Retailer-Driven Product Bundling in a Distribution Channel

Hemant K. Bhargava

January 27, 2012
Abstract

Retailer-Driven Product Bundling in a Distribution Channel

This paper studies product bundling in a distribution channel where a downstream retailer combines component goods produced by separate manufacturers acting independently of each other. The bundling literature offers deep insights about the economic benefits of bundling where an integrated firm creates a bundle of component products that it makes or controls, and for which unit costs are not impacted by choice of selling strategy. But, when the retailer bundles goods from separate manufacturers, the unit costs for the bundler (retailer) are, being the prices set by the manufacturers, no longer exogenous. This alters the economic balance with respect to bundling. We employ a model that encases a demand-side economic force in favor of bundling when applied to an integrated firm, and show that channel conflicts weaken the case for bundling. While pure bundling is better than pure components for the integrated firm, it is no longer so in the decentralized channel. When the retailer practices mixed bundling, then the bundle accounts for a smaller fraction of sales than would occur for the integrated firm. The culprit is a combination of horizontal channel conflict (each manufacturer wants a higher share of profits from bundle sales) and vertical conflict (incentive misalignment with respect to bundle vs. component sales). These conflicts lead to overpricing of component goods by manufacturers, weakening the effect of the demand-smoothing benefits of product bundling. The competitive interplay between the firms leads to lower profits for all. Price coordination between the firms could partially restore the role of bundling, and improve the firms’ profits as well as consumer surplus.
1 Introduction

This paper studies product bundling in a distribution channel. We examine the practice where a downstream firm (Retailer) sells a bundle of component goods made by multiple and independent upstream firms (Manufacturers). This structure is schematically described in the first panel of Fig. 1. It is observed in many industries including travel, technology, media, dining and entertainment. For example, theaters bundle multiple entertainment events from independent artists and entertainers into season passes. Travel sites such as Expedia bundle products (air transport, car rental, hotel, shows, etc.) from multiple providers. Firms that specialize in assembling products (e.g., PCs) bundle components from multiple manufacturers, as do system integrators (e.g., defense industry firms such as Raytheon) and information aggregators (e.g., Yahoo!). Bundling strategy can involve either pure bundling (selling just the bundle and not the component goods) or mixed bundling.

Since the early work of Stigler (1963), product bundling has been widely studied in the marketing and economics literatures, including bundling of complements and substitutes (Venkatesh and Kamakura, 2003), customized bundling (Wu et al., 2008), and bundling under competition (Bakos and Brynjolfsson, 2000). Common examples include Microsoft

![Figure 1: Different distribution structures for product bundling. The shaded boxes represent the firm making the bundling decision. The $c_i$’s are unit costs of component goods.](image-url)
Office, a package of multiple office productivity applications that could have stand-alone value, and “Happy Meals” bundles at fast-food restaurants. An important economic motivation for bundling is that consumer valuations for the bundle have less dispersion (relative to the mean) than valuations for component products (Adams and Yellen, 1976). This demand smoothing force is stronger when many goods are being bundled and when the distributions of consumer valuations for component goods are negatively correlated (but bundling can also be profitable under positive correlation), and it is most effective when unit marginal costs are very low relative to value (Schmalensee, 1984; McAfee et al., 1989; Bakos and Brynjolfsson, 1999). Venkatesh and Mahajan (2009) provide an excellent recent survey of the literature and practice.

Unlike the distribution channel structure of interest in this paper, the existing literature on bundling has studied a direct selling setting (reflected in the second panel of Figure 1) where the firm that designs the bundle selling strategy also makes or owns the component products. There are critical economic differences between the two settings. In the direct case, unit costs of the component goods are exogenous and not impacted by the choice of selling strategy. In the vertical channel, however, the unit costs for the firm (retailer) making the bundling decision are, being the prices set by the manufacturers, no longer exogenous. This alters the economic balance with respect to bundling. First, the separation of manufacturing and retailing functions leads to double marginalization (Spengler, 1950), hence the unit costs for the retail-level bundling decision are higher than for the integrated firm. Second, the retailer’s action of bundling can entice the manufacturers, seeking a higher share of the gains from bundling, to raise their prices. These forces are manifestations of vertical channel conflict. Third, manufacturers set prices of component goods independently, leading to higher prices as with “composite goods” (Cournot, 1929), an instance of horizontal channel conflict.¹ These two types of channel conflicts raise the retailer’s unit costs, rendering

¹Cournot’s model considered two component goods, made by different firms, that must be combined to create utility (e.g., “zinc + copper = brass”). While this is specifically about strong complements, the insight that independent manufacturers would overprice their components carries over more generally.
bundling less attractive.

Still, bundling carries an inherent advantage for all firms. The retailer desires bundling because the lower dispersion in consumer valuations enables it to extract more surplus. Manufacturers desire bundling because it leads to higher sales. Moreover, the retailer can suppress the manufacturers’ desire to raise prices by reserving the choice of selling strategy until after it observes manufacturer prices. If these prices are too high, the retailer would find it optimal to sell components (or, under mixed bundling, shift sales away from the bundle)—leading to lower profits for the manufacturers—hence this foreknowledge should exert downward pressure on manufacturers’ prices.

We will examine the problem in two parts. In the first, we study a contest between pure bundling and pure component selling, where the insights are sharper because the choices are stark. Then we extend the analysis to mixed bundling where the insights are more qualitative and manifested in the fraction of sales attributed to the bundle vs component goods. We show that, although bundling increases the total profits that the firms can earn in the decentralized channel, conflicts substantially weaken the benefits from bundling. Our two-good setting encases a demand-side economic force in favor of bundling when applied to an integrated firm: pure bundling is better than pure components; mixed bundling does better but resembles the pure bundle solution (the bundle still accounts for a majority of sales). Thus, this setting is unambiguously one where the integrated firm would find bundling attractive. We show that the demand-side motivation for bundling survives the vertical channel structure and double marginalization: bundling remains attractive in a bilateral monopoly. But this force is defeated when production of component goods is disaggregated into multiple manufacturers. The culprit is a combination of horizontal channel conflict

---

1 We will examine the problem in two parts. In the first, we study a contest between pure bundling and pure component selling, where the insights are sharper because the choices are stark. Then we extend the analysis to mixed bundling where the insights are more qualitative and manifested in the fraction of sales attributed to the bundle vs component goods. We show that, although bundling increases the total profits that the firms can earn in the decentralized channel, conflicts substantially weaken the benefits from bundling. Our two-good setting encases a demand-side economic force in favor of bundling when applied to an integrated firm: pure bundling is better than pure components; mixed bundling does better but resembles the pure bundle solution (the bundle still accounts for a majority of sales). Thus, this setting is unambiguously one where the integrated firm would find bundling attractive. We show that the demand-side motivation for bundling survives the vertical channel structure and double marginalization: bundling remains attractive in a bilateral monopoly. But this force is defeated when production of component goods is disaggregated into multiple manufacturers. The culprit is a combination of horizontal channel conflict
(each manufacturer wants a higher share of profits from bundle sales) and vertical conflict (incentive misalignment with respect to bundle vs. component sales). These conflicts lead to overpricing of component goods by manufacturers, whereupon the retailer no longer prefers pure bundling (more generally, under mixed bundling, the bundle commands a smaller share of overall sales). Manufacturers could raise their profits through price coordination, but such coordination would benefit the retailer as well, and could also increase consumer surplus and social welfare.

Our work combines three insights from separate literature streams: the demand-smoothing effect of bundling, double marginalization in the vertical channel, and overpricing of components by independent manufacturers. The distinctive feature of the analysis is the combination of bundle selling strategy and distribution channel structure. As noted above, past literature on bundling studies a direct producer-buyer distribution structure (e.g., a restaurant that packages a burger, chips and a drink; or a software firm that bundles multiple components). The literature on vertical channels and double marginalization models a retailer who essentially passes through the products of one or more manufacturers to the consumer, without considering the strategic lever of bundling these products. Similarly, the literature on composite goods does not consider an active bundling decision by an intermediate firm such as a retailer.\textsuperscript{cd} We tie these three streams by demonstrating that the double marginalization and component overpricing effects work together to weaken, and possibly even render ineffectual, the demand smoothing motivation for product bundling.

