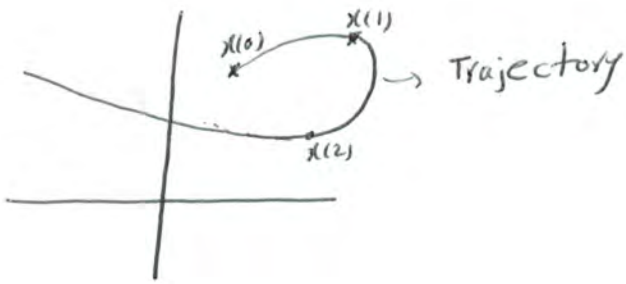
Dynamical systems:

$$\dot{x}_1 = x_1 - x_2 + 1 \qquad \dot{x}_2 = x_1^2 + x_2 \quad \longrightarrow \quad \dot{X} = f(X), \text{ given } x(0) \Rightarrow x(t) = \int_0^t f(x(\tau))\,d\tau + x(0)$$

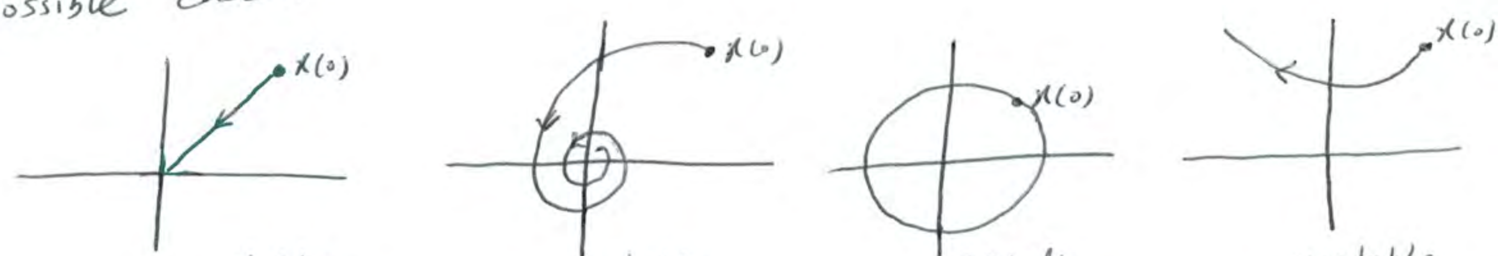


- X^* is an equilibrium if $f(X^*) = 0$
- By starting from x^*, the trajectory doesn't go anywhere

- Let's start with linear systems:

$\dot{X} = AX, \quad \Rightarrow$ Eq. point: $X = 0$

possible cases:



stable | stable | periodic (stable) | unstable

- Asymptotic global stability: $\forall x(0) \Rightarrow x(t) \longrightarrow 0$, stable $\Longleftrightarrow Re(\lambda(A)) < 0 \quad \forall \lambda(A)$
- stability $\Longleftrightarrow Re(\lambda(A)) \leq 0$
- Instability $\Longleftrightarrow \exists\, \lambda(A) : Re(\lambda(A)) > 0$

Another view: $\dot{X} = AX, \; X(0) = X_0 \Rightarrow X(t) = e^{At} X(0) = P e^{\Lambda t} P^{-1} x(0)$

<u>Lyapunov's method:</u>

Find an energy function $V(x)$:
- $V(x) \geq 0$, equality if $x = 0$
- $\dot{V}(x) = \frac{dV(x)}{dt} \leq 0$, equality if $x = 0$

- Consider $V(x) = x^T P x$

$V(x) \geq 0 \quad \forall x \quad \Longrightarrow \quad P > 0$

$\dot V(x) = \dot x^T P x + x^T P \dot x = x^T (A^T P + PA) x \leq 0 \quad \forall x \quad \Rightarrow A^T P + PA < 0$

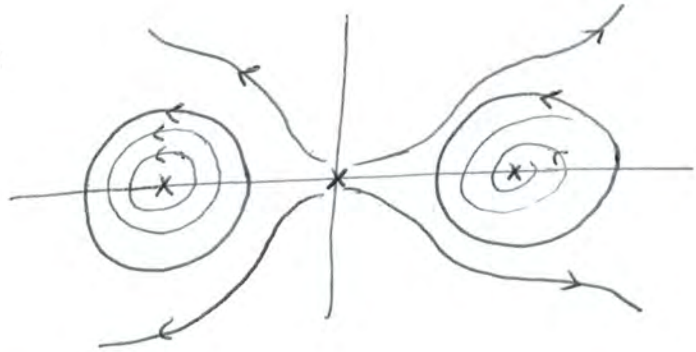
- Theorem: $\dot x = Ax$ stable iff the LMI

$\begin{cases} P > 0 \\ A^T P + PA < 0 \end{cases}$ is feasible

<u>Nonlinear systems:</u> $\dot X = f(X)$

Eq points: $f(X_*) = 0 \implies$ zero, one or multiple equilibria

One possible case:



$\Leftarrow$ vector field

- Attach the vector AX to any point $\underline{x}$.

