NONLINEAR PRICING AND NETWORK EXTERNALITIES IN TELECOMMUNICATIONS*

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This paper provides a mathematical framework for modeling demand and determining optimal price schedules in markets which have positive demand externalities and can sustain volume discounts. The theory addressing these aspects applies in particular to electronic communications networks where the benefit derived by a subscriber increases as more subscribers join the network. The paper extends the theory of nonlinear pricing to such markets. A special case of the results has been applied in a case study of a telecommunication network for the hearing impaired. In this context it is demonstrated that a nonlinear pricing strategy will motivate a monopolist supplier to lower the subscription fee. This makes the network affordable to more users and reduces the critical mass of subscribers needed for startup.

1. INTRODUCTION

Non-uniform pricing, i.e., when the unit price of a good depends on the quantity purchased, prevails in the markets for many goods and services and has been the subject of considerable economic research. Recent contributions in this area are due to Spence (10), Goldman, Leland and Sibley (3), Hirman and Sibley (5), Oren, Smith and Wilson (6), and Willig (12). Fundamental market characteristics that are necessary to sustain non-uniform pricing are the absence of a resale market and the ability of the supplier to monitor the purchase sizes of his customers. The telecommunication market is an important example in which both of these conditions prevail and non-uniform pricing is common.

![Diagram](image)

Fig. 1. Alternative Implementations of Nonuniform Pricing.

Operationally, non-uniform pricing can be offered in various ways. Fig. 1 illustrates three alternative types of schedules that are found in telecommunication markets. The first is block pricing in which the consumer pays different rates for subsequent blocks of consumption. In the second case, the consumer can choose among several two part tariffs, each consisting of a fixed charge and a corresponding marginal rate. The third case includes a "free volume" that comes automatically with payment of the fixed charge. In the second case the consumer has the options of choosing a higher fixed charge to obtain a lower marginal rate for service. For example, this may correspond to choosing a more expensive type of equipment that provides better efficiency and reduced marginal charges. The third case arises when the consumer can choose among alternative contracts consisting of a minimum purchase commitment at a fixed rate and a corresponding marginal cost above that commitment. For convenience of analysis, the nonuniform pricing literature typically deals with tariffs that change continuously with quantity. This is equivalent to assuming a continuum of blocks or two part tariffs.

An important characteristic of the telecommunication market which has not been addressed in the nonlinear pricing literature is the externality effect resulting from demand interdependencies. For one and two part tariffs, this has been studied by Artie and Averous (2), Littlechild (4), Rohlf (9), Squire (11), and Oren and Smith (8). This paper provides a framework for determining nonlinear price schedules and analyzing their implications in the presence of demand externalities. The models are developed for heterogeneous markets in which customers differ in their preferences and consumption levels which they select so as to maximize benefit minus cost. For the monopolist supplier case, the model allows the determination of the optimal profit maximizing nonlinear price schedules, the corresponding network size and the "critical mass" of subscribers needed to sustain the network at that price schedule.

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The results are specialized to a particular specification of the demand function which is used in a case study (1) of a telecommunication network for the hearing impaired (DEAFNET), developed by SRI International. Profitability and consumer welfare implications are discussed in this setting and compared for alternative price structures.

2. FORMULATION

Nonlinear pricing allows the supplier to increase his profits through price discrimination among customers having different demand functions. A fundamental assertion in constructing such pricing schedules is that the supplier knows the distribution of demand functions in the population by customer type. However, he is unable to discriminate directly among customers according to their type, either because he is prohibited from doing so (by regulation) or he cannot identify individual customers’ types. Consequently, he will attempt to discriminate according to amount purchased and will rely on the customers’ self-selection of purchase quantities. Throughout the paper we shall neglect income effects, and assume that the prices of all alternative modes of satisfying the consumers’ demand for communication services (other than the one under consideration) are fixed.

Following (3) an individual consumer’s demand may be characterized in terms of a function \( W(q,t) \), representing his willingness to pay for the first \( q \) units of consumption, \( W(0,t) = 0 \), or equivalently in terms of \( w(q,t) = \frac{W(q,t)}{q} \), which is the marginal willingness to pay for the \( q^{th} \) unit. It is assumed that \( w(q,t) \) is differentiable and satisfies \( \frac{dw}{dq} < 0, \frac{dw}{dt} > 0 \). The parameter \( t \) is an index that identifies customers’ types. Without loss of generality (see (6)), we may further assume that \( t \) is uniformly distributed on the interval \([0,1]\).