This paper contributes to both managerial practice and research in bundling, and is the first to describe and model the phenomenon of bundling in a distribution channel which separates component production and bundle-selling strategy. The model is rich enough to capture a combination of the demand-side economic motivation for bundling and the horizontal and

\textsuperscript{cd}Bakos and Brynjolfsson (2000) study a vertical channel with both upstream competition for content and downstream competition for consumers but these two processes are modeled separately of each other. The upstream analysis covers potential purchase of a single component product (thereby avoiding the complexity caused by intra-manufacturer price setting in the backdrop of inter-manufacturer-retailer dynamics) by two competing aggregators who already possess a collection of goods; their Proposition 2 shows that the already bigger aggregator wins this competition.
vertical conflicts in the channel setting, yet tractable enough to produce informative and credible insights. But as a first model of bundling in a decentralized distribution channel, it is neither perfect nor complete. It has several limitations but also presents many future research opportunities, discussed in §5. More generally, the component goods sold by the manufacturers may themselves be a bundle of goods. For example, cable TV and satellite firms bundle content created by multiple studios and programming networks, and each of these content providers pre-packages a bundle of content. Similarly, Netflix’s streaming service provides access to pre-bundled content from multiple studios and cable channels. This structure is similar to the first panel of Fig. 1 after relabeling the product that the manufacturer provides to the retailer. The across-manufacturer bundling, which is then carried out by the retailer, is the subject of this paper.

2 Model

We model a market where a retailer sells a bundle \( B \) of two component products \( i = 1, 2 \) made by independent manufacturers (M1 and M2) \(^{c1}\) who face unit costs \( c_1 \) and \( c_2 \) respectively. Consumer valuations \( v_i \) for product \( i \) are randomly distributed in an interval \([0, 1]\) according to a density function \( f_i \) with \( cdf \) \( F_i \). \(^{c2}\) The 2-goods setting is frequently employed to expose insights about bundling, including the early work on bundling Adams and Yellen (1976); Schmalensee (1984); Salinger (1995) and later explorations into the effect of correlation in product valuations (McAfee et al., 1989); super- or sub-additivity of bundle valuations (Venkatesh and Kamakura, 2003); and network effects (Prasad et al., 2010). \(^{c3}\) This paper adds a channel structure into the baseline model, and we assume that a consumer’s valuation for one product is independent of her valuation for the other (the assumption is relaxed in §4.4).

**Assumption 1 (Independence)** The density functions \( f_1 \) and \( f_2 \) are independent.

A consumer \( v \)’s valuation for the bundle is the sum of her valuation for the two components, \( z = v_1 + v_2 \). With Assumption 1, the density function \( g \) for bundle valuations is simply the
convolution of functions $f_1$ and $f_2$,

$$g(z) = (f_1 * f_2)(z) = \int_{-\infty}^{\infty} f_1(z - y)f_2(y) \, dy$$  \hspace{1cm} (1)

Let $p_1, p_2, p_B$ represent the per-unit prices set by the retailer for products $i = 1, 2, B$ in response to input costs $w_1, w_2$ (the prices set by the manufacturer). The general case is that the retailer pursues a mixed bundling strategy, i.e., it offers both the component goods and the bundle for sale with $p_1, p_2 \leq p_B$ and $p_B \leq p_1 + p_2$. Let $Q_i = Q_i(p_1, p_2, p_B)$ be the sales of each product given the retail prices $(p_1, p_2, p_B)$. Then the retailer and manufacturers’ profits are $\Pi_R, \Pi_1, \Pi_2$, respectively,

$$\Pi_R = (p_1 - w_1)Q_1 + (p_2 - w_2)Q_2 + (p_B - w_1 - w_2)Q_B$$ \hspace{1cm} (2a)

$$\pi_1 = (w_1 - c_1)(Q_1 + Q_B)$$ \hspace{1cm} (2b)

$$\pi_2 = (w_2 - c_2)(Q_2 + Q_B).$$ \hspace{1cm} (2c)

If the retailer pursued a pure components selling strategy, the firms’ equilibrium sales and profit are denoted with the superscript $^C$ and follow from Eq. 2 by setting $Q_B = 0$ (i.e., $p_B \geq p_1 + p_2$). Similarly, a pure bundle strategy is denoted with the superscript $^B$ and corresponds to $Q_1 = Q_2 = 0$ (i.e., $p_i > p_B$). For mixed bundling we denote the equilibrium solution with the superscript $^\ast$. For detailed computations, we make the following additional assumption about the density functions $f_i$ and the demand for component goods.

**Assumption 2 (Linear Demand for Components)** The functions $f_1$ and $f_2$ follow uniform density on $[0, 1]$. Component goods $i = 1, 2$ have linear demand $D_i(p) = 1 - p$. 
Figure 2: Sales under pure components, pure bundle, and mixed bundling.

With Assumption 2, the retailer’s sales levels under the pure components, pure bundling, and mixed bundling selling strategies are given below, and graphically depicted in Fig. 2.

**pure components**

\[
\begin{align*}
Q_1 &= (1 - p_1) \\
Q_2 &= (1 - p_2)
\end{align*}
\]  

(3)

**pure bundling**

\[Q_1 = Q_2 = Q_B = D_B(p) = \begin{cases} 
\frac{1}{2} (2 - p^2) & \text{if } p \leq 1 \\
\frac{1}{2} (2 - p)^2 & \text{o/w.}
\end{cases}\]  

(4)

**mixed bundling**

\[
\begin{align*}
Q_1 &= (1 - p_1)(p_B - p_1) \\
Q_2 &= (1 - p_2)(p_B - p_2) \\
Q_B &= (1 - p_B + p_1)(1 - p_B + p_2) - \frac{(p_1 + p_2 - p_B)^2}{2}.
\end{align*}
\]  

(5)

The total surplus (or social welfare, SW) under component sales is \( \int_{p_1}^{1} x \, dx + \int_{p_2}^{1} y \, dy \).

Under pure bundling, SW is \( 1 - \int_{0}^{p_B} \int_{0}^{p_B-x}(x+y) \, dy \, dx \) for \( p_B \leq 1 \), and \( \int_{p_B-1}^{1} \int_{p_B-x}^{1}(x+y) \, dy \, dx \) for \( p_B \geq 1 \). For mixed bundling, the total surplus is \( 1 - \int_{0}^{p_B-p_1} \int_{0}^{p_1}(x+y) \, dy \, dx - \int_{p_B-p_2}^{p_1} \int_{0}^{p_1}(x+y) \, dy \, dx \). Consumer surplus (CS) is SW - \( \Pi_1 - \Pi_2 - \Pi_R \).

The linear demand (or uniform distribution of valuations) formulation encapsulates the key economic driver for bundling, i.e., demand smoothing. The convolved demand curve, being flatter in the middle, enables extraction of more of the total surplus, and mixed
Table 1: Bundle selling dominates component selling for an integrated firm under Assumptions 1–2 and zero marginal costs.

<table>
<thead>
<tr>
<th>pure components</th>
<th>( p_1 )</th>
<th>( p_2 )</th>
<th>( p_B )</th>
<th>( Q_1 )</th>
<th>( Q_2 )</th>
<th>( Q_B )</th>
<th>( \Pi )</th>
<th>( CS )</th>
</tr>
</thead>
<tbody>
<tr>
<td>pure bundle</td>
<td>NA</td>
<td>NA</td>
<td>( \sqrt{\frac{2}{3}} )</td>
<td>NA</td>
<td>NA</td>
<td>( \frac{2}{3} )</td>
<td>0.5443</td>
<td>0.2742</td>
</tr>
<tr>
<td>mixed bundle</td>
<td>( \frac{2}{3} )</td>
<td>( \frac{2}{3} )</td>
<td>0.8619</td>
<td>0.0651</td>
<td>0.0651</td>
<td>0.5365</td>
<td>0.5492</td>
<td>0.3459</td>
</tr>
</tbody>
</table>

bundling is the profit-maximizing strategy for the integrated firm, which corresponds to the classical bundling problem studied in the literature (McAfee et al., 1989).\(^{c1}\) For simplicity in exposition, we use \( w_1, w_2 \) to denote the integrated firm’s marginal costs for goods 1-2.

Table 1 illustrates the outcomes under the special case of zero marginal costs, \( w_1 = w_2 = 0 \).

