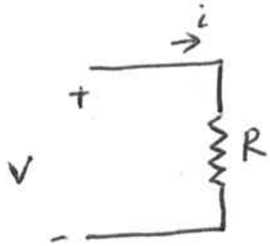
- i : Electric current (flow of electric charge)

- V : Voltage (electrical potential $\underbrace{\text{difference}}$)
  $\Downarrow$
  (needs a reference)

Electrical Components:

1- Resistor:

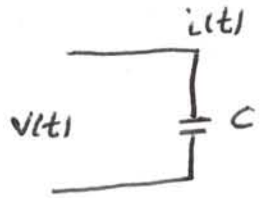


$V = iR \rightarrow$ ohm (Ω)

Volts (V) Amp (A)

R = Resistance, $G = \frac{1}{R}$ = Conductance

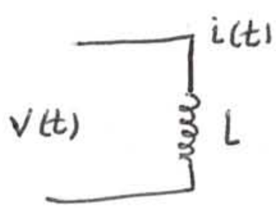
2- Capacitor:



$i(t) = C \frac{dv(t)}{dt}$

Farads (F)

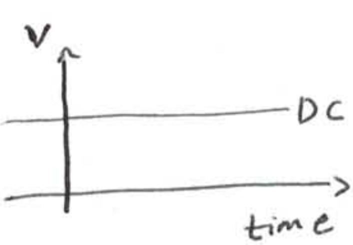
3- Inductor:

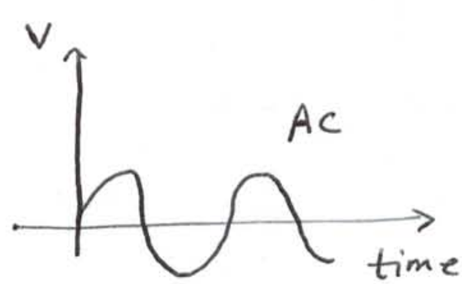


$V(t) = L \frac{di(t)}{dt}$

Henry (H)

DC versus AC:

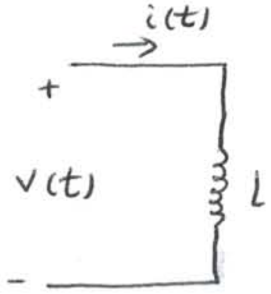




f : frequency (Hertz or Hz)

ω : angular $\underline{\underline{f}}$
$= 2\pi f$

Example:



$v(t) = V \sin(wt)$

$\Rightarrow i(t) =$ Vanishing term + sinusoidal term

- How to get the steady-state term:
  - Use Fourier transform

$v(t) \longrightarrow V, \quad i(t) \longrightarrow I$

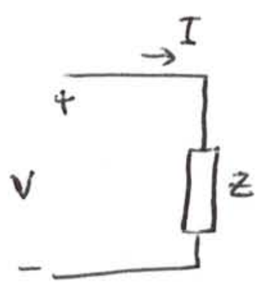
$\Rightarrow$

1 - Resistor $\quad V = IR$

2 - Inductor $\quad V = (2\pi f j) L I = (wjL) I$

3 - Capacitor $\quad I = (2\pi f j) C V = (wjC) V$

Impedance:



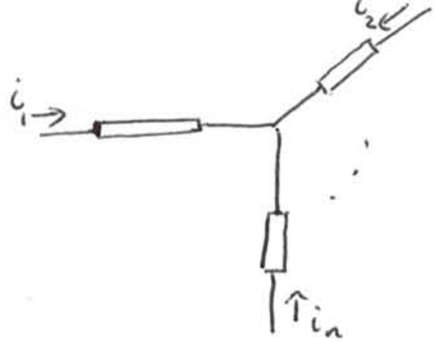
$Z = R + jX$

resistance, reactance

$V = ZI$

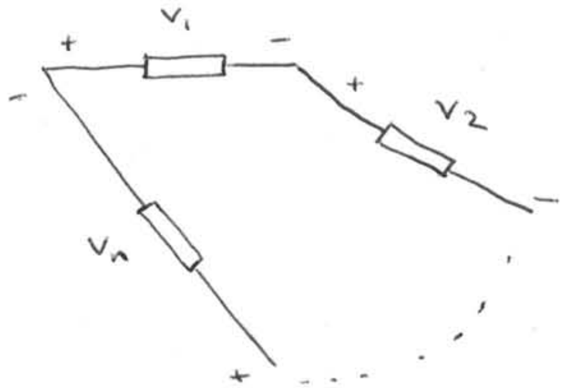
$Z_R = R, \quad Z_L = jwL, \quad Z_C = \frac{1}{jwC}$