The supplier chooses the tariff function \( R(q) \), defined as the total charge to a customer consuming \( q \), so as to maximize his net revenue. The supplier, who knows \( w(q,t) \), assumes that each consumer will select a consumption level that maximizes his consumer surplus, i.e., his benefit minus cost. The conditions this implies are referred to as the self-selection constraints. In general, the tariff function \( R(q) \) may have a jump at the origin, reflecting a fixed subscription charge. This may cause consumer surplus to be negative at the “optimal” consumption levels of some consumers, in which case they will choose not to subscribe to the service.

Demand externalities are incorporated in this model by allowing the willingness to pay function of each subscriber to depend on the identity of other subscribers. Thus, for any fixed set \( Y \subseteq [0,1] \), representing the set of \( t \) indices of all other customers who have subscribed, \( W(q,t,Y) \) and \( w(q,t,Y) \) will denote the corresponding willingness to pay functions. These functions are assumed to satisfy

\[
W(q,t,Y) > W(q,t,Y') \quad \text{and} \quad w(q,t,Y) > w(q,t,Y')
\]

for \( Y > Y' \).

i.e., increasing the number of subscribers increases the marginal and total willingness to pay for each purchase quantity. This monotonicity property implies that the demand externalities are always nonnegative, which assumes that congestion effects are negligible.

3. CUSTOMER’S SELF-SELECTION CONDITIONS

In this section we briefly summarize results which are formally proved in (6).

For any given set \( Y \subseteq [0,1] \) and tariff function \( R(q) \), let \( q^*(t,Y) \) denote the optimal consumption quantity for customer \( t \). That is,

\[
q^*(t,Y) = \arg \max_{q \geq 0} (W(q,t,Y) - R(q)) \quad (1)
\]

Assuming that \( q^*(t,Y) \) is well defined, a consumer \( t \) will subscribe if and only if his maximum consumer surplus (benefit minus cost) is non-negative, i.e., \( t \in Y^* \), where

\[
Y^*(Y) = \{ t \mid CS(t,Y) > 0 \}, \quad (2)
\]

while

\[
CS(t,Y) = W(q^*,t,Y) - R(q^*), \quad (3)
\]

with \( q^* \) given by (1).

The “equilibrium” subscriber set \( Y \) is then characterized as a fixed point of the set to set mapping defined by (2), or in other words a set \( Y \) satisfying the relation \( Y = Y^*(Y) \). To guarantee that such a set exists, we assume for each \( t \), \( Y \) that demand satisfies at some finite level \( q = Q(t,Y) \), so that it will not exceed \( Q(t,Y) \) even at zero marginal charge. This guarantees that \( q^*(t,Y) \) is finite and \( Y^* \) is well defined. The existence of a fixed point of \( Y^* \) then follows from its monotonicity. From a practical point of view, the above saturation assumption is quite reasonable, given some user cost (e.g., time) associated with consumption and the absence of a resale market. For example, people do not spend all their time making local telephone calls although they are free.

In practice, it is also unlikely for a tariff to have fixed charges at quantity levels other than zero. Therefore, we will further assume that \( R(q) \) is continuous for all \( q > 0 \). With that assumption one can show that an equilibrium subscriber set must be an interval \([0,Y]\). Accordingly, we simplify the notation, replacing \( Y \) with the marginal subscriber’s index, \( y \). The subscription conditions (2) and (3) can now be reduced to an explicit condition on the marginal subscriber \( y \).

\[
CS(y) = W(q^*(y,y),y,y) - R(q^*(y,y)) \geq 0 \quad (4)
\]

We can also replace (1) by the first order necessary conditions

\[
w(q^*,t,Y) = R'(q^*) \quad \text{if} \quad q^* > 0 \quad (5)
\]

and the monotonicity condition

\[
\frac{dY^*}{dt} \leq 0, \quad (6)
\]

which is equivalent to the second-order necessary condition for (1).
The monotonicity of $q^*(t,y)$ with respect to $t$ implies that the customer index $te[0,1]$ is precisely the customer's fracture ranking with regard to his consumption level and, therefore, can be inferred from empirical observations of communication volumes.