Pure bundling outperforms pure components; mixed bundling is best, but is closer to pure bundling than to pure components (the bundle contributes to a majority of product sales and profits). The solution for the more general case of positive costs is summarized in Lemma 1 below.\(^{c2}\)

**Lemma 1 (Integrated firm)** The optimal prices under the three selling strategies are

**Pure Components**

\[
p_i^C = \frac{1 + w_i}{2} \quad (i = 1, 2)
\]

**Pure Bundle**

\[
p_B = \begin{cases} \frac{1}{3}(w_1 + w_2 + \sqrt{6 + (w_1 + w_2)^2}) & \text{if } w_1 + w_2 < \frac{1}{2} \\ \frac{2}{3}(1 + w_1 + w_2) & \text{o/w.} \end{cases}
\]

**Mixed Bundle** \((i = 1, 2; j = 2 - i)\)

\[
p_i^* = 0.8603 + 0.5593 \ (w_1 + w_2)
\]

\[
p_i^* = \frac{1}{6} \left( 3p_B + (2 + w_i - w_j) - \sqrt{(3p_B - 2 + (w_j - w_i))^2 + 12w_j(1 - p_B)} \right)
\]

Among the three strategies, mixed bundling performs best. Pure bundling is better than pure components when costs are low, specifically when \((w_1, w_2) \in \Theta_1 \cup \Theta_2\), where \( W = w_1 + w_2 \).
and

$$\Theta_1 = \left\{ (w_1, w_2) : (W \leq \frac{1}{2}) \text{ AND } \left( \frac{W^3 + (6 + W^2)^\frac{3}{2} - 18W}{27} \geq \sum_{i=1}^{2} \frac{(1 - w_i)^2}{4} \right) \right\}$$

$$\Theta_2 = \left\{ (w_1, w_2) : (W \geq \frac{1}{2}) \text{ AND } \left( 2 \left( \frac{2 - W}{3} \right)^3 \geq \sum_{i=1}^{2} \frac{(1 - w_i)^2}{4} \right) \right\}.$$ 

3 Mixed Bundling in Vertical Channel

We seek to examine how the attractiveness of bundling changes as we transition from a direct channel setting, where a single integrated firm makes and sells the two products, to the vertical channel structure where the retailer bundles component goods made by separate manufacturers. Specifically, for the mixed bundling strategy discussed in this section we will inquire (i) how much of a profit increase is delivered by a mixed bundle selling strategy over a pure components strategy, and (ii) how the proportion of sales changes between the bundle and component goods. In each case we will compare the outcomes to the corresponding outcomes for an integrated firm (covered in Table 1). To facilitate the first comparison, we compute a pure component-selling benchmark in the vertical channel. This solution is specified in Table 2, and is derived by aggregating the solution of two separate two-stage pricing games, each of which features one manufacturer (stage 1) and retailer (stage 2).

<table>
<thead>
<tr>
<th></th>
<th>Manufacturers</th>
<th>Retailer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prices</td>
<td>$w_i = \frac{1 + c_i}{2}$</td>
<td>$p_i = \frac{3 + c_i}{4}$</td>
</tr>
<tr>
<td>Firm’s profit</td>
<td>$\Pi_i = \frac{1}{8}(1 - c_i)^2$</td>
<td>$\Pi_R = \frac{1}{16}((1 - c_1)^2 + (1 - c_2)^2)$</td>
</tr>
<tr>
<td>Industry profit</td>
<td>$\Pi_1 + \Pi_2 + \Pi_R = \frac{3}{16}((1 - c_1)^2 + (1 - c_2)^2)$</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Pure components solution in vertical channel.

The mixed bundle pricing problem in the vertical channel is solved as a two-stage game. The retailer sets prices $p_1^*, p_2^*, p_B^*$ in stage 2, after observing its input costs (i.e., the prices $w_1, w_2$ set by manufacturers 1 and 2). Stage 1 involves a simultaneous pricing game between manufacturers 1 and 2 who set their prices knowing the retailer’s price reaction strategy...
derived in stage 2. Formally, the retailer’s stage 2 price-optimization problem is

$$\text{Maximize } \Pi_R = (p_1 - w_1)Q_1 + (p_2 - w_2)Q_2 + (p_B - w_1 - w_2)Q_B$$

subject to

$$w_1 < p_1 < \min\{1, p_B\}, \quad w_2 < p_2 < \min\{1, p_B\}, \quad w_1 + w_2 < p_B < p_1 + p_2,$$

with $Q_1, Q_2, Q_B$ as in Eq. 5. This stage 2 problem is equivalent to the classical problem of bundling by an integrated firm, except that the retailer’s unit costs are now the prices $w_1, w_2$ set by the manufacturers. Subject to these prices, the retailer’s optimal price functions are the $p_B^*$ and $p_i^*$ specified in Lemma 1.

3.1 Mixed Bundling Equilibrium

In stage 1, manufacturers 1 and 2 anticipate the retailer’s pricing rules given in Lemma 1, and their sales quantities ($\hat{Q}_1 = Q_1 + Q_B$, and $\hat{Q}_2 = Q_2 + Q_B$, respectively) are obtained by substituting the retailer’s prices into Eq. 5. After algebraic rearrangement and simplifications we can reduce these expressions to,

$$\hat{Q}_1 = (0.6164 - 0.5772w_1 - 0.3550w_2) + 0.0161 f(w_1, w_2) - 0.0394 f(w_2, w_1)$$

$$+ 0.0188 (w_1 + 3.9503w_2) \times (f(w_1, w_2) + f(w_2, w_1))$$

where $f(w_1, w_2) = \sqrt{0.3374 + 0.7876(w_1 + w_2) + 4w_2 - 3.0809w_1w_2 + 0.4595(w_1^2 + w_2^2)}$. The expression for $\hat{Q}_2$ is symmetric. The manufacturers’ equilibrium prices $w_1^*, w_2^*$ are obtained by solving a simultaneous game where manufacturers seek to maximize individual profit $\Pi_i(w_1, w_2) = (w_i - c_i)\hat{Q}_i$.

$$\begin{cases} w_1^* = \arg \max_{w_1} \left( \Pi_1(w_1, w_2) = (w_1 - c_1)\hat{Q}_1 \right) \\
 w_2^* = \arg \max_{w_2} \left( \Pi_2(w_1, w_2) = (w_2 - c_2)\hat{Q}_2 \right) \end{cases}$$

The nonlinearity of the $\hat{Q}_i$ terms and the expanded profit expressions prevents a direct analytical solution to Eq. 8. However, it is possible to compute a near-exact approximation.
of the equilibrium outcome by deriving an accurate linear approximation for $\hat{Q}_i$,

$$
\hat{Q}_1 \approx 0.5539 - 0.5189w_1 - 0.0554w_2 \\
\hat{Q}_2 \approx 0.5539 - 0.5189w_2 - 0.0554w_1.
$$

(9)

With this approximation, the manufacturers’ profit functions are quadratic and the pricing game of Eq. 8 yields best-response functions that are linear in the two variables. The system of two simultaneous equations therefore has a unique equilibrium solution summarized below. More details regarding this argument are provided in the proof.

**Lemma 2 (Mixed bundling in vertical channel)** The equilibrium solution in the decentralized vertical channel is the $p_1^*, p_2^*, p_B^*$ given in Lemma 1, with $w_1^*, w_2^*$ given by

$$
w_1^* = 0.5067 + 0.5014c_1 - 0.0268c_2 \\
w_2^* = 0.5067 + 0.5014c_2 - 0.0268c_1.
$$

**Corollary 1** Each manufacturers best-response price $\Omega_j(w_i)$ is a decreasing function of the other manufacturer’s price, specifically, $\Omega_j(w_i) = 0.5337 + 0.5c_j - 0.0534w_i$.

### 3.2 Channel Structure Reduces Attractiveness of Bundling

We apply Lemma 2 to identify how the channel structure impacts the attractiveness of bundling. First, we compute the extent to which mixed bundling increases profit in the vertical channel (over a component-selling strategy) and compare this against the corresponding increase for the integrated firm. For the special case of $c_1 = c_2 = 0$, manufacturer prices are $w_1^* = w_2^* = 0.5067$, while (from Lemma 1) the retail prices are $p_1^* = p_2^* = 0.7778$, $p_B^* = 1.4271$. Each manufacturer earns 0.1317, the retailer’s profit is 0.1248, and the combined firms’ profit is 0.3882, which is significantly below the profit for the integrated firm (29.3% lower). Moreover, compared with component-selling, mixed bundling in the vertical channel delivers only a modest 3.5% increase in industry profits from 0.375 to 0.3882, as against a 9.84% gain for the integrated firm. The results hold true in general for positive marginal costs. For instance, if the costs were $c_1 = c_2 = 0.2$, bundling increases industry profit by only 3.248% under the channel structure, compared with the 6.57% gain for the integrated firm. Hence the profitability metric indicates that the channel structure negatively impacts the attractiveness of bundling.
Next, we evaluate the question using a different metric, the fraction of bundle sales \( Q_B/(Q_1 + Q_2 + Q_B) \) in equilibrium, which captures the extent to which bundling is part of the selling strategy. The sales levels for the two component products and the bundle (at \( c_1 = c_2 = 0 \)) are 0.1443, 0.1443 and 0.1147 respectively. Comparing retail prices against the solution for the integrated firm given in Table 1, the bundle price is significantly higher, while component prices are only minimally higher, so that the retailer primarily sells component goods rather than the bundle. This is vividly illustrated in Fig. 3, which demonstrates that the share of the bundle as a fraction of overall sales drops substantially from 80.6590% under the integrated-firm structure to 28.4442%. The same property also holds in the more general case of positive marginal costs, and is depicted in Figure 4.

