and $-S_{21}$:



- If $|V_1| \neq |V_2|$, then the circles do not intersect:

$$\frac{|V_1|^2 + |V_2|^2}{|Z|} > \frac{|V_1||V_2|}{|Z|} \times 2$$

- As θ_{12} increases from zero, active power send/received increases.

- Max active power received happens at $\theta_{12} = \measuredangle Z$

- max active power sent happens at $\theta_{12} = 180^\circ - \measuredangle Z$

- $R = 0$ and $Z = jX$: $P_{12} = -P_{21} = \frac{|V_1||V_2|}{X} \sin\theta_{12}$

$$Q_{12} = \frac{|V_1|^2}{X} - \frac{|V_1||V_2|}{X} \cos\theta_{12}$$

$$Q_{21} = \frac{|V_2|^2}{X} - \frac{|V_1||V_2|}{X} \cos\theta_{12}$$

$\Rightarrow$ ultimate transmission capacity $= \frac{|V_1||V_2|}{X}$

- ways to increase capacity:

1- Increase voltage level

2- Decrease X (series compensation)