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THE TRANSITION TO NONDEPLETABLE ENERGY: SOCIAL PLANNING AND MARKET MODELS OF CAPACITY EXPANSION

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As depletable energy becomes increasingly scarce and expensive, energy markets will turn to a multitude of nondepletable sources of energy. The rate at which this transition occurs will depend on the quantities of fuels available, production costs, government policies, and the choices of major producers. In this paper, we analyze the transition to nondepletable fuels, first in the context of a social planning model, then using a Stackelberg model to represent the dynamic game between owners of depletable and nondepletable fuels. We show that the pace of capacity expansion in the nondepletable sector has a strong influence on socially optimal energy prices and production rates. With the Stackelberg model, we characterize the strategy a market-dominating producer of depletable energy will use against the nondepletable sector. Numerical implementations of this model allow us to compare the socially optimal and market-determined outcomes. Our results show how a dominant depletable-energy producer can manipulate the nondepletable sector by pricing at the marginal cost of backstop output during an early phase of resource depletion.

Depletable energy sources, primarily crude oil and natural gas, have been the mainstay of world energy consumption for decades. As these types of energy become more scarce and their costs rise, other sources of energy and new technologies for utilizing energy will be developed. Uncertainty about future energy demand and supply (both conventional and unconventional) makes it difficult to predict the nature and timing of the transition from one technology to the next. Many of the technologies that will eventually play a major role in energy production have not yet been invented. So it is impossible to describe in any detail the evolution of energy production in the long run. In particular, predictions of the market shares of individual unconventional energy sources are subject to wide margins of error.

The concept of a *backstop* energy source was invented[1] (Nordhaus 1979), to simplify the analysis of long-run energy issues. The backstop is a hypothetical energy source that can produce unlimited quantities of energy at a constant cost. That is, the unit cost of backstop energy is assumed to remain the same no matter how high the current *rate* of production or the total *cumulative* production.

The backstop is an abstraction of the entire range of energy sources and technologies that must eventually replace depletable energy. The backstop concept is powerful because it provides a realistic solution to the problem of how best to plan consumption of the fixed stock of depletable energy. The backstop provides, in effect, a long-run steady state for the energy economy: without it, available energy sources must inevitably be exhausted and economic collapse may be unavoidable. Since the supply of backstop energy is unlimited (by assumption), its unit cost sets a ceiling for the price of depletable energy. Typically, then, an efficient consumption plan for depletable energy is characterized by a price which rises monotonically to the backstop cost. At the moment the price reaches this level, the stock of depletable energy is exhausted and the transition to the backstop occurs.

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The standard model of this transition is a social planning model in which the production rates of the two homogenous resources, depletable and backstop energy, are determined over time to maximize the present value of utility net of extraction and production costs. Two notable features of this model are, first, that energy prices never rise above the cost of the backstop, and second, that the transition from one source to the other occurs instantaneously; i.e., backstop output jumps from zero to its steady state level at the transition date.

In this paper, we focus on two important aspects of the interaction between depletable and nondepletable energy. The first is the effect of capital costs on the evolution of nondepletable energy production. The available evidence indicates that capital costs for new energy technologies are high, and that they rise rapidly with the rate of investment. So even if the potential supply of backstop energy is unlimited, its cost may not be constant over successive vintages of capacity. One question we seek to answer, then, is what is the optimal rate of investment in backstop capacity, and what is the resulting price path of energy?

The second issue we address is the strategic interaction between the established depletable energy sector and a nondepletable energy sector whose potential for expansion constantly threatens the depletable energy sector's market share. Spokesmen for OPEC, for example, repeatedly have expressed their concern that high OPEC prices will encourage development of new energy sources. And potential producers of unconventional energy (e.g., shale oil and tar sands) similarly have expressed their fear that OPEC might retaliate against them by driving the world oil price below their breakeven level. We model the interaction between these two sectors as a Stackelberg game, with the depletable energy sector as the leader and the backstop sector as the follower.

The paper is organized as follows. In Section 1 we lay out the basic backstop capacity model, using a social planning framework. The analytic solution to this model is then outlined. In Section 2 we develop the Stackelberg model in which the backstop capacity model is used for the follower, and the leader is the depletable resource sector. Finally, in Section 3 we present the results of numerical implementations of these two models. These numerical examples allow us to compare the Stackelberg and social planning solutions, and to study the sensitivity of the models to backstop capacity costs. In Section 4, we summarize our results.

1. Backstop Capacity Expansion: Social Planning Model

1.1. Model Formulation

The model and analysis presented in this section were originally developed in Powell (1983). A closely related discrete time model was developed by Switzer and Salant (1980) and recently published in French (Switzer and Salant 1986). For completeness of our discussion, we present a brief description of the continuous time formulation and analysis. This will provide a point of reference for the Stackelberg model developed in Section 2.

We assume that a social planner controls the rate of production of all energy resources. These resources are of two types: depletable energy, which is costless[2] to produce but available in a limited amount, and backstop energy, which has a constant production cost and unlimited (physical) availability. The two resources are perfect substitutes; that is, consuming a unit of each provides the same gross utility to society. Since our interest is primarily in the development of backstop capacity, we make a distinction between the operating cost of the backstop, and the cost of creating backstop capacity. In keeping with the spirit of the backstop concept we assume that backstop operating costs are constant, regardless of the scale of production or the vintage of capital used. The cost of investment for the backstop, on the other hand, is not constant. Rather, we assume that beyond some critical rate of investment (per period), the marginal cost of backstop capacity increases.