Finally, the impact of the channel structure can be understood more sharply by examining the manufacturers’ pricing behavior. First, consider the optimal prices for the integrated firm (Lemma 1): the price of each good under component-selling is independent of the cost of the other (i.e., \( \frac{\partial p_1}{\partial w_2} = 0 \)), whereas the prices are co-dependent on the cost of both goods under mixed bundling. Second, consider the same property for the optimal prices in the channel structure. From Lemma 2, each manufacturer’s equilibrium price depends only on
its own cost and is insensitive to the price set by the other manufacturer. This property holds in essence for the price-response functions as well. For instance, from Corollary 1, the price-response elasticity for manufacturer \( i \)'s price to \( w_j \) (computed at \( c_1 = c_2 = 0 \)) is in \([0, 0.07]\) for all \( w \in [0, \frac{2}{3}] \), which is very low relative to that under pure bundling (\( \in [0, 0.5] \)). This behavior is the hallmark of a component-selling rather than bundling regime.

Hence, all these metrics—profits, the share of the bundle as a fraction of overall sales, and the nature of the manufacturers’ pricing rules—indicate that the mixed bundling solution in the vertical channel moves closer to component-selling and away from bundling. While the integrated firm gets a substantial profit increase by choosing bundling over component-selling, such gains are not realized in the decentralized channel because manufacturers set prices in a way that reduces the market share of the bundled good. We conclude that the competitive interplay between firms in the decentralized channel is detrimental to bundling and suppresses the majority of the potential gains from demand-smoothing. The findings from this section are summarized below.

**Proposition 1 (Mixed Bundling in the Vertical Channel)** The decentralized vertical channel structure weakens the benefits from bundling. Mixed bundling leads to a smaller percentage increase in profit when a retailer bundles component goods from independent manufacturers, relative to when a firm bundles its own products. Mixed bundling in the
decentralized vertical channel performs more like component-selling than bundling, in terms of the fraction of bundle sales (smaller) and manufacturers’ price reaction functions (independent of the other manufacturer).

3.3 Vertical vs. Horizontal Channel Conflicts

We have demonstrated that the channel structure negatively affects the attractiveness of bundling. Although demand smoothing creates a large pool of relatively high-value buyers in the retail market (and the retailer predictably discounts the bundle relative to component pricing), the presence of this pool is exploited by manufacturers who set higher wholesale prices. This greedy approach raises the bundler’s (i.e., retailer’s) unit costs over the integrated structure. Consistent with intuition about the effect of product costs on the benefits of bundling (Schmalensee, 1984; McAfee et al., 1989; Bakos and Brynjolfsson, 1999), bundling is no longer as attractive. These outcomes are driven jointly by two types of channel conflict, vertical (between the manufacturers and retailer) and horizontal (among manufacturers). This section explores the relative influence of these two types of conflict.

Vertical conflict has a direct negative effect on bundling, by increasing the bundler’s (retailer’s) unit costs for the two goods. There are two causes. First, double marginalization, or the manufacturers’ need to earn a margin on sales of component goods. Second, the adverse impact to manufacturers from setting a low price: they earn lower margins, and fail to realize a proportionate increase in sales because the retailer does not proportionately lower the market price. This can be validated by running a simple thought experiment (for this illustration, assume $c_1 = c_2 = 0$), in which manufacturers reduce their prices from the equilibrium level of 0.5067 to $w_i = 0.5$. The retailer would react with a small reduction in prices, $p_i$ to 0.7752 and $p_B$ to 1.4196. But while the retailer’s profit increases 2.7827%, manufacturers see only a small increase of 0.1% (industry profits increase 1% to 0.3920). The imbalance is amplified under a larger drop in prices (e.g., dropping to $w_i = 0.25$ would yield a windfall for the retailer while reducing the manufacturers’ profit).

The horizontal conflict occurs in the stage 1 simultaneous pricing game where manu-
facturers set prices for the component goods. Here, each manufacturer is aware that its price (increase) can be masked into the bundle price charged by the retailer; and conversely, that its own interests are adversely affected by overpricing by the other manufacturer. For instance, a price increase by one content provider in a television bundle would cause only a small decrease in its own demand (and demand of all providers in the bundle) because the retailer’s price increase would be smaller in percentage terms. This behavior is akin to composite goods (Cournot, 1929, Ch. IX), where the total component price becomes a public resource which leads to free-riding by each manufacturer, and causes manufacturers’ prices in equilibrium to be even higher than the level caused by vertical channel conflict. This explains why the thought experiment discussed above (dropping $w_i$’s to 0.5) is not an equilibrium outcome: even though it raises profits for every firm (and employs the retailer’s optimal price-reaction rule), these price levels give each manufacturer an incentive to unilaterally raise its own price and increase its own profit. Hence, the horizontal conflict between manufacturers is especially influential in reducing the attractiveness of bundling in the decentralized channel.

The effects of the two types of channel conflict can be separated by evaluating a hypothetical intermediate structure, a bilateral monopoly in which a single manufacturer makes goods 1 and 2, and the retailer employs a mixed bundling strategy in selling these goods. Comparison of this structure with the integrated firm isolates the role of vertical conflict. Because there is no horizontal conflict in the bilateral monopoly, any difference in outcomes must be on account of vertical conflict. While our goal is primarily to develop insights, the bilateral monopoly structure does occur in practice, and includes applications that are often treated under a direct-selling setting (e.g., the Microsoft Office bundle, designed by Microsoft but sold through retailers, and home theater audio-video systems).

**Proposition 2 (Mixed Bundling under Bilateral Monopoly)** Each manufacturer prices the component goods at $w_i^* = 0.4822 + 0.5c_i$ while, subject to these prices, the retailer’s pricing of the component goods and bundle is as given in Lemma 1. The bundle selling strategy retains its allure despite the vertical conflict in this structure.

Under the bilateral monopoly structure, vertical conflict does lead to higher unit costs
for the bundler, but the effect is moderated relative to the decentralized channel structure. With \( c_1 = c_2 = 0 \), we get \( w_i = 0.4822 \), and the product sales are \( Q_1 = Q_2 = 0.1462 \) and \( Q_B = 0.1265 \). The manufacturer’s profit from the two goods is 0.263030, and the retailers’ profit is 0.1388. Mixed bundling raises industry profit, relative to component-selling, by 7.1419 % (with \( c_1 = c_2 = 0 \)); recall the corresponding numbers 3.5% for the decentralized channel structure and 9.84% for the integrated firm. Hence, this analysis suggests that the majority of potential gains from bundling are preserved despite vertical conflict, and that horizontal conflict is the more destructive force with respect to bundling.

The bilateral monopoly structure can be viewed as a decentralized channel structure in which manufacturers coordinate on price. Consistent with intuition, such coordination increases the manufacturer’s profits. It also eliminates the free-riding (on price) which occurred under non-coordinated pricing. This leads to lower \( w_i \) prices and hence increase the retailer’s profit as well. For instance, for \( c_1 = c_2 = 0 \), the retailer’s profit increases from 0.1257 under the decentralize channel to 0.1388 (each manufacturer has a smaller increase in profit, from 0.1313 to 0.1315). Finally, as is obvious from the outcome for the integrated firm, coordination between all the firms would lead to even higher industry profit and, because of lower prices in the retail market, higher consumer surplus. While the benefits of price coordination have been suggested previously for complementary products (Lichtman, 2000, e.g., platforms and applications), we see that price coordination between firms can increase consumer (and producer) surplus even in the absence of product complementarity.

**Proposition 3 (Pareto-improving Collaboration between Firms)** Collaboration between manufacturers, or between the manufacturers and retailer, can increase all of their profits and total surplus. Moreover, price coordination can increase surplus of each firm as also consumer surplus.

Finally, the relative roles of vertical and horizontal conflict can be made more sharp by comparing the two extreme selling strategies, pure bundling vs. pure components. This comparison is useful because mixed bundling hides the workings of one essential characteristic of bundling in a vertical channel: the fact that prices set by individual manufacturers are partially masked by the overall bundle price set by the retailer, which gives each manufacturer
an incentive to free-ride and price above an otherwise-efficient level. This characteristic is only mildly present under mixed bundling, because each manufacturer must also contend with the effect of its price increase on the demand for its own component good. In order to make this effect more salient, we analyze the case where the retailer can only employ a pure bundling strategy. In this case, because the retailer sells only the bundle, the free-riding behavior will be magnified and can be isolated more clearly. We do this in the next section.

4 Pure Components vs. Pure Bundling

This section examines pure bundling primarily in order to isolate the role of channel conflicts, but the analysis of pure bundling also has practical relevance. In reality, a retailer’s bundling strategy might be restricted to pure bundling because it can be predicted from historical actions, is necessary to induce manufacturer participation, or because this is the only reasonable business practice. An example is the movie distribution firm Netflix which offers rental access at a flat price to its entire library which comprises movies from several different studios (the component-selling alternative would be a separate price for access to movies from each studio). The component studios are aware of the stickiness of this all-you-can-eat business model when they set prices for their content. Other information aggregators such as news services also have this characteristic. The limitation to pure bundling can also occur in practice when technological challenges make mixed bundling infeasible, or when firms anticipate antitrust concerns (under mixed bundling) regarding the bundle discount relative to component prices (Fang and Norman, 2006). Pure bundling avoids these concerns because there are no component prices to compare with.