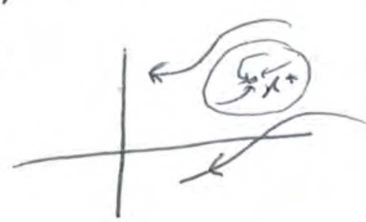
- Region of attraction:

Given an eq. x_*, let $\underline{S}$ be the largest set surrounding x_* such that $\forall\ x(0) \in \text{int}(S) \implies x(t) \longrightarrow x_*$ as $t \to \infty$

- An equilibrium x_* is globally stable if $S = \mathbb{R}^n$

- x_* is locally stable if $\exists\ \varepsilon$:

$B_\varepsilon \subseteq S$



- How to check local stability? Linearization

- How to check global stability? Lyapunov

Local stability at x_*:

$$\dot{x} = f(x) \longrightarrow \dot{x}(t) = \underbrace{f(x_*)}_{0} + \underbrace{\frac{df(x)}{dx}\Big|_{x=x_*}}_{A} (x(t) - x_*) + O(x(t) - x_*)$$

$$\longrightarrow \dot{(x(t) - x_*)} = A(x(t) - x_*) + \text{h.o.t} \xrightarrow[\text{Linearization}]{} \dot{x} = Ax$$

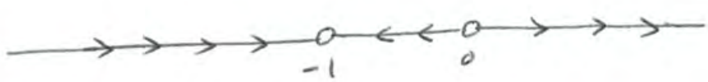
eq: $x = 0$ (shifted)

Stable if $Re(\lambda(A)) < 0 \quad \forall \lambda(A)$,

Unstable if $\exists \lambda(A) : Re(\lambda(A)) > 0$

Example: $\dot{x}_1 = x_1^2 + x_1$

- Eq $x_1 = 0 \Rightarrow \dot{x}_1 = x_1$ unstable
- Eq $x_1 = -1 \Rightarrow \dot{x}_1 = -x_1$ stable



Global stability:

- what if we have two eq. pt x_1^*, x_2^* : no global stability
- assume there is a unique equilibrium x^*:

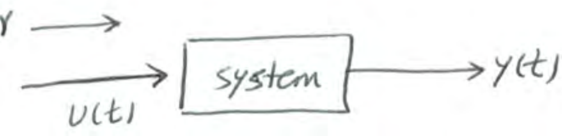
Search for $V(x)$ s.t.
- $V(x) > V(x^*) \quad \forall x \in \mathbb{R}^n \setminus x^*$
- $\dot{V}(x) = \nabla V^T f(x) < 0 \quad \forall x \in \mathbb{R}^n \setminus x^*$
- $V(x) \rightarrow \infty$ as $\|x\| \rightarrow \infty$
-

Control of unstable systems:

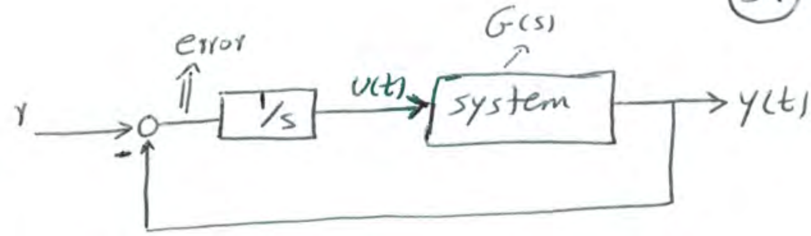
Linear systems: $\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$ or $Y(s) = (C(sI - A)^{-1}B + D)\, U(s)$

- How to design $u(t)$ to satisfy certain properties?
- Example: For a single-input, single-output system, design $u(t)$ such that $y(t) \rightarrow r$ as $t \rightarrow \infty \quad \forall r$ (Tracking problem)