4. CRITICAL MASS AND EQUILIBRIUM USER SETS

While our assumptions so far guarantee a unique quantity selection $q^*(t,y)$ satisfying condition (3), further assumptions are needed to characterize the values of $y$ that will satisfy condition (4). Specifically, these additional assumptions concern the network externality effect which manifests itself through the dependence of $W(q,t,y)$ on $y$. For simplicity, we will assume that for any quantity $q$, $W(q,t,0) = W(q,1,y) = 0$ for all $t, y([0,1])$. That is, the service has no value with zero subscribers, and the lowest volume customer ($t = 1$) derives zero benefit.

![Fig. 2. The utility for consumption level $q$, as a function of customer index and network size.](image)

Fig. 2 illustrates a typical utility function $W(q,t,y)$ and the corresponding $W(q,y,y)$, which is the cross section of $W(q,t,y)$ along the diagonal $t = y$. In this illustration $W(q,y,y)$, which is the willingness to pay of the last subscriber, is unimodal. However, as shown in Oren and Smith (8), one can construct examples where $W(q,y,y)$ has multiple local maxima. For a large class of functions $W(q,y,y)$ it is shown in (6) that the roots of the equation $CS(y) = 0$ will occur in pairs of adjacent roots between which the function $CS(y)$ is nonnegative. Fig. 3 illustrates a typical form of $CS(y)$ in relation to the marginal subscriber's willingness to pay $W(q^*(t,y),y,y)$. In view of condition (4), only values of $y$ in the intervals $[y_1,y_2]$ and $[y_3,y_4]$ define viable subscription levels, since only in these intervals has the marginal subscriber the incentive to subscribe.

The points $y_1$ and $y_3$ define "Critical Mass" subscription levels. If, for example, the subscription level is below level $y_1$, the marginal subscriber has a negative consumer surplus and will therefore leave the network. The same is true for his predecessor, so a chain reaction of subscription cancellations will follow until the network reaches a stable equilibrium (at $y = 0$). On the other hand, if the network reaches the subscription level $y_1$, then subsequent customers can obtain a positive consumer surplus by subscribing and the network will expand spontaneously to its next "equilibrium user set" $[0,y_2]$. Beyond $y_2$ again any additional customer will have no incentive to subscribe unless the network reaches, by some means, level $y_3$, from which it will again expand spontaneously to $y_4$. Similar phenomena and concepts have been described previously by Arthur and Averous (2), Robles (9) and by Oren and Smith (8), in the context of other tariff structures.

5. THE MONOPOLY PROFIT MAXIMIZING TARIFF

As shown in the previous section, the consumption level of each subscriber is influenced by the marginal tariff and the network size, while the equilibrium network size is determined by the last subscriber's fee. Both the usage levels and network size affect the revenues of a monopoly supplier, who controls these quantities by choosing the marginal price schedule $R(q)$ and the fixed charge $C(q)$. In this section, we derive conditions for the optimal price schedule that maximize the equilibrium net revenues of a monopoly supplier, whose per-customer supply cost is given by a function $C(q)$. We assume that $C(q)$ is increasing and continuously differentiable for $q > 0$, but may have an upward jump $k = C(0^+)$ at the origin.

We will pursue our analysis by first deriving conditions for the optimal marginal tariff $R'(q)$, $q > 0$, conditional on a given equilibrium user set $y$. Then we will obtain conditions for the optimal network size $y$. To simplify the analysis we will assume from here on that the function $W(q,y,y)$ is unimodal in $y$. This implies that there may be at most one nonempty equilibrium user set and one corresponding critical mass level.

For any given equilibrium network size $y$, a monopoly supplier will choose an price schedule which maximizes his net revenue $\pi(y)$, where

$$\pi(y) = \int_0^y (R(q^*(t,y)) - C(q^*(t,y)))dt$$

(7)

The function $q^*(t,y)$ satisfies the consumer self-selection conditions (5), (6) and $R(q)$ must satisfy the boundary condition
\[ R(q^*(y,y)) = W(q^*(y,y),y,y) \]

stating that the smallest purchaser breaks even.

