Due date: Thursday April 16.

1. Consider the “optimal selling” problem discussed in class. Specifically, you receive offer \( Y_t \) at times \( t = 1, \cdots, T \). You need to decide, after receiving the offer, whether to accept or reject it. Offers which have been rejected can not be accepted at future time points. Once an offer has been accepted, the game ends. If no offer has been accepted by time \( T \), then the final offer \( Y_T \) must be accepted. The goal is to maximize the discounted expected selling price. Assume an interest rate of \( r \) (compounded) and that \( Y_1, \cdots, Y_T \) are iid with cumulative distribution function \( F(y) \).

(a) We showed in class that the optimal policy was to accept offer \( Y_t \) if it exceeded the threshold \( \alpha_t = \frac{1}{1+r} E[V_{t+1}(Y)] \) and that
\[
\alpha_t = \frac{1}{1+r} \left\{ \alpha_{t+1} F(\alpha_{t+1}) + \int_{\alpha_{t+1}}^{\infty} y dF(y) \right\}.
\]
Assuming that \( Y_i \) is exponential with mean 8 with \( T = 10 \), compute \( \alpha_1, \cdots, \alpha_T \). Compute the value of \( \alpha_t \) when \( t \to -\infty \). Assume that \( r = 0.02 \).

(b) Repeat part (a) when the mean is 14.

(c) Consider now the case when the mean is either 8 or 14 but its actual value is not known to the decision maker. Let \( \rho_0 = P[\text{mean is 8}] = 1 - P[\text{mean is 14}] \) be the prior on the mean and \( \rho_t \) denote the associated posterior (associated with observations \( Y_1, \cdots, Y_t \)). Using simulation, estimate the expected profit if the seller uses the approximation to the Bayesian approach discussed in class with prior \( \rho_0 = 0.5 \), and the offers happen to be have mean 8 (of course, this last fact is not known by the decision maker). Repeat the computation for several values of \( \rho_0 \) to get a sense of the sensitivity of the resulting selling price to the prior.

2. Consider a situation involving a blackmailer and his victim. In each period, the blackmailer has a choice of (a) accepting a lump sum payment of \( R \) from the victim and promising not
to blackmail again, or (b) demanding a payment of $u$, where $u \in [0, 1]$. If blackmailed, the victim will either (1) comply with the demand and pay $u$ to the blackmailer. This happens with probability $1 - u$. Alternatively (2) the victim refuses to pay with probability $u$ and reports the blackmailer to the police. Once known to the police, the blackmailer can not ask for any more money. The blackmailer wants to maximize the expected amount of money he gets over $N$ periods by an optimal choice of payment demands $u_k$. (Note that there is no additional penalty for being denounced to the police). Write a DP algorithm and find the optimal policy.