Tracking:



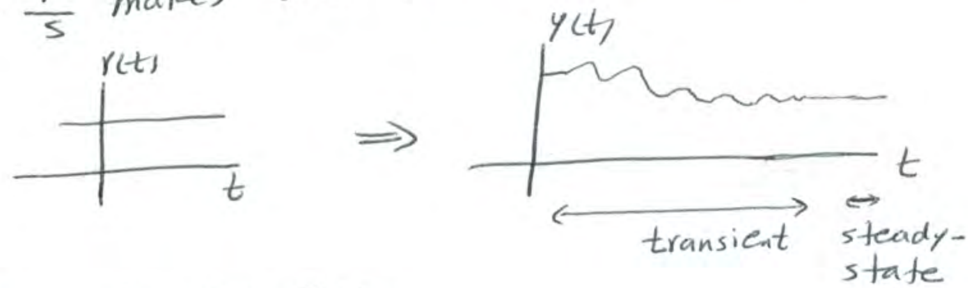
$\Rightarrow$



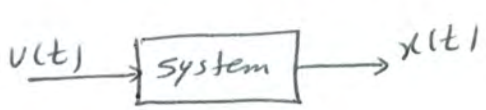
$$r(s) = \frac{r}{s} \Rightarrow e(s) = \frac{r}{s} \times \frac{1}{\frac{1}{s}G(s)+1} = \frac{r}{s+G(s)}$$

$$\Rightarrow \lim_{t\to\infty} e(t) = \lim_{s\to 0} s\,e(s) = \frac{0}{G(0)} \Rightarrow \text{If } G(0) \neq 0, \text{ then}$$

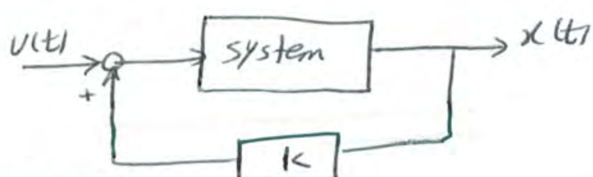
the controller $K(s) = \frac{1}{s}$ makes the system track any constant reference:



- Consider the system $\dot{x} = Ax + Bu$, where $\exists\, \lambda(A) : Re(\lambda(A)) > 0$. How to stabilize the system?

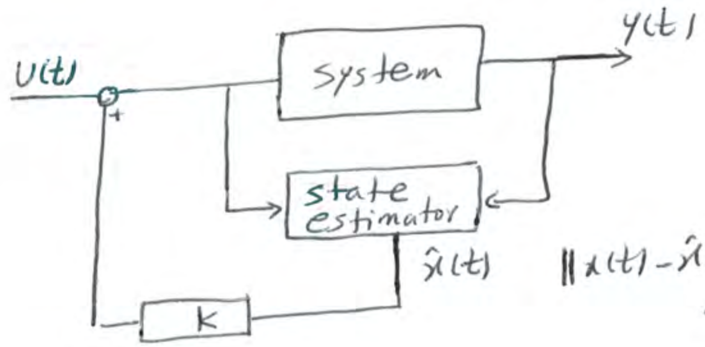


$\Rightarrow$



- Design K such that $A+BK$ Hurwitz.
- For almost all values of (A,B), there exists a stabilizing feedback $u(t) = Kx(t)$
- what happens if $x(t)$ is not measurable?

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + Du(t) \end{cases}$$



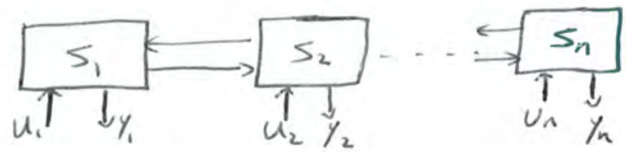
$\|x(t) - \hat{x}(t)\| \to 0$ as $t \to \infty$

state estimator:

Luenberger observer

Centralized control v.s. Decentralized control:

$$\begin{cases} \dot{X}_i = \sum_{j=1}^{n} A_{ij} X_j + B_i U_i \\ y_i = C_i X_i \end{cases}, \quad i=1,2,\dots,n$$



For simplicity, assume that $y_i = x_i$

- Stabilization problem: Find K s.t. $A + BK$ Hurwitz, $B = \begin{bmatrix} B_1 & & 0 \\ & \ddots & \\ 0 & & B_n \end{bmatrix}$

- Issue: If K is a full matrix, every subsystem should send its state to all subsystems.