Finding a tariff \( R(q) \) that maximize \( \pi(y) \), subject to the constraints (5), (6) and (8), is a calculus of variations problem. It can be solved indirectly by determining the function \( t^*(q,y) \), which is the optimally induced assignment of subscribers to purchase quantities. Using (5) we then obtain

\[ R(q) = K(y) + \int_0^q w(t,t^*(q,y),y)dt, \]

where \( K(y) \) is a constant of integration determined by (8).

When the monotonicity constraint (6) is inactive, \( t^*(q,y) \) is defined implicitly by the first order necessary condition

\[ tw_q(q,t,y) + w(q,t,y) - c(q) = 0, \]

where \( c(q) \equiv C'(q) \). Regions of \( t \) over which (6) is binding correspond to linear segments in \( R(q) \), which can be determined following the approach described in [3].

An intuitive representation of (10) can be obtained by defining the aggregate demand function \( N(p,q,y) \), which is the fraction of customers that will buy at least \( q \) units at marginal price \( p \) given market penetration \( y \). Let \( p(q) = R(q) - w(q,t^*(q,y),y) \), then (10) can be expressed as

\[ 1 - 1/e_{np}(q,y) = c(q)p(q), \]

where \( e_{np}(q,y) \) is the price elasticity of \( N(p,q,y) \) defined as

\[ e_{np}(q,y) = -(pN(p,q,y)/dq)/N(p,q,y). \]

Condition (11) is exactly the classical profit maximizing monopoly condition, parametric on \( q \) and \( y \). In other words, for any market penetration level \( y \), the monopoly profit maximizing price schedule \( p(q) \) can be determined by treating each \( q \)th purchase unit as a separate market having a demand function \( N(p,q,y) \).

6. OPTIMAL NETWORK SIZE AND FIXED CHARGE

The optimal equilibrium network size \( y \) can now be determined from the first order necessary condition \( dy/dx = 0 \) where \( x = t^*(q,y) \). This condition can be reduced to the form

\[ R(q^*(y,y)) - c(q^*(y,y)) + \int_0^{q^*(0,y)} w(q,t,y) dt = 0, \]

where \( e_{wy} \) is the partial elasticity of the willingness to pay function with respect to customer index for the marginal subscriber, i.e.,

\[ e_{wy} = -t(\partial w(q,t,y)/\partial t)/w(q,y) \]

\[ q^*(y,y) = y \]

We note that the externality effect, which is reflected by the sensitivity of \( w(q,t,y) \) with respect to \( y \), is captured by the integral term in (13). Since \( dw/dy \geq 0 \) this term will be nonnegative and will have a similar effect as reducing the supplier's provision cost \( C(q^*(y,y)) \) for the marginal subscriber. This induces a profit maximizing monopoly to reduce his fixed charge in order to achieve a larger network size (see (6) for a comparative static analysis). If there was no externality effect at all, i.e., \( dw/dy = 0 \), then the integral term in (13) would vanish and (13) would reduce to the classical monopoly profit maximization condition for the marginal subscriber's charge \( R(q^*(y,y)) \).

Another important observation that follows from (13) concerns the issue of cross subsidy. In the absence of network externality, since \( e_{wy} \geq 0 \), it is evident from (13) that \( R(q^*(y,y)) > C(q^*(y,y)) \). In other words, the charge paid by the last subscriber is no less than the cost of supply for his optimal consumption quantity. This is not necessarily true in the presence of network externalities and it is possible to have a whole range of \( t \) for which \( R(q^*(t,y^*)) < C(q^*(t,y^*)) \). Customers in this range will obtain the service below cost. The occurrence of these phenomenon depend on the strength of the externality effect. If this effect is sufficiently strong, the potential increase in demand and willingness to pay of the large users, resulting from increased network size, induces the supplier to subsidize the low-volume users, offering them basic subscription below installation cost. The incentive for doing this is that the additional communications traffic from the larger users to these low-volume users will generate revenues that will more than cover the subsidy. The senders of this extra traffic also increase their surplus as a result.

7. A SPECIAL CASE AND APPLICATION

For a more concrete illustration of the results, we use the following specifications of marginal utility, satisfaction volume and cost functions

\[ w(q,t,y) = 2w_0(1 - q/Q(t,y)) \quad q \leq Q(t,y) \]

\[ = 0 \quad q > Q(t,y) \]

where \( Q(t,y) = 2T y(2 - y)(1 - t) \)

and \( C(q) = k + cq \).