4.1 Pure Bundling Equilibrium

The sequence of events in this game is that first the retailer conveys a pre-commitment to bundling, then manufacturers set their component prices, and finally the retailer sets the price for the bundle. Lemma 1 yields the retailer’s optimal pricing rule $p^B$. Each firm sells $Q^B$ units (see Eq. 15), obtained by substituting $p^B$ into Eq. 4. In the first stage, manufacturers
set their prices while taking into account the retailer’s pricing rule and their own sales volume $Q^B$. Normalizing the manufacturers’ marginal costs $c_1, c_2$ to zero, the Nash Equilibrium for manufacturer prices is obtained by solving

$$
\begin{align*}
& w_1^B = \arg \max_{w_1} \left( \Pi_1(w_1, w_2) = w_1 D_B(p_B^B(w_1, w_2^B)) \right) \\
& w_2^B = \arg \max_{w_1} \left( \Pi_2(w_1, w_2) = w_2 D_B(p_B^B(w_1^B, w_2)) \right)
\end{align*}
$$

where each manufacturer’s best-response price (with respect to the other manufacturer’s price) is the better of the optimal values within the two sub-intervals given by $(w_1 + w_2) \gtrless \frac{1}{2}$.

**Proposition 4 (Pure Bundling Regime)** When the retailer pre-commits to bundling products $i = 1, 2$ from independent firms $M_1, M_2$, both the retailer and manufacturers earn lower profit than under a pure components strategy. The pricing game has a unique equilibrium $(w_1^B, w_2^B, p_B^B)$ with

<table>
<thead>
<tr>
<th>Prices</th>
<th>Manufacturer</th>
<th>Retailer</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_1^B = w_2^B = \frac{1}{2}$</td>
<td>$p_R^B = \frac{4}{3}$</td>
<td></td>
</tr>
</tbody>
</table>

and each manufacturer’s optimal price response is a decreasing function of its competitor’s price ($w_i^B = \frac{2 - w_j}{3}$).

Comparing with the pure components solution given in Table 1, both the manufacturers and retailer earn lower profit under the bundle selling strategy. Manufacturers set high wholesale prices, hence the retailer sets a higher retail price, leading to lower sales level and lower profits for all. These inter-firm effects are illustrated by the recent market evolution of Netflix, the leading movie-rental firm in the US. Netflix experienced massive growth from 2007-2010, simultaneously in subscrib manufacturers who set high wholesale prices ($w_i$ is $\frac{1}{2}$ under both pure components and pure bundling). This greedy high-priced approach raises the retailer’s input costs, leading to high retail price, lower sales level and lower profits for all.ers and profitability, because of the market’s rapid acceptance of Internet-based movie streaming. Netflix had acquired multi-year streaming rights from studios, at very low prices, when streaming was a rarity. With these contracts set for renewal around 2011, it became evident that the combination of horizontal and vertical channel conflicts, combined with
the public knowledge regarding the popularity of streaming content, would cause studios to demand substantially higher fees from Netflix. Indeed, during 2010-2011, Netflix cited potential price increases by movie studios as one of the big risks facing its movie streaming business, and experienced a dip in both its subscriber base and profitability when increased costs forced it to raise subscription prices.

4.2 Relative Role of Vertical vs. Horizontal Conflict

<table>
<thead>
<tr>
<th>components</th>
<th>$w_i$</th>
<th>$p$</th>
<th>$Q_i$</th>
<th>$\Pi_M$</th>
<th>$\Pi_R$</th>
<th>$\Pi = \Pi_M + \Pi_R$</th>
<th>$CS$</th>
</tr>
</thead>
<tbody>
<tr>
<td>bundle</td>
<td>$\frac{1}{3}, \frac{1}{3}$</td>
<td>$\frac{10}{9}, \frac{32}{81}, \frac{32}{81}$</td>
<td>$64, 243, 729$</td>
<td>$128, 729, 729$</td>
<td>$320, 729, 729$</td>
<td>$\frac{85,333}{729}$</td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Bundle selling dominates component selling for a bilateral monopoly.

In §3.3, we examined the relative role of vertical and horizontal conflicts by evaluating the intermediate bilateral monopoly structure. Table 3 lists the bilateral monopoly solution under pure bundling. Again, any difference in outcomes with the integrated firm is attributed to the effects of double marginalization or vertical channel conflict. Proposition 5 summarizes the idea that vertical conflict, alone, does not negate the demand-side motivation for bundling. While double marginalization in the vertical channel does lower the firms’ profits, bundle selling nevertheless increases profit for both the manufacturer and retailer than component sales. This result confirms and sharpens the corresponding result under mixed bundling (Proposition 2). Additionally, the sharp contrast between the bilateral monopoly and decentralized channel outcome (Proposition 4) attests that horizontal conflict was the primary culprit behind the failure of bundling in the decentralized channel.

**Proposition 5 (Bilateral Monopoly)** In a bilateral monopoly with bundling, the manufacturer prices each component at $w_i = \frac{1}{3}$ and the retailer sets $p_B = \frac{10}{9}$. Total manufacturer profit ($\frac{64}{243}$) and the retailer’s profit ($\frac{128}{729}$) are higher than under the component selling regime.

---

Based on analysis of 2011 SEC filings. Movie consumption costs presently account for about 10% of Netflix’s cost to serve an average subscriber.
4.3 Strategic Choice of Bundle or Component Sales

§4.1 studied the impact of the channel structure on a retailer whose only product is a bundle of component goods from multiple manufacturers. With this single-product design, the retailer was vulnerable to overpricing by manufacturers, and the potential of bundling to raise firms’ profits was wasted. Might this potential be restored if the retailer were to choose whether to sell a bundle or the components after observing manufacturers’ prices? The expectation in this analysis is that (i) the retailer would choose bundling only if manufacturers set prices low enough, and (ii) knowledge of this selection rule would steer manufacturers towards lower prices in order to realize the bundling (i.e., surplus-maximizing) outcome.

Let $\Theta$ be the low-price region of $(w_1, w_2)$’s for which the retailer prefers to bundle the products. Figure 5 provides a visual depiction of the sequence of decisions made by the manufacturers and retailer. An equilibrium outcome in the overall game is a strategy profile $<w_1, w_2, BC, p_1, p_2, p_B>$ where $BC$ is $B$ or $C$ representing the retailer’s choice of Bundling or Components, respectively. By convention, $p_1$ and $p_2$ are null (sufficiently high) when the retailer picks $B$ while $p_B$ is null when the retailer picks $C$. Since the retailer deterministically sets prices and strategy after observing $w_1$ and $w_2$, any outcome of the game is fully described by the pair of manufacturer prices $(w_1, w_2)$. Analysis of this game reveals that the equilibrium outcome is component-selling rather than bundling.
Proposition 6 (Component-selling Equilibrium) The two-stage game has a unique equilibrium solution in which the manufacturers set prices $w_1 = w_2 = \frac{1}{2}$, and the retailer sells the component products at prices $p_1 = p_2 = \frac{3}{4}$. Each firm earns $\frac{1}{8}$, the total profit across all three firms is $\frac{3}{8}$, and total consumer surplus is $\frac{1}{16}$, with total system surplus $\frac{7}{16}$ (of a possible 1).

As with the pre-committed retailer (Proposition 4), the decentralized structure negates the demand-side motivation for bundling. Even though the retailer holds the threat of switching to the component-selling regime, manufacturers set their prices higher than the level needed to induce the retailer to bundle. When manufacturers set their prices low enough to induce the retailer to bundle ($(w^1, w^2) \in \Theta_2$), the retailer’s bundle pricing rule awards them a lower profit than under their optimal component sales price. And if manufacturers price high enough to make bundling attractive to them, then the retailer earns higher profit from selling the component goods separately. This misalignment of incentives prevents the emergence of a bundling equilibrium even though bundling has the potential to increase all firms’ profit.

4.4 Extensions

The salient insight with respect to pure bundling is that the demand-smoothing benefit of product bundling, which dominates when a firm owns the components it is bundling, loses its power when a firm bundles component goods made by independent and separate upstream manufacturers; under mixed bundling the result is qualitatively the same, that the channel structure weakens the attractiveness of bundling. This result raises the following question: If the demand-side effect were stronger, would it make bundling more likely, or would it simply precipitate the efforts of each firm to garner greater profits? We consider this question by examining two factors that strengthen the demand-side motivation for bundling: negative correlation in valuations for components and a larger number of component goods being bundled.