- Impossible in several cases:
  - large-scale (many subsystems)
  - geographically distributed

- This strategy is called <u>centralized</u>.

- Decentralized control: Local controller $U_i = f_i(y_i)$
  e.g. $U_i = K_i y_i$

$\Rightarrow$ Design K s.t. $A + B \begin{bmatrix} K_1 & & \\ & \ddots & \\ & & K_n \end{bmatrix}$ Hurwitz.

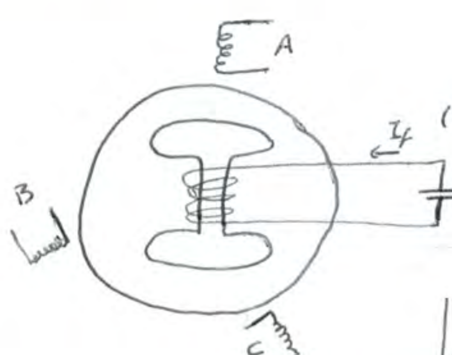
Almost always impossible unless K is replaced by a dynamic controller

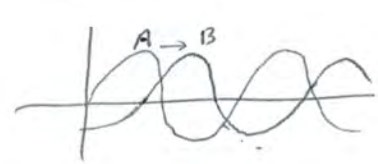
- Power grid naturally needs decentralized control.

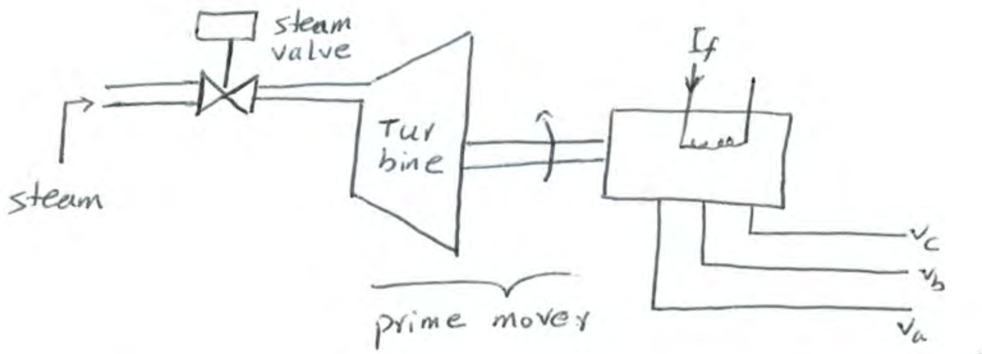
Synchronous Generator:

- fixed stator (connected to the grid)

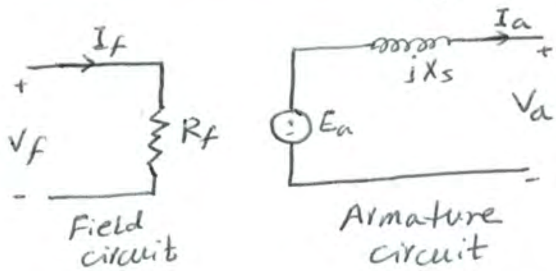
- Moving rotor (Connected to a DC excitation system)







Equivalent Circuit:



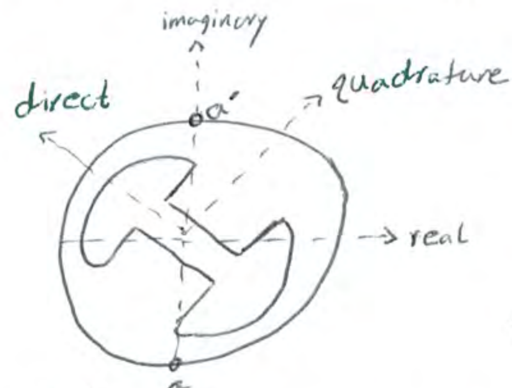
$$P_G = \frac{|E_a||V_a|}{X_s} \sin \delta_m \;, \quad Q_G = \frac{|E_a||V_a|}{X_s} \cos \delta_m - \frac{|V_a|^2}{X_s}$$

$\delta_m = \measuredangle E_a - \measuredangle V_a$, $\delta_m > 0$ generator, $\delta_m < 0$ motor

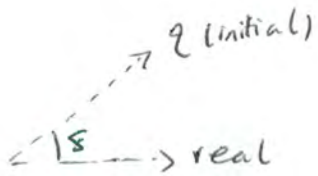
$\delta_m = 0 \Rightarrow P_G = 0$, $Q_G = \dfrac{|V_a|(|E_a| - |V_a|)}{X_s}$ $\longrightarrow$ synchronous condensor (better than capacitor bank)

$\rightarrow$ (not turbine spins freely)

Axes:



real-imag : fixed, q-d : rotating



$\Rightarrow \theta = \delta + \pi/2 + w_0 t$

(Assume that q-d is rotating at a fixed speed w_0, where w_0 is the grid frequency).