The rationale for this assumption is simple. Recall that the backstop represents an entire industry, which is expected to grow very fast over a short time span (see Energy Modeling Forum 1982) under the standard model. Such growth will place severe demands on the supply of skilled labor (e.g., engineers) and on scarce materials (e.g., steel pipe). At some rate of growth, these demands will begin to drive up the prices of these key inputs. Eventually, an absolute scarcity of some input may impose a hard constraint on the feasible rate of expansion of the backstop sector. For simplicity, we will assume that the investment cost function is a continuous and monotonically increasing function of the rate of investment; thus, backstop investment shows decreasing returns to scale.

The social planner's problem is to choose the optimal rates of production of depletable and backstop energy, as well as the rate of investment in backstop capacity. We assume the existence of a concave social utility function defined on total energy production. Furthermore, we assume the planner's objective is to

maximize the present value of social utility net of production and investment costs. The operative constraints are, first, that total production of depletable energy not exceed the fixed stock; second, that backstop capacity increases each period by the rate of backstop investment;[3] and third, that backstop production is limited in each period by the current level of backstop capacity. Formally, the planner's problem can be written as follows.

Social Planning Model

$$\underset{\{x_t, y_t, v_t\}}{\text{Max}} \int_0^\infty e^{-rt}[u(x_t + y_t) - cy_t - f(v_t)]\, dt \qquad (1)$$

subject to

$$\dot{S}(t) = x(t); \quad S(0) = 0, \quad S(t) \leq \bar{S}$$

$$\dot{K}(t) = v(t); \quad K(0) = 0$$

$$y(t) \leq K(t)$$

$$x(t) \geq 0, \quad y(t) \geq 0, \quad v(t) \geq 0$$

where

$x(t)$ = depletable resource production
$y(t)$ = backstop production
$v(t)$ = backstop investment
$K(t)$ = backstop capacity
$S(t)$ = cumulative production of depletable energy
$\bar{S}$ = stock of depletable energy
u = social utility function
f = backstop investment cost function
c = backstop operating cost
r = social discount rate.

1.2. Summary of Model Solution

The solution to the social planning model (1) can be described in terms of four phases (see Figure 1). During an initial period, Phase I, only depletable energy is produced, its marginal value (or price) rises exponentially, and backstop investment is zero. The rate of depletable energy production declines steadily. In Phase II, the price of energy continues to rise exponentially until it reaches the level of backstop operating costs. Again, only depletable energy is produced, but backstop investment is positive, even though the growing capacity in the backstop sector is kept idle. Phase III begins as the price of energy rises above backstop operating costs (this is the overshoot result of Switzer and Salant 1980). Backstop production begins, and initially jumps to the level of backstop capacity created during Phase II. Depletable energy production drops by an offsetting amount, as the price of energy continues to rise smoothly. Backstop investment continues, although at a slower rate than during Phase II. At the end of this phase, the energy price reaches its maximum level and depletable energy production ceases with the exhaustion of its stock. During the final period, Phase IV, backstop capacity and output continue to grow and the price declines asymptotically to the backstop operating cost. The steady state level of backstop capacity and output, and the corresponding price of energy, are determined by the requirement that the price of energy must cover both the operating and the capital costs of the last unit of backstop investment.

A number of properties shown by this model are also evident in related models. The phenomenon of prices overshooting the backstop cost has been demonstrated by Dasgupta and Stiglitz (1981) for the case where the arrival data of the backstop is uncertain, and by Oren and Powell (1985) for the case where the backstop cost is uncertain. Chao (1981) considers capital and operating costs separately and notes that simultaneous production from depletable and nondepletable sources is possible. Fuller and Vickson (1986) consider the optimal development of Canadian tar sands and show that the cost of conventional oil will eventually exceed that of tar sands-derived oil. Both demand and tar sands capacity are exogenous in their model. The analysis presented here establishes conditions under which backstop capacity is created and left idle before the price reaches the level of operating costs. It also establishes that while total energy production is continuous, the rate of output of depletable energy drops discontinuously when backstop production begins. A final feature of this model is its characterization of the path of backstop capacity expansion: rising continuously to a maximum rate which corresponds to the date of exhaustion of depletable energy, then declining asymptotically to zero as the steady state level of capacity is approached.

1.3. Solution of Backstop Capacity Model

We now indicate how the major properties of the social planning model are derived. The Hamiltonian for model (1) is

$$H = u(x_t + y_t) - cy_t - f(v_t) + \lambda_s(t)x(t) + \lambda_k(t)v(t) + \mu_t[K_t - y_t] \qquad (2)$$

where the adjoint variables $\lambda_s(t)$ and $\lambda_k(t)$ measure the effect on the objective of changes in the cumulative extraction of depletable resources and backstop capacity, respectively. The variable $\mu(t)$ reflects the impact of the constraint that backstop production cannot exceed capacity.