The demand smoothing effect becomes stronger when demands are anti-correlated ($\rho < 0$), creating a greater pool of consumers willing to buy the bundle at relatively high price.
Will manufacturers exploit these higher reservation prices and set even higher prices for their individual components, thereby destroying the retailer’s incentives to sell a bundle? To examine this, consider the retailer’s decisions for products with dependent demand. As demonstrated by McAfee et al. (1989), there are no explicit forms for the optimal prices of the bundle under dependent demand (but these are needed in the decentralized channel to fully formulate the first-stage manufacturers’ pricing problem). Reisinger (2006) employs an innovative extension of Matutes and Regibeau (1992)’s analysis of bundling two complements in a duopoly (each firm makes both components), leading to analytical solutions for general $\rho$. However, every consumer purchases a bundle (the only question is which component from which firm), making the analysis orthogonal to the key questions analyzed in our model. The general problem in analyzing bundling of components with dependent demand is the lack of closed-form terms for the sum of two dependent random variables (Makarov, 1981; Arbenz et al., 2011). Due to this, we follow McCardle et al. (2007)’s approach of evaluating the extreme case of $\rho = -1$.

For $\rho = -1$, all consumers value the bundle at 1, hence the retail price under the bundling regime is 1. Given $w_i$’s, the retailer will choose to bundle rather than sell components when

$$(1 - w_1 - w_2) \geq \left( \frac{1 - w_1}{2} \right)^2 + \left( \frac{1 - w_2}{2} \right)^2$$

i.e., $w_i \leq \sqrt{2} - 1 \approx 0.3912$. And, indeed, manufacturers will prefer to set this price and induce a bundling equilibrium than deviate and earn the component selling profit. Because the incremental profit from bundling is monotonic (decreasing) in correlation $\rho$ (Gürl et al., 2009), we conclude the following.

**Proposition 7** Let $\rho$ be the correlation in consumer valuations for component goods 1 and 2. Then there exists $\tilde{\rho} \in (-1, 0)$ such that a bundling equilibrium emerges whenever $\rho < \tilde{\rho}$.

Next, consider the case where the retailer bundles components from $N > 2$ manufacturers. Several authors have examined bundling of large numbers of goods, including Bakos and Brynjolfsson (1999) who employ limit analysis, and Fang and Norman (2006) who study finite number of goods by applying peakedness of distributions (Proschan, 1965). With
independent product valuations, each consumer’s valuation of the bundle gets arbitrarily close to $\frac{N}{2}$ as $N$ increases. In a bundling regime, the retailer would set bundle price at $\approx \frac{N}{2}$, and pick the bundling strategy when

$$\left(\frac{N}{2} - \sum_{i=1}^{N} w_i\right) \times 1 \geq \sum_{i=1}^{N} \left(\frac{1 - w_i}{2}\right)^2$$

which yields the condition (by considering symmetry in manufacturer pricing) $w_i \leq -1 + \sqrt{2} \approx 0.4142$. And, indeed, each manufacturer earns a higher profit ($0.4142 \times 1$) than the component-selling profit they would earn on deviating from this price. Hence we have shown that with large $N$, bundle selling would be more profitable than components even in the absence of any supply-side economic motivation.

**Proposition 8** There exists $\hat{N}$ such that for all $N > \hat{N}$, there is a bundling equilibrium when the retailer has an opportunity to bundle $N$ components from different manufacturers.

## 5 Conclusion

This paper has analyzed an important factor that is ignored in the vast literature on bundling, i.e., that bundling is often practiced in a decentralized channel in which production and price-setting of components is managed by separate manufacturers acting independently of each other, and the selling strategy (bundle or components) is determined by a downstream retailer. Industry outcomes under this structure follow from a complex intertwining of two horizontal and vertical channel conflicts with the economic motivation for bundling. We have developed an analytically tractable model for understanding the linkage between these various economic forces, and are able to address a number of novel and challenging issues that arise in this unique multi-echelon distribution channel.

Our analysis demonstrates that the demand-side motivation (lower dispersion in bundle valuations across consumers)—which is sufficient to induce bundling in the direct channel—is no longer sufficient in the decentralized channel. We show that this effect is caused not by the vertical conflict alone (i.e., double marginalization) but by horizontal conflict caused by the disaggregation in component production and pricing. The latter feature leads to
It seems like an annual rite: to usher in the new year, cable providers and networks squabble over programming fees.

Figure 6: A few examples of carriage disputes in the TV industry (Jan-Sep 2010).

overpricing of components by manufacturers, reducing the retailer’s incentives to offer a bundle in the market. Firms face a Prisoner’s Dilemma in pricing: all would earn higher profits if manufacturers set prices lower than their component-selling optimal and the retailer priced below the level that maximized its own profit. Consequently, many industries feature bundling solutions which exhibit price and revenue-sharing tensions among participants, where these participants perceive an opportunity to raise prices and attain higher profits in the short-term but at the risk of destroying the bundle outcome in the market. Such conflicts have been most conspicuous in the TV industry in the form of carriage fee disputes, as illustrated by recent news headlines in Figure 6. Our results extend in the predictable way to alternative demand structures (correlation between component demands) or number of component goods. Specifically, the negative effects of channel conflict can be overcome when the demand-smoothing force is stronger, for instance when component demands have negative correlation or when the firm can bundle a large number of components. Bundle sales will also occur when a retailer can employ mixed bundling. However, even under this
strategy, the channel conflicts negate much of the economic potential of bundling.

Our model leaves open additional directions that have been investigated in the direct-selling setting and might be of interest in future work on bundling in a vertical channel. These include bundling of vertically differentiated products (Banciu et al., 2010), bundling of complements and substitutes (Venkatesh and Kamakura, 2003), and bundling under competition either between retailers or between manufacturers making components that are substitutes for each other. It would also be useful to study formal incentive-compatible revenue-sharing mechanisms that can improve profits for all parties (and possibly consumer surplus) by achieving better price coordination in the decentralized channel. Industry-specific arrangements may also be relevant, such as, in the TV industry, advertising. Since ad revenue accrues primarily to the manufacturers (content owners) they have an increased incentive to maximize subscribers rather than margins, pushing the outcome towards lower component prices and lower bundle price.

Finally, industry structure is often more complex than assumed in our simple model with two single-product manufacturers. TV bundles feature hundreds of channels, not just two, though most of them from about 6-10 programming networks and studios. Many of the big studios pre-bundle more than a dozen channels in their negotiations with the retailers. For example, Disney executives negotiate for the inclusion of certain less-popular channels in exchange for the right to carry ESPN, similarly News Corp. charges a bundle price for the collection of FOX channels. Another complicating factor is that some retailers are also manufacturers who provide their own content (e.g., Comcast several networks including E! The Style Network, G4, and the Golf Channel).\footnote{http://www.comcast.com/corporate/about/pressroom/comcastcabelnetworks/comcastcabelnetworks.html} Incorporating any of these features into our model would be challenging but would highly enrich the analysis. Our model is a useful starting point and provides an analysis framework that is computationally tractable and insightful. We hope that it will spur substantial new work in this exciting area.
Appendix

A.1 Integrated Firm

**Proof of Lemma 1.** The optimal price under component selling \( p_C^i = \frac{1}{2} + \frac{w_i}{2} \) follows trivially from the first-order conditions for the linear demand formulation for each good. Good \( i \) sells \( Q_C^i = \frac{1}{2} - \frac{w_i}{2} \) units, and the firm’s profit is \( \frac{(1-w_1)^2 + (1-w_2)^2}{4} \). For pure bundling, Venkatesh and Kamakura (2003) solve for the optimal bundle price under symmetric marginal costs.

The optimal pure-bundle price under asymmetric costs is obtained by maximizing the profit function \( \Pi_B^R = (p - W)D_B(p) \), where \( W \) is the per-unit cost of the bundle, \( W = w_1 + w_2 \).

Consider the profit function for the two price segments \([0, 1]\) and \([1, 2]\).

\[
p \leq 1: \quad \Pi_B^R = (p - W)(1 - \frac{p^2}{2}) \text{ yields } p^* = \frac{1}{3}(W + \sqrt{6 + W^2}). \quad \text{This is valid } (p \leq 1) \text{ only for } W \leq \frac{1}{2}.
\]

\[
p \geq 1: \quad \Pi_B^R = \frac{1}{2}(p - W)(2 - p)^2 \text{ yields } p^* = \frac{2}{3}(1 + W), \quad \text{valid only when } W \geq \frac{1}{2}.
\]

By definition, \( \Pi_B^R = \max\{\Pi_B^R(p \leq 1), \Pi_B^R(p \geq 1)\} \). Comparing the two profits, the switchover point is \( W = \frac{1}{2} \). The quantity sold and profits are obtained by substituting into Eq. 4. Algebraic comparison of pure component and pure bundle profits yields the \( \Theta \) regions.