$\Rightarrow \delta = \measuredangle E_a$.

- $|E_a|$ is proportional to V_f in the steady state.
- swing equation: $M\ddot{\delta} + D\dot{\delta} + \underbrace{P_G}_{\text{generated}} = \underbrace{P_m}_{\text{mechanical}}$

steady state: $P_m = P_G$

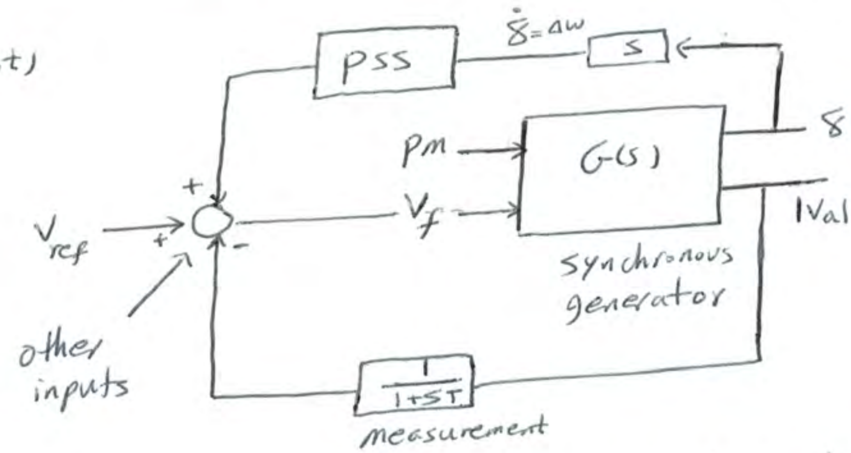
observations: - excitation system controls bus voltage magnitude

- mechanical power controls active power.

- Both control frequency.
($\delta(t)$ depends on P_m, P_G, and P_G depends on $|E_a|$).

Voltage control + PSS (power system stabilizer):

$(\omega = \dot{\theta} = (\dot{\tfrac{\pi}{2} + \delta + \omega_0 t})$
$\Rightarrow \Delta\omega = \dot{\delta})$

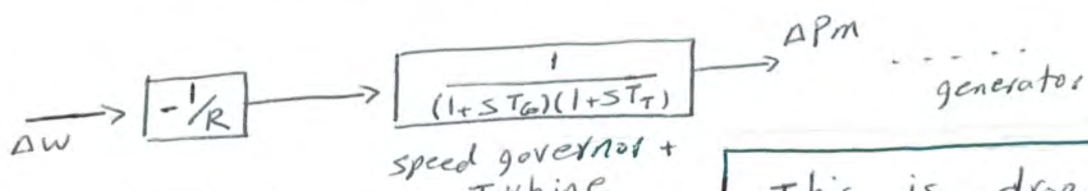


- Loads fluctuate in real-time so the generator should respond to them.

- This is done via speed governor.

- <u>Decentralized control:</u> sense a speed deviation ($\Delta\omega$) and then convert it into appropriate valve action to change P_m.

- Intuition: $\Delta\omega > 0 \rightarrow$ reduce speed.
$\Delta\omega < 0 \rightarrow$ increase speed.



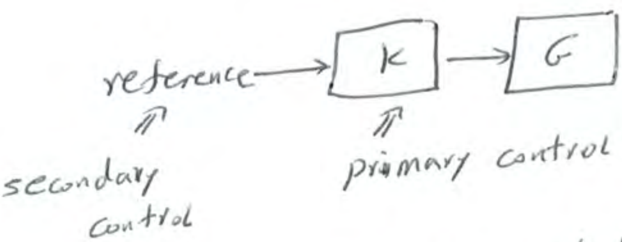
$T_G = 0.1$ second, $T_T \simeq 2$ seconds

$\boxed{\text{This is droop control}}$

$\Delta P_m = -\frac{1}{R} \Delta w$ in steady state.