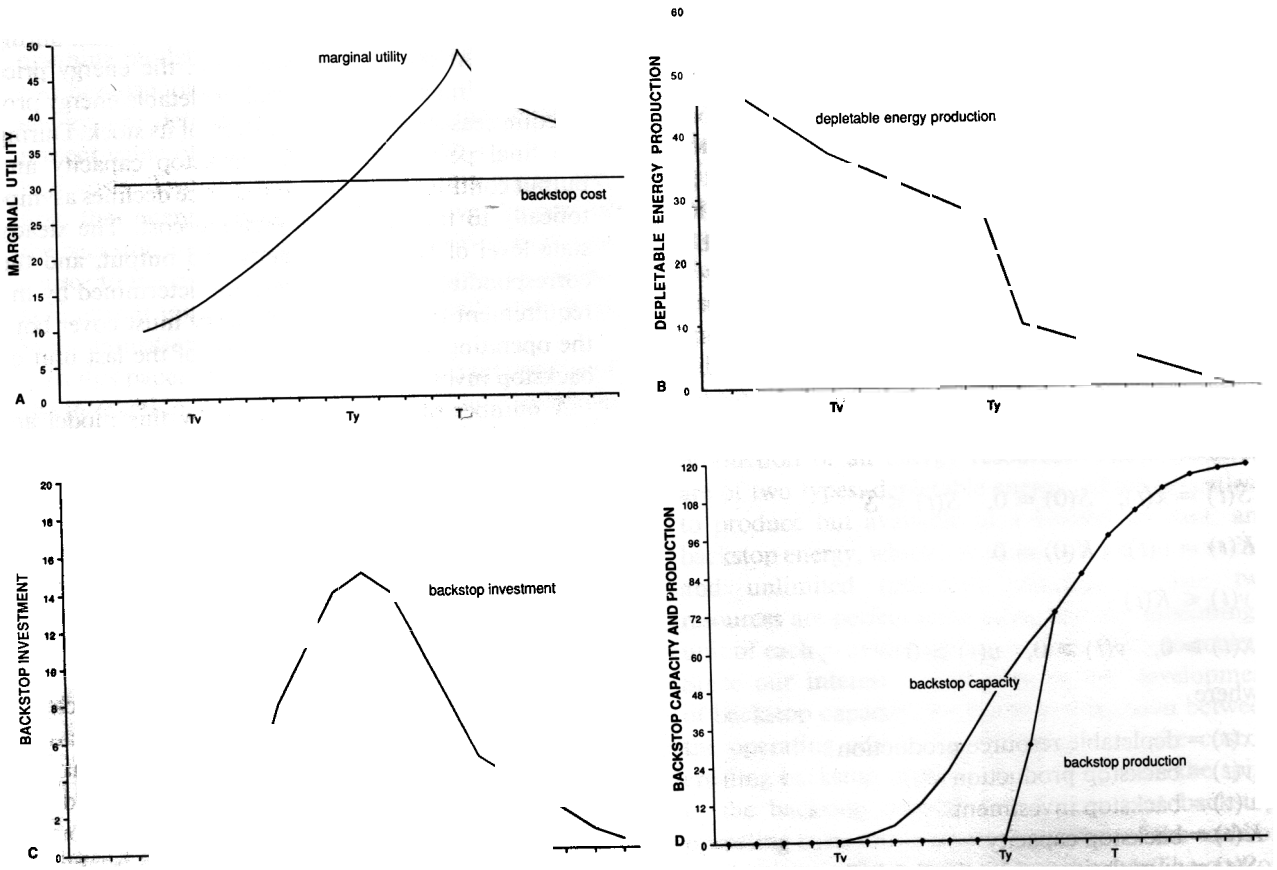

Figure 1. Social planning model.

The complete set of necessary conditions for optimality are given in (3) through (8):

$$u'(x_t + y_t) + \lambda_s(t) = 0 \quad \text{if } x_t > 0$$
$$\leqslant 0 \quad \text{if } x_t = 0 \tag{3}$$

$$u'(x_t + y_t) - c - \mu(t) = 0 \quad \text{if } y_t > 0$$
$$\leqslant 0 \quad \text{if } y_t = 0 \tag{4}$$

$$f'(v_t) = \lambda_k(t) \quad \text{if } v_t > 0$$
$$\geqslant \lambda_k(t) \quad \text{if } v_t = 0 \tag{5}$$

$$\dot{\lambda}_s(t) = r\lambda_s(t) \tag{6}$$

$$\dot{\lambda}_k(t) = r\lambda_k(t) - \mu(t) \tag{7}$$

$$\mu(t) \geqslant 0, \quad \mu(t)[K(t) - y(t)] = 0, \quad y(t) \leqslant K(t). \tag{8}$$

These conditions are also sufficient for optimality (Kamien and Schwartz 1982) under our assumptions that u is concave and f is convex.

We begin our analysis of the backstop capacity model (1) by observing that the necessary conditions (3) and (6) together imply that whenever the production of depletable energy is positive, marginal utility must rise at the rate of interest. That is, when $x(t) > 0$

$$u'(x_t + y_t) = -\lambda_s(0)e^{rt}. \tag{9}$$

A number of properties of the optimal path of depletable energy production follow from this result. First, we can see that production of depletable energy must begin at time zero, since it is costless and all units of backstop energy cost at least c. Next, we note that production of depletable energy cannot extend over an infinite horizon, since the marginal utility of energy (from 9) would eventually exceed the cost of backstop energy. We denote the date of exhaustion of the stock of depletable energy by T. Finally, it can be shown that the marginal utility of energy must exceed c at T, and some backstop capacity must be created before T. Both these properties follow from the necessity of continuity in the path of marginal utility. Marginal utility is continuous throughout since, from (3), it is continuous over intervals where $x > 0$, and from (4), where $y > 0$. And, as we have shown, these two phases overlap.

We now analyze the behavior of backstop production. We have seen that marginal utility rises exponentially on [0, T], while depletable energy is being produced. At some time during this interval, u' crosses c from below. We denote this date by T_y. We can show that if any backstop capacity exists after T_y, output must equal capacity. This follows from (4) and (8), because if $y(t) = 0$ when $K(t) > 0$, (8) implies $\mu(t) = 0$. But this contradicts (4) since $u' > c$. Likewise, backstop production must be zero prior to T_y, when $u' < c$. Otherwise, from (4) $\mu(t) < 0$, which contradicts (8). So T_y, the date when u' passes through c, is also the date when backstop output switches from zero to full capacity.