Finally, for mixed bundling, its dominance over other selling strategies under Assumptions 1–2 is established by several authors including McAfee et al. (1989); Eckalbar (2010). The mixed bundle pricing problem has, however, been addressed only with computational and graphical techniques, and without an explicit form for optimal prices (see e.g., Salinger (1995); Schmalensee (1984); Venkatesh and Kamakura (2003); Ibragimov and Walden (2010); Eckalbar (2010)). Recently, a highly accurate pseudo-analytical solution was developed by Anonymous (2011), and we employ this solution in our computations. The solution analytically specifies \( p_1^*, p_2^* \) as exact functions of \( p_B^* \), and it derives an accurate approximation of \( p_B^* \) as a linear function of marginal costs of goods 1 and 2 \((p_B^* = 0.8603 + 0.5593(w_1 + w_2))\). The \( p_B^* \) approximation is highly accurate and achieves over 99.5% of the exact optimal profit.
A.2 Mixed Bundling Equilibrium in Vertical Channel

Proof of Lemma 2. We start the stage 1 analysis by computing the sales quantities for each manufacturer. These terms ($\hat{Q}_1 = Q_1 + Q_B$, and $\hat{Q}_2 = Q_2 + Q_B$, respectively) are computed by substituting the retailer’s pricing rules given in Lemma 1 into Eq. 5. Algebraic rearrangement and simplification yields Eq. 7. Because these $\hat{Q}_i$ terms are highly nonlinear, the two-player game in Eq. 8 precludes direct analytical solution. However, the Eq. 7 expressions for $\hat{Q}_i$ can be replaced with a highly accurate linear approximation, either (i) a Taylor series approximation of the expressions, or (ii) a linear regression estimate using the true values of $\hat{Q}_i$ over a vector of values for $w_1$ and $w_2$ respectively. For the Taylor expression, we compute it around $(w_1 = 0.5, w_2 = 0.5)$, which are the equilibrium prices for the special case of zero marginal costs (this can be verified through brute force computation for the special case), yielding the functions

$$\hat{Q}_1 \approx 0.5510 - 0.5143w_1 - 0.0624w_2$$
$$\hat{Q}_2 \approx 0.5510 - 0.5143w_2 - 0.0624w_1. \quad (12)$$

The alternative approximation via linear regression yields

$$\hat{Q}_1 \approx 0.5567 - 0.5234w_1 - 0.0484w_2$$
$$\hat{Q}_2 \approx 0.5567 - 0.5234w_2 - 0.0484w_1. \quad (13)$$

The accuracy of each of these approximations is depicted in Fig. 7, which depicts the percentage deviation from the actual value of $\hat{Q}_1$. Note that the equilibrium prices under positive marginal costs will generally be above 0.5 (which corresponds to zero costs). The Taylor approximation for $\hat{Q}_1$ works best when $w_2$ is small (below 0.5; and a symmetric statement holds for the approximation for $\hat{Q}_2$), and the linear approximation works well for higher values of $w_2$. We develop an additional, hybrid, approximation by averaging the two approximation functions. This yields the $\hat{Q}_1, \hat{Q}_2$ functions stated in Eq. 9, and the hybrid approximation is fairly robust (within 3% of the exact value throughout the region). The alternative approximations are even closer in terms of the equilibrium values they produce for $w^*_1, w^*_2$ after solving the simultaneous pricing game.
Substituting the linear approximations for $\hat{Q}_1$ and $\hat{Q}_2$ into the manufacturers’ profit functions, Eq. 8 can be solved analytically to produce the Nash equilibrium prices $w_1^*, w_2^*$. Manufacturer $i$’s best-response function to $j$’s price and in anticipation of the retailer’s pricing rules, is

$$\Omega_1(w_2) = 0.5337 + 0.5c_1 - 0.0534w_2$$
$$\Omega_2(w_1) = 0.5337 + 0.5c_2 - 0.0534w_1$$

Solving the pair of simultaneous equations yields the equilibrium price functions.

**Proof of Proposition 2.** The manufacturer’s profit function in stage 1 of the game is $(w_1 - c_1)\hat{Q}_1 + (w_2 - c_2)\hat{Q}_2$. From first-order conditions for the two decision variables $w_1, w_2$, the optimal prices satisfy the two symmetric simultaneous equations $w_i = 0.5337 + 0.5c_i + 0.0534c_j - 0.1068w_j$. Solving these simultaneous equations yields the unique solution, $w_1^* = 0.4822 + 0.5c_1; w_2^* = 0.4822 + 0.5c_2$. Bundling does substantially increase the firms’ profit relative to the pure component-selling solution, as illustrated for the $c_1 = c_2 = 0$ case in the text.

**Proof of Proposition 3.** The bilateral monopoly solution validates the claim about price coordination between manufacturers. The integrated firm solution validates the claim about vertical price coordination across the channel.
A.3 Pure Bundling

A.3.1 Pre-Committed Retailer

Proof of Proposition 4. Plugging \( p^B \) into Eq. 4, each firm sells \( Q^B = D_B(p^B_B(w_1, w_2)) \) units,

\[
Q^B = \begin{cases} 
\frac{2}{3} - \frac{w_1 + w_2}{9}((w_1 + w_2) + \sqrt{6 + (w_1 + w_2)^2}) & \text{if } w_1 + w_2 < \frac{1}{2} \\
\frac{2}{9}(2 - w_1 - w_2)^2 & \text{o/w.}
\end{cases}
\]

(15)

The first-stage Nash Equilibrium in manufacturer prices is derived by plugging \( Q_B \) into the system of equations in Eq. 10. Each manufacturer’s best-response price is the better of the optimal values within the two sub-intervals given by \( W \gtrless \frac{1}{2} \). Thus we need to solve these two simultaneous conditional equations. However, let’s examine the manufacturer price behavior within the lower-price interval \( W \leq \frac{1}{2} \).

\[
\frac{d\Pi_i}{dw_i} = \left( \frac{2}{3} - \frac{w_i}{9}(W + \sqrt{6 + W^2}) \right) - \frac{w_i}{9} \left( 2W + \sqrt{6 + W^2} + \frac{W^2}{\sqrt{6+W^2}} \right)
\]

= (is monotonically decreasing in \( w_i \))

= \[ \frac{2}{3} - \frac{1}{6} - \frac{w_i}{9} \left( \frac{36}{10} \right) \]

= \[ \frac{1}{2} - \frac{1}{6} - \frac{w_i}{9} \left( \frac{36}{10} \right) \]

= \[ \frac{3}{10} + \frac{2w_i}{9} \]

\[ > 0. \]

Since each manufacturer \( i \)'s profit is increasing in its own price the highest profit within this interval is at the extreme point, leading us to the second interval \( (w_1 + w_2 + c) \geq \frac{1}{2} \). Thus, \( Q_B \) must equal \[ \frac{2}{9}(2 - w_1 - w_2)^2 \]. The Nash Equilibrium in manufacturer prices is obtained by plugging this into Eq. 10 which yields the following system of equations.

\[
\begin{align*}
\frac{2}{9}(2 - w_1 - w_2)^2 - \frac{4}{9}w_1(2 - w_1 - w_2) &= 0 \\
\frac{2}{9}(2 - w_1 - w_2)^2 - \frac{4}{9}w_2(2 - w_1 - w_2) &= 0.
\end{align*}
\]

This system has a unique solution given in the result.

Proof of Proposition 5. The retailer’s profit under component sales in a bilateral monopoly is \( (p_1 - w_1)(1 - p_1) + (p_2 - w_2)(1 - p_2) \), yielding the standard solution for linear demand: \( p_i = \frac{1+w_i}{2} \). Plugging this back into the manufacturer’s pricing problem, \( w_i = \frac{1}{2} \),
then \( p_i = \frac{3}{4}, Q_i = \frac{1}{4} \). The manufacturer earns \( \frac{1}{8} \) for each product, and the retailer earns a total of \( \frac{1}{8} \) from the two products.