Kirchhoff's Laws: KCL, KVL



KCL: $i_1 + i_2 + \cdots + i_n = 0$

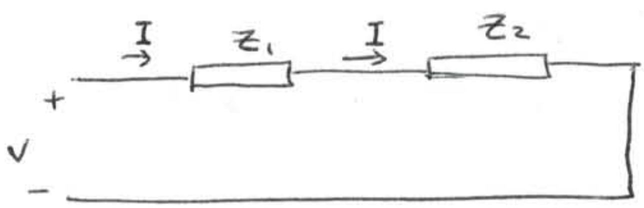
(conservation of charge)



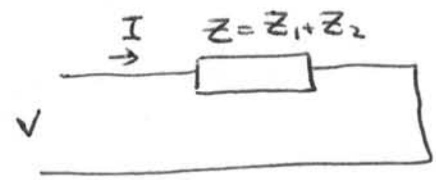
KVL: $V_1 + V_2 + \cdots + V_n = 0$

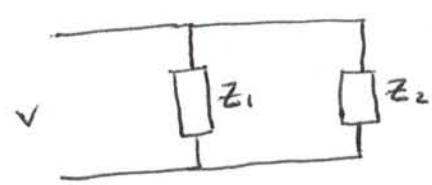
(Conservation of energy)

- Equivalent Impedance:

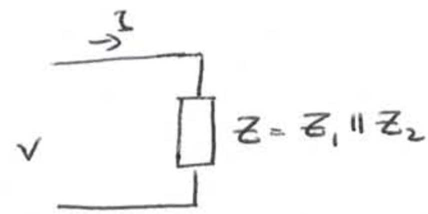


$\simeq$





$\simeq$

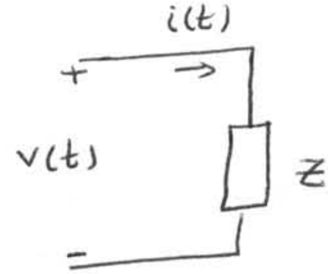


Total current $= \dfrac{V}{Z_1} + \dfrac{V}{Z_2} = \dfrac{V}{Z} \Rightarrow \dfrac{1}{Z} = \dfrac{1}{Z_1} + \dfrac{1}{Z_2}$

Admittance:

$$Y = \frac{1}{Z} = \frac{1}{R + Xj} = \underbrace{G}_{\text{conductance}} + j\,\underbrace{B}_{\text{susceptance}}$$

- Instantaneous power:



$$p(t) = v(t)\, i(t)$$

Assume that:

$$v(t) = V_{max} \cos(\omega t + \theta_V)$$
$$i(t) = I_{max} \cos(\omega t + \theta_I)$$

$$\Rightarrow p(t) = \frac{1}{2} V_{max} I_{max} \left(\cos(\theta_V - \theta_I) + \cos(2\omega t + \theta_V + \theta_I) \right)$$

Note that:

- First term is constant.
- second term has zero average.

- phasor Representation:

$$v(t) = V_{max} \, Re\left(e^{j\omega t} e^{j\theta_V} \right) \Longrightarrow \underset{\text{phasor}}{V} = \frac{V_{max} e^{j\theta_V}}{\sqrt{2}}$$

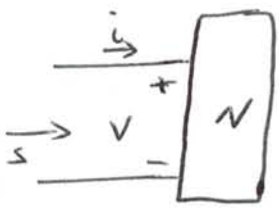
$$\Rightarrow V = |V| \cos\theta_V + j|V| \sin\theta_V$$

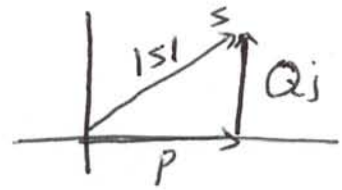
- Active power: $P = \frac{1}{T} \int_0^T p(t)\, dt = \frac{1}{2} V_{max} I_{max} \cos\varphi$