The third control variable to analyze is the rate of backstop investment. We have remarked that $\lambda_k(t)$ measures the marginal value of a unit of backstop capacity at time T. Equation 5, which determines $v(t)$, simply requires that (for $v_t > 0$) the marginal cost of investment equals its marginal value. And if the marginal cost of the smallest unit of investment exceeds its return, investment is zero. To understand how backstop investment changes over time we analyze the behavior of $\lambda_k(t)$. First we define a useful auxiliary variable

$$\eta(t) = \mu(t) - (u'(x_t + y_t) - c). \tag{10}$$

Then by integrating (7) forward in time and eliminating $\mu(t)$ using (10), we have

$$\lambda_k(t) = \int_t^\infty e^{r(t-\tau)}[u'(x_\tau + y_\tau) - c + \eta(\tau)]\, d\tau. \tag{11}$$

Equation 11 shows that $\lambda_k(t)$, the marginal value of a unit of backstop capacity at t, has two components. One is the present value of a marginal unit of output, $u' - c$, measured over the infinite lifetime of a unit of capacity created at t. The other component relates to the multiplier $\eta(t)$. From (4) and our earlier demonstration that backstop output turns positive when $u' > c$, we know $\eta(t) = 0$ when $u' > c$, and $\eta(t) > 0$ when $u' < c$. Thus, when $u' > c$, the value of $\lambda_k(t)$ comes solely from the utility component. But backstop investment may have value even when $u' < c$; i.e., when the *direct* utility gained from backstop investment is negative. This value arises from the fact that the convex investment cost function penalizes too rapid rates of investment. The social planner will choose to create backstop capacity even when its direct value is negative, and current capacity is idle, because the cost of delaying investment and creating capacity more rapidly at a later date is higher.

We now can establish a key property of this model: backstop investment begins before T_y, the date when marginal utility first reaches the operating cost of the backstop. Let T_v represent the date when backstop investment begins, and assume this occurs after T_y. If investment begins at T_v, then $\lambda_k(t) < f'(0)$ for $t < T_v$ and $\lambda_k(t) > f'(0)$ after T_v. But from (11), we see that $\dot{\lambda}_k(t) < 0$ for $t > T_y$ since $u' > c$ and $\eta(t) \geq 0$. Thus, if $\lambda_k(T_v) = f'(0)$, $\lambda_k(t) > f'(0)$ over some interval prior to T_v, which contradicts the assumption that investment begins after T_y.

Summarizing our results to this point, we have shown that marginal utility rises exponentially on [0, T] while depletable energy is being produced. At some date T_v, when u' is still below c, backstop investment begins. At a subsequent date T_y, when u' first reaches c, backstop production turns positive. Thereafter, u' always exceeds c and backstop output equals its capacity. From T_v to T_y backstop capacity is positive, investment in the backstop continues, but all backstop capacity sits idle.

Two issues remain to be discussed. The first is the behavior of the model after exhaustion of the depletable resource at T. Since $u'(T) > c$ and $K(T) > 0$, it follows that marginal utility can only fall after T, since backstop capacity in this model never depreciates. In fact, it can be shown by using (11) that backstop investment remains positive after T, as does backstop production, and the marginal value of energy approaches a lower asymptote given by $u' = c + rf'(0)$. This condition requires that the last unit of backstop capacity built must return an amount that covers operating costs plus the amortized cost of capacity.

The second issue concerns the path followed by backstop investment. It follows from (11) that the rate of investment must be declining after T_y, since $\eta(t) = 0$ and $u' > c$. But from T_v to T_y, $\eta(t) \geq 0$ and $u' < c$. Thus we cannot use (11) directly to infer the behavior of the rate of investment. However, an indirect argument (the details of which are given in Powell, p. 128 ff.) can be used to establish that backstop investment is increasing up to T_y. Since investment increases up to T_y and decreases thereafter, it must reach its maximum rate at that date.

2. Backstop Capacity Expansion: Stackelberg Model

2.1. Model Formulation

In the previous section, we analyzed a model in which the socially optimal rates of investment and production of backstop energy were determined along with the rate of production of depletable energy. All energy sources were assumed to be under the control of a single, benevolent agent. Now we turn to a model in

which the same variables are determined by a market in which several agents compete. We assume depletable resources are owned by a single agent with the power to influence market price. Backstop resources are also owned by a single agent, but one that lacks market power.[4] To reflect the differences in power between these two competing sectors we model their interaction as a Stackelberg game,[5] with the depletable energy sector as the leader and the backstop sector as the follower. Both players are assumed to have perfect foresight. The crucial difference between them is that the backstop sector, as the follower, takes the depletable energy sector's production rate as given (or, equivalently, the price of energy), while the depletable energy sector takes into account both the effect its production has on price through the demand curve, and the effect it has on backstop investment and production.

The leader in this model is pulled in two directions. Its monopoly power leads it to prefer a low rate of production so as to garner high revenues. But high prices convey a signal to the backstop sector that the return to investment is high. And the higher backstop capacity and output, the lower are the leader's profits. So the existence of the backstop sector must force the leader to choose higher output rates and lower revenues than it otherwise would.

The backstop sector here, as the follower, is essentially controlled by the leader. Whatever price path the leader sets determines backstop investment and output. The backstop sector is passive in this model, while the depletable energy sector is free to choose whatever price path it can sustain, subject to the constraints imposed by market demand and the backstop sector.