For bundle sales, write \( W = w_1 + w_2 \), then the equilibrium outcome under bundling is a pair \((W^B, p^B)\)

\[
p^B(W) = \arg \max_p (p - W)D_B(p) \quad \text{Retailer Profit}
\]

\[
W^B = \arg \max_W W D_B(p^B(W)) \quad \text{Manufacturer Profit}
\]

with a sales level

\[
Q_B = \begin{cases} 
\frac{2}{3} - \frac{W}{9}(W + \sqrt{6 + W^2}) & \text{if } W \leq \frac{1}{2} \\
\frac{2}{9}(2 - W)^2 & \text{if } W > \frac{1}{2}.
\end{cases}
\]

The (single) manufacturer sees his profit as \( \Pi_M = WQ_B \), and will seek the higher profit between the two \( p^B \) intervals corresponding to \( W \gtrless \frac{1}{2} \). However, let’s examine the manufacturer price behavior within the lower-price interval \( W \leq \frac{1}{2} \).

\[
\frac{d\Pi_M}{dW} = Q_B - \frac{W}{9} \left( 2W + \sqrt{6 + W^2} + \frac{W^2}{\sqrt{6 + W^2}} \right)
\]

\[
= \frac{2}{3} - \frac{W}{9} \left( 3W + 2\sqrt{6 + W^2} + \frac{W^2}{\sqrt{6 + W^2}} \right)
\]

\[= \text{(is monotonically decreasing in } W)\]

\[= \frac{2}{3} - \frac{1}{18} \left( \frac{3}{2} + 5 + \frac{1}{10} \right) \quad \text{at } W = \frac{1}{2}
\]

\[= \frac{3}{10} > 0.
\]

The manufacturer’s profit is increasing throughout the \( W \leq \frac{1}{2} \) hence the optimal \( W \) is in the higher-priced region corresponding to \( p^B > 1 \). Maximizing \( \Pi_M = W \cdot \frac{2}{9}(2 - W)^2 \) yields \( W^B = \frac{2}{3} \) (i.e., \( w_1 = w_2 = \frac{1}{3} \)), \( p^B = \frac{10}{9} \), \( Q_B = \frac{32}{81} \). Plugging this into the profit functions completes the result.

\[
\blacksquare
\]
A.3.2 Strategic Choice by Retailer

Proof of Proposition 6. For \( i = 1, 2, R \), define \( \Pi^C_i(w_1, w_2) \) as player \( i \)'s profit when the retailer sells Components and picks the best (for retailer) component prices corresponding to \((w_1, w_2)\). The bundle regime follows a similar notation with \( \Pi^B_i(w_1, w_2) \). Then the pair of prices \((w_1, w_2)\) constitutes a bundling equilibrium if the retailer prefers to bundle at these prices, and each manufacturer prefers its own price over any other price \( w \) (including prices that induce the retailer to sell as components).

\[
\begin{align*}
\Pi^B_R(w_1, w_2) & \geq \Pi^C_R(w_1, w_2) \quad \text{AND} \quad \\
\Pi^B_1(w_1, w_2) & > \max_w \{ \Pi^B_1(w, w_2), \Pi^C_1(w, w_2) \}, \\
\Pi^B_2(w_1, w_2) & > \max_w \{ \Pi^B_2(w_1, w), \Pi^C_2(w_1, w) \}. \\
\end{align*}
\]  

Similarly, \((w_1, w_2)\) constitutes a component equilibrium if the retailer prefers a components selling strategy, and each manufacturer prefers its own price over any other price \( w \) (including prices that induce the retailer to bundle).

\[
\begin{align*}
\Pi^C_R(w_1, w_2) & \geq \Pi^B_R(w_1, w_2) \quad \text{AND} \quad \\
\Pi^C_1(w_1, w_2) & > \max_w \{ \Pi^B_1(w_1, w), \Pi^C_1(w_1, w) \}, \\
\Pi^C_2(w_1, w_2) & > \max_w \{ \Pi^B_2(w_1, w), \Pi^C_2(w_1, w) \}. \\
\end{align*}
\]  

The first half of both Eq. 18 and Eq. 19 represents the Retailer, while the second half (in curly braces) covers simultaneous price-setting by manufacturers, each of whom picks its best price taking the competitor’s price as a given, but while anticipating the retailer’s bundling/components strategy (and prices) as a function of the pair of input prices. We optimize and compare the retailer’s optimal profits conditional on each of the selling strategies.

\[
\Pi_R = \max \left\{ \begin{array}{ll}
\max_p (p - w_1 - w_2) D_R(p) & \text{retailer sells bundle} \\
\left( \frac{1-w_1}{2} \right)^2 + \left( \frac{1-w_2}{2} \right)^2 & \text{retailer sells components}
\end{array} \right. 
\]
For bundling, the optimal retail prices for the two cases of \( W \geq \frac{1}{2} \) follow from the proof of Proposition 4, while \( Q_B \) follows from Eq. 5, and the optimal profit is simply \( (p_B - W)Q_B \). The retailer’s profit under a component selling strategy, given prices \( w_1, w_2 \), is \( \Pi^C_R = \frac{(1-w_1)^2 + (1-w_2)^2}{4} \), with \( p_i^C = \frac{1}{2} + \frac{w_i}{2} \) and \( Q_i^C = \frac{1}{2} - \frac{w_i}{2} \). Comparing the component and bundle profits, the retailer picks bundle selling if manufacturer prices \( (w_1, w_2) \) are in \( \Theta = \Theta_1 \cup \Theta_2 \) where (with \( W = w_1 + w_2 \))

\[
\begin{align*}
\Theta_1 &= \left\{ (w_1, w_2) : (W \leq \frac{1}{2}) \text{AND} \left( \frac{W^3 + (6 + W^2)}{27} - 18W \geq \sum_{i=1}^{2} \frac{(1-w_i)^2}{4} \right) \right\} \quad (21a) \\
\Theta_2 &= \left\{ (w_1, w_2) : (W \geq \frac{1}{2}) \text{AND} \left( 2 \left( \frac{2-W}{3} \right)^3 \geq \sum_{i=1}^{2} \frac{(1-w_i)^2}{4} \right) \right\} \quad (21b)
\end{align*}
\]

and then her pricing rule and profits are

\[
\begin{array}{c|c|c}
\text{Price } p_B^R & W \leq \frac{1}{2} & W \geq \frac{1}{2} \\
\hline
\text{Quantity } Q_B^R & \frac{1}{3}(W + \sqrt{6 + W^2}) & \frac{2}{3}(1 + W) \\
\text{Profit } \Pi_B^R & \frac{2}{3} - \frac{W}{9}(W + \sqrt{6 + W^2}) & 2 \left( \frac{2-W}{3} \right)^2 \\
\end{array}
\quad (22)
\]

Otherwise, the retailer sells components at \( p_i = \frac{1+w_i}{2} \), with \( Q_i = \frac{1-w_i}{2} \).

Turning to the first stage of manufacturer pricing, consider the strategic thinking of Manufacturer M2 who earns \( w_2 \cdot Q_2 \) or \( w_2 \cdot Q_B \). For an M1 price of \( w_1 \), M2 can either pick the best price \( w_2 \) such that \( (w_1, w_2) \) is in \( \Theta = \Theta_1 \cup \Theta_2 \) and earn a bundle profit \( \Pi^B_R(w_1, w_2) \), or the best price such that \( (w_1, w_2) \notin \Theta \) and earn component profit \( \Pi^C_i(w_1, w_2) \) (no more than \( \frac{1}{8} \)). However, we can rule out \( (w_1, w_2) \in \Theta_1 \). (1) If, to the contrary, there is a bundle equilibrium \( (w_1, w_2) \) in which \( W = w_1 + w_2 < \frac{1}{2} \), then each manufacturer assumes the retailer’s “low price” rule (see Eq.22), with \( Q^B = \frac{2}{3} - \frac{W}{9}(W + \sqrt{6 + W^2}) \). First-order conditions for the manufacturer’s problem produce \( w_1 = w_2 = w \), and \( W = \sqrt{13 - 3} \approx 0.7781 \) which violates the requirement \( W \leq \frac{1}{2} \). Hence there cannot be an interior bundling equilibrium in \( \Theta_1 \). (2) The absence of a component equilibrium \( (w_1, w_2) \) in which \( W \leq \frac{1}{2} \) is trivial, because each \( w_i = \frac{1}{2} \) under component selling. With the elimination of \( \Theta_1 \), we can restrict the retailer’s actions to either the “high bundle price” rule \( p_B = \frac{2}{3}(1 + W) \) or component price \( p_i = \frac{1-w_i}{2} \).
With \( W = (w_1 + w_2) \geq \frac{1}{2} \), the retailer would (under the bundle strategy) employ the “high price” rule. Given \( w_i \), Mj’s best-response price is either \( w_j = \frac{2 - w_i}{3} \) (anticipating the retailer’s “high price” rule, and conditional on the retailer preferring to bundle at these prices) or its best price that induces a component-selling outcome by the retailer. The bundling case has a possible Nash equilibrium solution \( w_1 = w_2 = \frac{1}{2} \), with the retailer’s price at \( \frac{4}{3} \) (see the proof of Proposition 4). But this requires two conditions, (i) the Retailer should prefer to bundle at these prices, and (ii) M2 should prefer this price (with a bundle outcome) than a component-selling price. But the first condition fails because it translates to \( \frac{2}{27} \geq \frac{1}{8} \). Hence the only possible outcome is component-selling.

References