$$= Re(V I^*)$$

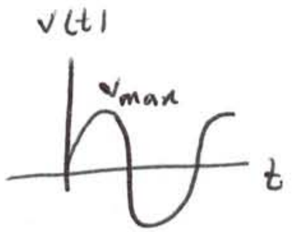
- Reactive power: $Q = Im(V I^*)$

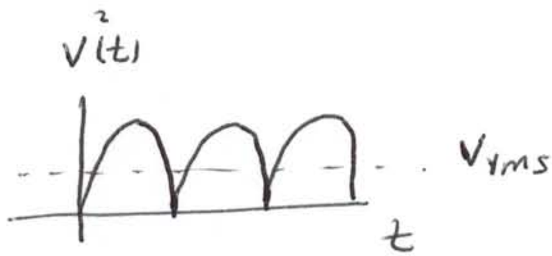
- Complex power: $S = V I^* = P + Qi$
- Apparent power: $|S| = \sqrt{P^2 + Q^2}$
- Power Factor: $\cos\varphi = \cos(\measuredangle S)$





why $\sqrt{2}$?





$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v(t)^2\,dt} = \frac{V_{max}}{\sqrt{2}}$$

<u>Units:</u>

P: watts (W, kW, MW)

Q: Voltamperes reactive (VAr, kVAr, MVAr)

S: Volt amperes (VA, kVA, MVA)

|S|: volt amperes (VA, kVA, MVA)

Examples:

- Resistor: $S = V I^* = (Z I) I^* = |I|^2 R$

$$\Rightarrow P = |I|^2 R, \quad Q = 0$$

- Inductor: $S = (jwL\,I)\,I^* = jwL\,|I|^2$

$$\Rightarrow P = 0, \quad Q = wL\,|I|^2$$

- Capacitor: $S = \left(\frac{I}{jwC}\right) I^* = -\frac{1}{wC}\,j\,|I|^2$

$$\Rightarrow P = 0, \quad Q = \frac{|I|^2}{wC}$$

Note that:
- Resistor consumes active power (converts to heat)
- Inductor consumes reactive power
- capacitor generates reactive power

Connection between $p(t)$ and P/Q ?

- Inductor: Assume that $i(t) = \sqrt{2}\,|I|\cos(wt+\theta)$

$$\Rightarrow v(t) = L\frac{di(t)}{dt} = -\sqrt{2}\,w\,L\,|I|\sin(wt+\theta)$$

$$\Rightarrow p(t) = v(t)\,i(t) = -wL\,|I|^2\sin(2wt+2\theta)$$

$$= -Q\sin 2(w+\theta)$$

Reactive power: goes back and forth in the network

Max magnitude: Q

- General case: $i(t) = \sqrt{2}\,|I| \cos\omega t$

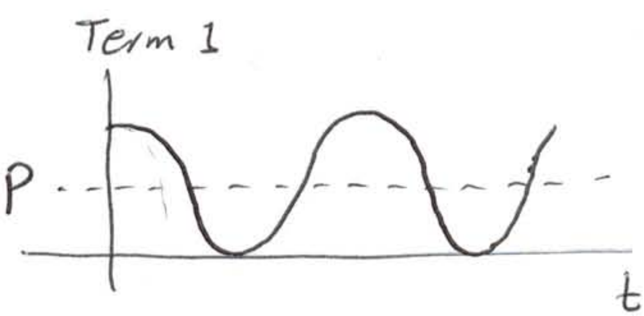
$$\Rightarrow p(t) = v(t)\, i(t) = \left[\sqrt{2}\,|I|\,|Z| \cos(\omega t + \measuredangle Z)\right] i(t)$$

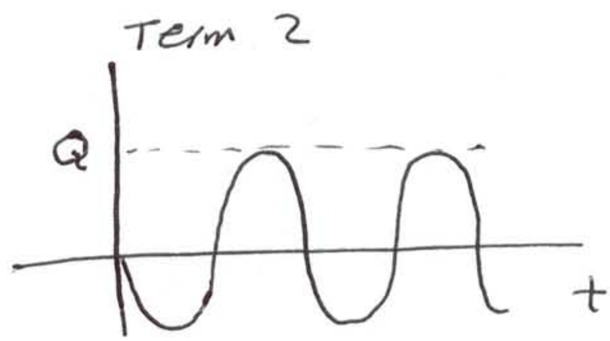
$$= |Z|\,|I|^2 \left(\cos(\measuredangle Z) + \cos(2\omega t + \measuredangle Z) \right)$$

$$= |Z|\,|I|^2 \left(\cos(\measuredangle Z) + \cos(2\omega t) \cos\measuredangle Z - \sin(2\omega t) \sin\measuredangle Z \right)$$

$$= P(1 + \cos 2\omega t) - Q \sin 2\omega t$$

$$= \text{Term } 1 + \text{Term } 2$$





- Two ways to get S:

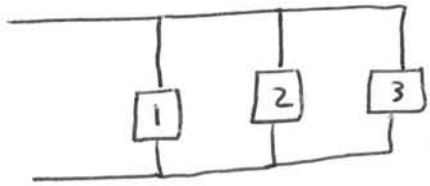
1. $S = V I^* = (Z I) I^* = |I|^2 Z$

$$\Rightarrow P = R|I|^2, \quad Q = X|I|^2$$

2. $S = V I^* = (V)\left(\frac{V}{Z}\right)^* = \frac{|V|^2}{Z^*}$

$$\Rightarrow P = G|V|^2, \quad Q = B|V|^2$$

- Example:



- Three Loads are connected across a 480 V (rms) Line
- Load 1 absorbs 12 KW and 6.667 KVAr.
- Load 2 absorbs 4 KVA at 0.96 PF leading. → ($\underline{I}$ leads $\underline{V}$)
- load 3 absorbs 15 kw at unity power factor.
- Find equivalent impedance ?

$$S_1 = 12 + j\,6.667 \text{ KVA}$$

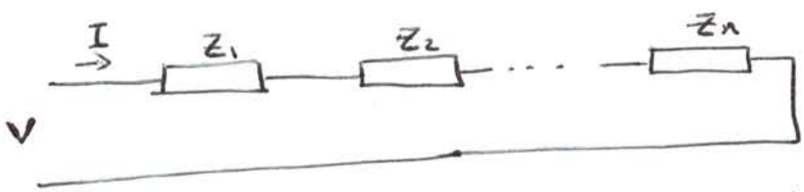
$$S_2 = 4\,(0.96 - j\sin(\cos^{-1}0.96)) \text{ KVA}$$

$$S_3 = 15\,(1 + j0) \text{ KVA}$$

$$S_i = \frac{|V|^2}{Z_i^*} \Rightarrow \text{Find } Z_i \Rightarrow \frac{1}{Z} = \sum \frac{1}{Z_i}$$

Conservation of Energy:

1:

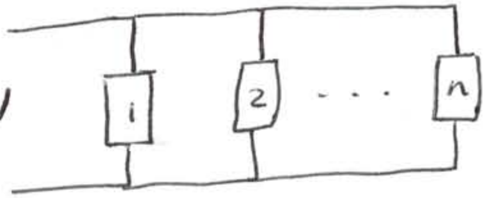


$$S_i = Z_i |I|^2 \Rightarrow \sum S_i = (\sum Z_i)|I|^2$$

$$= (\sum Z_i)\, I\, I^*$$

$$= V I^* = S$$

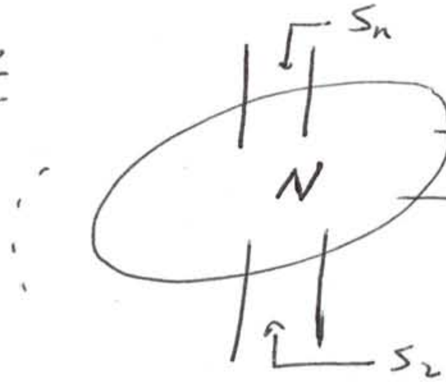
2: V



$$S_i = V I_i^* = \frac{|V|^2}{Z_i^*}$$

$$S = VI^* = V\left(\sum I_i\right)^* = \sum V I_i^* = \sum S_i$$

3: General case

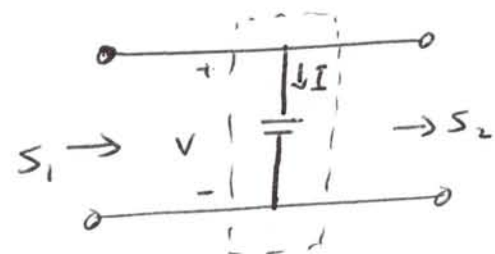


$$\boxed{\sum S_i = 0}$$

N: passive network (say R, L, C, …)

(Active network generates active power)

Example:



$$S_2 - S_1 = -j\omega C|V|^2$$

$$\Rightarrow \quad P_2 = P_1, \quad Q_2 = Q_1 + \omega C|V|^2$$

- Capacitor (bank) at a bus injects reactive power into the network.

Example:

$$S_2 - S_1 = j\omega L|I|^2$$

- short transmission line consumes reactive power.