This model rests on five important assumptions. First, we assume that all energy resources, depletable and nondepletable, are controlled by either the backstop or depletable energy sectors. Thus, we ignore the so-called competitive fringe, producers of depletable energy who act as price-takers.[6] Second, we assume the depletable energy sector has market power, while the backstop sector acts as a price-taker (at least as long as the leader controls any resources). Third, as the Stackelberg leader, the depletable energy sector takes into account the reaction of the backstop sector to its own plans. The backstop sector, on the other hand, acts as if its decisions have no effect on the leader. Fourth, we assume both players have perfect foresight. Finally, we make the technical assumption that the depletable energy sector acts as the leader only up to an exhaustion date of its choosing. This is not restrictive, since the exhaustion date can be as early or as late as the leader chooses. Once the leader has exhausted its resources, it leaves the market to the backstop sector, which carries on as a monopoly.

We base our model of the follower on the backstop model developed in Section 1. The backstop sector's objective is to maximize the present value of profits, with the future price path of energy ($P(t)$) taken as given (i.e., fixed by the leader). A constant operating cost and convex investment cost function are assumed. Finally, we assume that the leader depletes its resources entirely by some date T. After that date, the backstop sector is left on its own, with whatever capacity it inherited at T.

The formal statement of the follower's problem is as follows.

Stackelberg Model—Follower

$$\underset{\{y_t, v_t\}}{\text{Max}} \int_0^T e^{-rt}[(P_t - c)y_t - f(v_t)]\,dt + \Phi[T, K(T)] \quad (12)$$

subject to

$$\dot{K}(t) = v(t); \quad K(0) = 0$$

$$0 \leq y(t) \leq K(t), \quad v(t) \geq 0.$$

where

$$\Phi[T, K(T)] = \underset{\{y_t, v_t\}}{\text{Max}} \int_T^\infty e^{-rt}[(P_t(y_t) - c)y_t - f(v_t)]\,dt$$

subject to

$$\dot{K}(t) = v(t), \quad K(T) = K_T.$$

The leader's problem can be written very simply if we leave the constraints imposed by the follower implicit. Thus the leader in this model solves the following.

Stackelberg Model—Leader

$$\underset{\{x_t\}, T}{\text{Max}} \int_0^T e^{-rt}[P(x_t + y_t)x_t]\,dt \quad (13)$$

subject to

$$\dot{S}(t) = x(t), \quad S(0) = 0, \quad S(T) = \bar{S}$$

and the constraints imposed by the backstop sector.

2.2. Solution for Backstop Sector

The analysis of the backstop sector in this model closely parallels the procedures used for the social planning model. (Details of the arguments made in

this and the following sections can be found in Powell, Chapter 5.) The relationship between price and backstop operating cost determines backstop output, with no output when $P(t) < c$ and output at capacity when $P(t) > c$. $P(t) = c$ is a singular arc, along which backstop output is indeterminate. Backstop investment is again determined by the present value of $P(t) - c$. Since the leader controls the price path, the actual behavior of investment only can be determined when the equilibrium price path is determined.

2.3. Solution for Depletable Energy Sector

The usual procedure for solving Stackelberg games analytically is to derive the first order conditions for the follower with the values of the leader's control variables as given, and then to solve the leader's problem with the follower's first order conditions as constraints. This procedure runs into two difficulties in the model developed here. One arises from the above mentioned indeterminacy in backstop production when $P(t) = c$. The basic Stackelberg paradigm (see Basar and Olsder 1982, p. 126) requires that the response of the follower to any of the leader's possible strategies be unique. What is needed here is an agreement between the leader and follower on how to divide up total demand during any interval when $P(t) = c$. Similar problems have arisen in other applications of the Stackelberg paradigm to resource markets (e.g., Marshalla).

A second source of difficulty in developing an analytical solution to this model is that several of the leader's constraints are nondifferentiable. Thus backstop output is determined by

$$v_t = \text{Max}[0, (f')^{-1}[\lambda_k^f(t)]] \tag{14}$$

where λ_k^f is the shadow price to the follower of backstop capacity. This is a constraint involving both state and control variables and it is not generally differentiable at $v(t) = 0$.

In order to provide illustrations of the qualitative behavior of these models we have formulated equivalent discrete time versions and solved them numerically using the general constrained nonlinear optimization program MINOS/Augmented (Murtagh and Saunders 1977, 1980). A discussion of these numerical results follows.

3. Numerical Examples

3.1. Formulation

A common set of parameters was used for all the examples. We took for market demand the linear function

$$P(t) = 100 - 0.5(x_t + y_t).$$

The social planner's objective function was taken to be the social surplus generated by this demand curve. The stock of depletable resources was set at 1000. Finally, the backstop operating cost was set at 30 and the backstop investment cost function[7] was

$$f(v_t) = \tfrac{1}{2}v_t^2.$$

The formulation of the social planning problem in discrete time is straightforward. The results reported below are based on the solution of the nonlinear program

$$\underset{\{x_t, y_t, v_t\}}{\text{Max}} \sum_{t=1}^{T} (1 + r)^{-t}[u(x_t + y_t) - cy_t - f(v_t)] \tag{15}$$

subject to

$$\sum_{t=1}^{T} x(t) = \bar{S}$$

$$K(t + 1) = K(t) + v(t) \quad t = 1, 2, \ldots, T -$$

$$y(t) \leq K(t) \quad t = 1, 2, \ldots, T$$

$$x(t), \quad y(t), \quad v(t) \geq 0 \quad t = 1, 2, \ldots, T.$$

In contrast to the social planning model, converting the Stackelberg model to discrete time involves a number of steps and several additional assumptions. The first step is to solve an appropriate discrete time version of the follower's problem (12). The first order conditions from this solution must then become constraints in the leader's problem. The follower's problem can be stated as