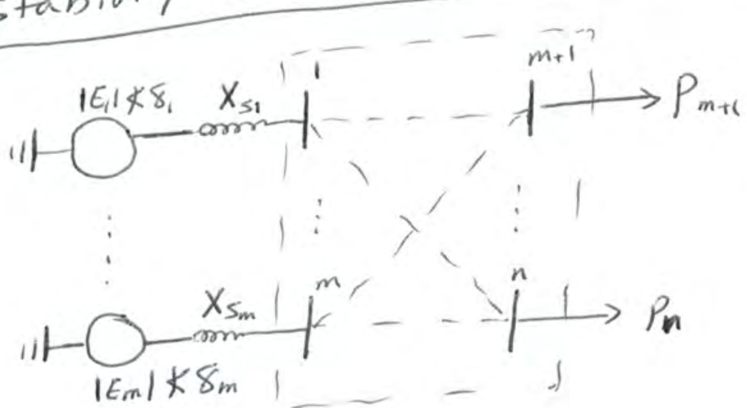
- Each generator should be able to provide 5% droop control ($R = 0.05$ p.u.)
- 5% change in the frequency allows us to go from no load to full load.

- The system is operating at a steady-state but a big Load suddenly joins.
  - The frequency starts dropping very abruptly.
  - Speed governor stabilizes the network at a lower frequency.
  - Automatic generation control (AGC): The energy management system (EMS) solves an economic dispatch problem and changes the references of controllers to get the network back to the nominal frequency.



There is no stability guarantee!

Stability of swing equations:



$i = 1, \dots, m$:

$$M_i \ddot{\delta}_i + D_i \dot{\delta}_i = P_{M_i} - P_{G_i}$$

$$\Rightarrow P_{G_i} = |E_i|^2 \hat{G}_{ii} + \sum_j |E_i||E_j| \left(\hat{B}_{ij} \sin(\delta_i - \delta_j) + \hat{G}_{ij} \cos(\delta_i - \delta_j) \right)$$

$i = m+1, \dots, n$:

P_i = an algebraic equation

⇒ we have a mix of algebraic and differential equations in δ_i, w_i and $\dot{w}_i$.

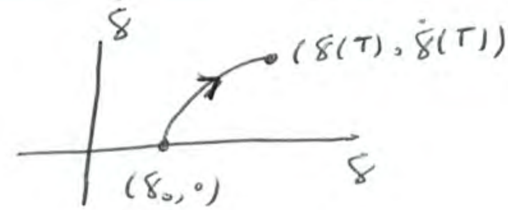
Example:
- A generator is at eq. point $(\delta, \dot{\delta}) = (\delta_0, 0)$ at time $t=0$, and is connected to an infinite bus.
- Fault occurs at $t=0$ and circuit breaker opens.
- At $t = T$, fault clears and breaker recloses.
- Question: Does the system get back to the same eq point?

$$M\ddot{\delta} + D\dot{\delta} + P_G(\delta) = P_m$$

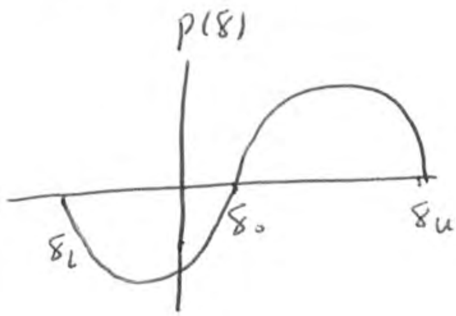
$0 \le t \le T \Rightarrow M\ddot{\delta} + D\dot{\delta} = P_m \Rightarrow$ There is a trajectory for $(\delta, \dot{\delta})$

If $D = 0 \Rightarrow \delta(t) = \frac{P_m}{2M} t^2 + \delta_0$, $\dot{\delta}(t) = \frac{P_m}{m} t$