$$\underset{\{y_t, v_t\}}{\text{Max}} \sum_{t=1}^{T} (1 + r)^{-t}[(P_t - c)y_t - f(v_t)] \tag{16}$$

subject to

$$K(t + 1) = K(t) + v(t) \quad t = \text{'}, 2, \ldots, T - 1$$

$$0 \leq y(t) \leq K(t) \quad t = 1, 2, \quad , T$$

$$v(t) \geq 0 \quad t = 1, 2, \ldots, T.$$

If we introduce the multipliers $\mu(t)$ and $\eta(t)$ corresponding to the constraints $y(t) \leq K(t)$, and $y(t) \geq 0$, respectively, we can write the first order conditions

for this problem as

$$P(t) - c - \mu(t) = 0 \tag{17}$$

$$f'(v_t) = \lambda(t+1) \tag{18}$$

$$\lambda(t+1) = (1+r)\lambda(t) - \mu(t) \tag{19}$$

$$\mu(t) \geq 0, \quad \mu(t)[K(t) - y(t)] = 0 \tag{20}$$

$$\eta(t) \geq 0, \quad \eta(t)y(t) = 0 \tag{21}$$

together with the constraints in (16).

Again, these conditions are not sufficient to determine backstop output when $P(t) = c$. Since the depletable energy sector is the more powerful in our model, we will assume that when it sets $P(t) = c$ it implicitly threatens to lower the price still further unless the backstop sector shuts down. Thus, backstop output will always be zero when $P(t) = c$.[8]

Furthermore, we assume that at prices equal to or below the backstop operating cost, the demand elasticity is such that the leader's revenues rise with increasing price. This is not an unrealistic assumption because operating costs are generally a small fraction of the total costs of producing alternative energy. Under this assumption the leader never chooses to set a price below c, because its revenue is greater at higher prices, and hence, no competing backstop production will come on the market until $P(t) > c$.

We now can simplify the follower's first order conditions. First, we eliminate $\eta(t)$ since $y(t) \geq 0$ is guaranteed. Then we use (17) to eliminate $\mu(t)$ in (19). The resulting first order conditions for backstop investment[9] are

$$f'(v_t) = \lambda(t+1) \tag{22}$$

$$\lambda(t+1) = (1+r)\lambda(t) - (P_t - c). \tag{23}$$

Finally, we guarantee that $y(t) = 0$ when $P(t) = c$, and $y(t) = K(t)$ otherwise, by imposing the following pair of constraints

$$\begin{aligned} &P(t) \geq c \\ &(y_t - K_t)(P_t - c) = 0. \end{aligned} \tag{24}$$

Now we come to the implementation of the leader's problem. The leader's objective is simply

$$\underset{\{x_t\}}{\text{Max}} \sum_{t=1}^{T} (1+r)^{-t} P(x_t + y_t)x_t. \tag{25}$$

The leader faces the standard resource constraint

$$\sum_{t=1}^{T} x(t) \leq \bar{S}$$

as well as the constraints that summarize the reaction of the backstop sector to the leader's behavior. The leader's problem is stated in full below:

$$\underset{\{x_t\}}{\text{Max}} \sum_{t=1}^{T} (1+r)^{-t} P(x_t + y_t)x_t \tag{26}$$

subject to

$$\sum_{t=1}^{T} x(t) = \bar{S}$$

$$K(t+1) = K(t) + v(t) \qquad t = 1, 2, \ldots, T-1$$

$$f'(v_t) = \lambda(t+1) \qquad t = 1, 2, \ldots, T-1$$

$$\lambda(t+1) = (1+r)\lambda(t) - (P_t - c) \qquad t = 1, 2, \ldots, T-1$$

$$P(t) \geq c \qquad t = 1, 2, \ldots, T$$

$$(y_t - K_t)(P_t - c) = 0 \qquad t = 1, 2, \ldots, T$$

$$y(t) \leq K(t) \qquad t = 1, 2, \ldots, T.$$

3.2. Base Case

Base case results for both the social planning and Stackelberg models are shown in Figure 2. Examining the results for the social planning model first, in Figure 2A we see that the price (or marginal utility) of energy rises from an initial level around 22 to a maximum of 37 by period 12. As we expect from our earlier discussion, this is approximately the date at which the depletable resource stock is exhausted. Figure 2B shows depletable energy output declining gradually for the first six periods from an initial level of 155, then declining steeply to zero by period 11. Investment in backstop capacity (Figure 2C) begins in the first period and reaches its maximum at approximately the date that price equals operating cost.

Figure 2D shows that a capacity of over 62 units is in place by the time backstop output begins at period 5. Of course, backstop output is zero prior to this because the price is below operating costs. Thereafter, production is always at capacity since the price exceeds operating costs. Capacity continues to expand, driving the price back down to the level of operating costs by period 16. With some allowances made for the discrete nature of this example, it is evident that all of the essential properties of the social planning model are exhibited here.

The corresponding results for the Stackelberg model are also shown in Figure 2. First, Figure 2A shows that the leader chooses a price path which falls from an initially high level to the level of backstop costs at period 4. For the next four periods the price is pegged at 30: the leader chooses to stave off backstop

(26)

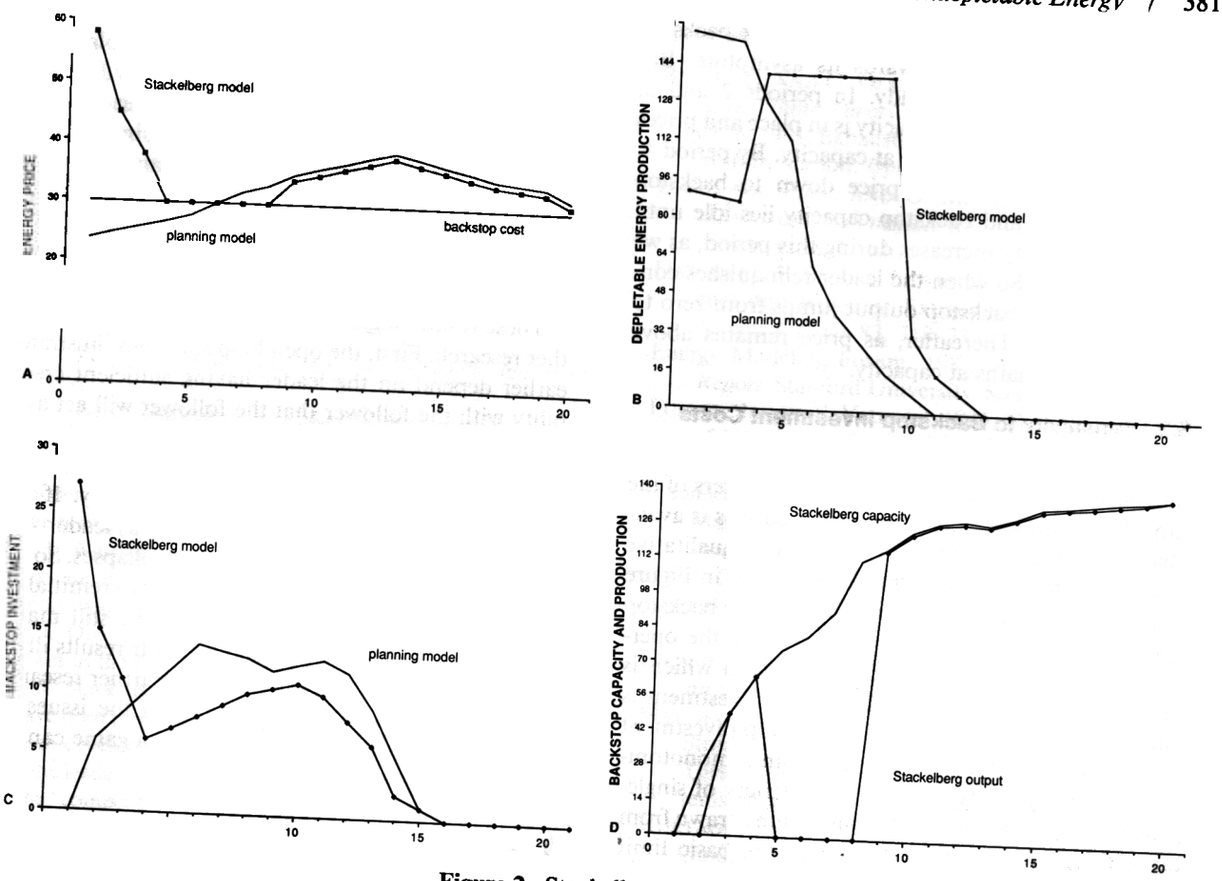

Figure 2. Stackelberg model.

production and delay investment by keeping the price at the level of operating costs for a substantial period of time. After period 8, prices are allowed to rise a few units above 30; by the last period, the expansion of backstop capacity and output has brought the price back to 30. The production path the leader chooses, which generates this price path, is shown in Figure 2B. For the first three periods, leader production is low. Then output jumps to 140 units for periods 4 through 8: this is the interval during which the leader takes the whole market and backstop output is zero. By period 8 most of the leader's resources are exhausted; production falls to zero by period 12. The leader's strategy consists of an initial interval during which high revenues are secured by low rates of production, followed by a period of limit-pricing in which the backstop is totally shut out of the market. The leader can afford to allow prices to be high in early years because the threat of a subsequent period of zero profits causes the backstop sector to invest in capacity cautiously.

Backstop investment is shown in Figure 2C. The rate of investment in this case is high but falling in the first two periods; thereafter, investment increases slowly from about 8 to 10.5 units per year. At period 8 investment reaches its maximum; it drops to zero by period 15. The most notable conclusion we can draw from these results is that backstop investment is positive until the steady state level of capacity is achieved, even when the leader excludes backstop production from the market for a lengthy period. The return to backstop investment can be positive even when backstop output is zero. This is a result, again, of the rising cost of backstop investment. The leader cannot stave off the backstop forever, and when the leader's reserves run out, the price of energy will rise above the level of backstop operating costs. The return on backstop investment will then be positive, but since there is a cost penalty for investing too fast, it pays the backstop sector to increase its capacity before it is actually needed; i.e., to add to capacity even when existing capacity is standing idle.

The paths of backstop capacity and output that are (indirectly) determined by the leader are shown in Figure 2D. Since backstop investment is positive until

very near the terminal date, it follows that backstop capacity grows steadily towards its asymptote. But backstop *output* is not steady. In periods 2 and 3, when a small amount of capacity is in place and prices are high, backstop output is at capacity. By period 4, the leader has driven the price down to backstop operating costs, and backstop capacity lies idle until period 8. Capacity increases during this period, as we have remarked. So when the leader relinquishes control at period 8, backstop output jumps from zero to over 100 units. Thereafter, as price remains above cost, output remains at capacity.