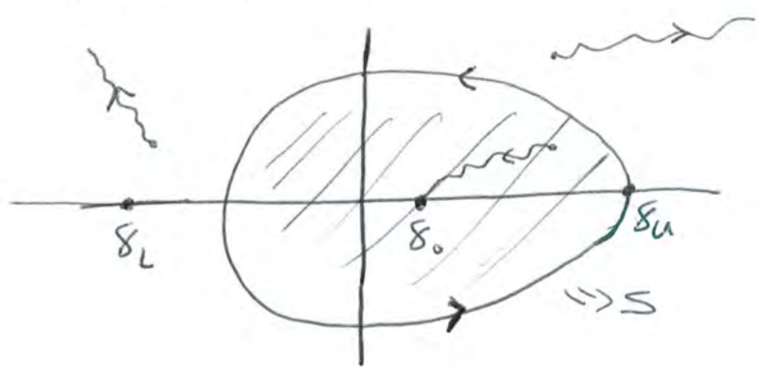
$\Rightarrow \delta - \delta_0 = \frac{M}{2P_m} \dot{\delta}^2 \Rightarrow$



- $t > T$: $p(\delta) = P_G(\delta) - P_m$



$\dots, \delta_L, \delta_0, \delta_u, \dots$ equilibria

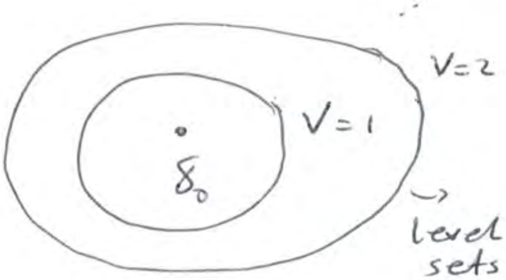


stability question:
are we still inside $\underline{S}$ at $t = T$?

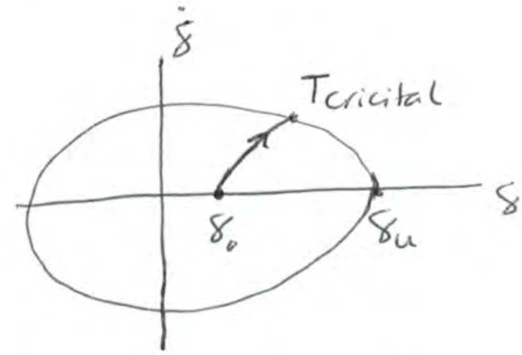
$$M\ddot{\delta} + D\dot{\delta} + P(\delta) = 0$$

$$\underset{\text{energy function}}{V(\delta,\dot{\delta})} = \underbrace{\tfrac{1}{2}M\dot{\delta}^2}_{\text{kinetic}} + \underbrace{\int_{\delta_0}^{\delta} p(u)\,du}_{\text{potential}} \Rightarrow V(\delta_0, 0) = 0$$

$$\dot{V}(\delta,\dot{\delta}) = M\dot{\delta}\ddot{\delta} + p(\delta)\dot{\delta} = -D\dot{\delta}^2 \le 0$$



$\Rightarrow$



stability: $T < T_{critical}$ or $\frac{1}{2}M\dot{\delta}_T^2 + \int_{\delta_0}^{\delta_T} p(u)\,du <$

$$\underbrace{\tfrac{1}{2}M\dot{\delta}_u^2}_{0} + \int_{\delta_0}^{\delta_u} p(u)\,du$$

$$\Rightarrow \boxed{\tfrac{1}{2}M\dot{\delta}_T^2 < \int_{\delta_T}^{\delta_u} p(u)\,du}$$ (condition for returning to the first equilibrium)

- Example: synchronous generator connected to a synchronous motor driving a mechanical load.

$$\begin{cases} \underset{\text{inertia}}{M_1}\ddot{\delta}_1 + \underset{\text{damping}}{D_1}\dot{\delta}_1 + P_{G_1}(\delta_1 - \delta_2) = P_{m_1} \\ M_2\ddot{\delta}_2 + D_2\dot{\delta}_2 + P_{G_2}(\delta_2 - \delta_1) = P_{m_2} \end{cases}$$

$P_{G_1} + P_{G_2} = 0$, if $\frac{D_1}{M_1} = \frac{D_2}{M_2}$ (note that $P_{m_1} + P_{m_2} = 0$ due to equilibrium).

$$\Rightarrow \ddot{\delta}_{12} + \frac{D_1}{M_1}\dot{\delta}_{12} + \left(\frac{1}{M_1} + \frac{1}{M_2}\right)P_{G_1}(\delta_{12}) = \left(\frac{1}{M_1} + \frac{1}{M_2}\right)P_{m_1}, \quad \delta_{12} = \delta_1 - \delta_2$$

looks like a single machine: for one value of δ_{12} ($|\delta_{12}| < \frac{\pi}{2}$), we have stability.