3.3. Sensitivity to Backstop Investment Costs

Space limitations preclude an extended discussion of the sensitivity of these results to the parameters of the model. A complete set of sensitivity analyses is available in Powell. Generally speaking, the qualitative features of the Stackelberg model exhibited in Figure 2 hold through a wide range of values for backstop operating and capital costs. High values of the operating cost can lead to an investment path which is single peaked. Similarly, the pattern of investment is quite sensitive to the slope of the backstop investment cost function, with results ranging from a monotonically declining investment rate to a variety of single-peaked shapes. Otherwise, the conclusions drawn from our results are robust. In particular, the basic limit pricing strategy of the leader is robust across all parameter values tested.

3.4. Dynamic Inconsistency

The Stackelberg solution we analyze here can be characterized as an "open loop" or "binding contract" solution because both players are assumed to agree upon courses of action at time zero and to faithfully carry out those plans as the game unfolds. In the absence of binding contracts, and it is generally conceded that international agreements are seldom binding, one must ask whether one or both players has an incentive to deviate from the open loop strategy during the game. If so, then the open loop strategy is dynamically inconsistent. Unfortunately, dynamically consistent Stackelberg solutions are "almost insuperably difficult to solve" (Newberry 1981).

Powell discusses a set of numerical examples that illustrate how the leader alters its optimal plan if allowed to do so during the game. The procedure takes the current values of the state variables as initial conditions at some time during the game, and recalculates the open loop solution from that point on. The result repeats the type of strategy used in the original open loop strategy: the leader again sets a high initial price, but threatens the backstop sector with a subsequent limit-pricing interval. This takes place during periods when the initial strategy calls for limit pricing, not for a high price. In effect, the leader tricks the follower into reacting cautiously to an initial threat (i.e., the backstop chooses low initial rates of investment because of the threat of later limit pricing), and then later, if allowed, can renege on that threat. The leader, in fact, can repeat the trick, as long as the follower remains gullible.

These results suggest some interesting ideas for further research. First, the open loop solutions illustrated earlier depend on the leader having sufficient credibility with the follower that the follower will act as if the leader were sure to adhere to its original plan. Presumably the leader can fool the follower once, but not infinitely often without losing credibility. If the leader loses credibility, it loses its status as leader—in effect, the entire Stackelberg structure collapses. So we can imagine the leader having the power to reinitialize its solution at infrequent intervals, while still maintaining credibility with the follower. Our results illustrate what happens in one such case. Further research is needed to develop models in which the issues of trust and credibility between actors in a game can be analyzed realistically.

4. Summary

The standard abstract model of the transition from depletable to nondepletable resources predicts that energy prices will never rise above the cost of nondepletable energy, and the transition from one source to the other will occur instantaneously. In this paper, we construct a social planning model in which the rate of investment in the nondepletable energy sector is determined along with the optimal rates of production of depletable and nondepletable energy. We show that both resources are used simultaneously, and that the price of energy rises above the operating cost of nondepletable sources. Investment in capacity in the nondepletable sector begins prior to the date when price equals operating cost, but capacity remains idle until this point is reached.

We then used this model as the foundation for a Stackelberg market model in which all depletable resources are owned by the dominant player, and nondepletable resources are owned by the follower. Numerical solutions of this model show that the leader's strategy is to allow high prices initially, but to control backstop capacity expansion by threatening to set the price at the operating cost of the nondepletable sector during an intermediate period. Since the

nondepletable sector makes no profit during this period, its rate of capacity expansion in early years, when prices are high, is limited. In the Stackelberg model, prices are generally above the socially optimum level in early years, and below that level in later years. Capacity in the nondepletable sector expands initially at a faster rate than is socially optimal, but output from this sector is delayed longer.

Notes

1. For a textbook treatment, see Dasgupta and Heal (1979).

2. Depletable energy is assumed to have a zero extraction cost solely for simplicity of presentation. See Heal (1976) and Oren and Powell (1985) for a discussion of backstop models in which the cost of one or both energy sources varies with cumulative production.

3. We assume zero depreciation of backstop capacity because its inclusion would complicate matters with no additional insight.

4. One might think of the leader as OPEC and the follower as the U.S. shale oil industry.

5. Stackelberg models for depletable resource markets have been proposed by Marshalla (1978) and Gilbert (1978). Both develop models in which a depletable resource cartel is the leader and all remaining depletable resource producers (the "competitive fringe") together are the follower. Gilbert's analysis is particularly interesting because he studies the effects of capacity constraints on fringe production. Our model differs in that the follower is the backstop sector (there is no competitive fringe), and backstop capacity is endogenous.

6. In Gilbert, exogenous capacity constraints on the competitive fringe play a central role. Gilbert shows that when fringe capacity is expanding, the price set by the depletable resource sector may fall over time. In Section 3, we show how capacity operating costs set a floor for the price path, which typically falls from time zero toward the level of backstop operating costs, then for a period of time stays constant at that level, and finally, rises again as depletable resources near exhaustion.

7. Assuming a quadratic investment cost function probably overstates the penalty associated with rapid backstop capacity expansion. However, our results show that the actual rates of backstop investment chosen in the model are generally well above zero, suggesting that the properties of the investment cost function at low rates of investment may not be critical.

8. In effect, we are excluding from the leader's feasible set an output less than $P^{-1}(c)$ at $P = c$. The leader, who makes additional profit on all units up to $P^{-1}(c)$, will choose maximal output at $P = c$.

9. If f is convex and $P(y)y$ is concave, these conditions are necessary and sufficient for optimality (see Kamien and Schwartz).

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