The parameter \( w_0 \) is the average willingness to pay per unit for all customer types and network sizes, while \( T \) may be interpreted as the average volume per subscriber that would be sent in a free maximal network (\( y = 1 \)).

From condition (10) we obtain in this case

\[ q^*(t,y) = 2Ty(2 - y)(1 - t) \]

where \( y = (1 - c/2 w_0) \).

After some laborious algebra, this leads to

\[ R(q) = \left( w_0/3 \right) \left( 6a - 4(1-y)y\sqrt{q(y,y)} \right)^{1/2} \]

\[ + \gamma^2(1 - y)^2 q(y,y) \]

where \( \gamma = 1 - c/2 w_0 \) and \( a = \left( w_0 / 3 \right) (6a - 4(1-y)y\sqrt{q(y,y)})^{1/2} \).
The optimal network size $y^*$ is determined from (13) which can be reduced in this case to the polynomial equation.

$$(1 - y)(12 - 15y + 5y^2)/6 = k/4y^2 T_0.$$  

(21)

Finally, the critical mass level $y_c$ corresponding to $y^*$ is obtained from the breakeven condition

$$R(q^*(y_c, y_c)) = W(q^*(y_c, y_c), y_c, y_c),$$  

(22)

where $R(q^*)$ is given by (20) with $y = y^*$. In this particular example one can show (see (8)) that (22) is satisfied if and only if

$$Q(y_c, y_c) = Q(y^*, y^*),$$  

(23)

from which $y_c$ can be easily determined.

A comparison of these results with those obtained in (8) for the flat rate and two part tariffs (using the same specifications) reveals several distinct advantages for nonlinear pricing.

We demonstrate these observations below in the context of an actual case study (1) sponsored by the National Telecommunication and Information Administration (NTIA) to examine the commercial feasibility of a nationwide communication system for the hearing impaired at a cost comparable to telephone rates. The system under consideration conceived at SRI International is based on regional nodes interconnected nationwide through a Value Added Network (VAN), such as GTE Tele- net, for example. Each of these nodes contains computer facilities that multiplex the subscribers' terminals, providing them timeshared access to the VAN. A small scale network (DEAFNET) based on this principle was already built, establishing its technological feasibility.

The goal of the economic analysis was to obtain rough estimates of profitability, social benefit, critical mass and typical charges under various tariff structures. The analysis was based on the following inputs assumptions:

- The potential market (maximal network size) is two million subscribers consisting of the deaf population, their relatives, friends, and concerned institutions.
- Willingness to pay for the services ($w_0$), averaged over all potential subscribers, is assumed to be 20 cents per call, in addition to any charges for the terminal and for the basic telephone connection.
- Average potential usage in a maximal network, $(T)$, is 200 calls per month per subscriber, assuming free usage. This is close to the current calling rate (local plus long distance) of the average telephone subscriber in the U.S. today.
- Provision cost per subscriber ($k$) is assumed to be $8.00 per month. This is an optimistic estimate based on projected low cost of computer technology. The cost was assumed to be insensitive to usage which is quite realistic once the network is set up (similar calculations for $k = $20.00 are given in (1)).

The table below summarizes the analysis results corresponding to the above inputs. These numbers should be viewed as a preliminary analysis of economic feasibility rather than a financial analysis in a business sense. A more complete analysis of profit and loss potential would include tax considerations, startup financing costs and many other factors.

### Economic Analysis of a Telecommunication Network for the Hearing Impaired

<table>
<thead>
<tr>
<th>Tariff Structure</th>
<th>Flat Rate</th>
<th>Two Part</th>
<th>Non-Linear</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equil. Network Size</td>
<td>1.14M</td>
<td>1.46M</td>
<td>1.76M</td>
</tr>
<tr>
<td>Critical Mass</td>
<td>0.50M</td>
<td>0.32M</td>
<td>0.13M</td>
</tr>
<tr>
<td>Mo. Subscription Fee</td>
<td>$28.16</td>
<td>$8.00</td>
<td>$0.08</td>
</tr>
<tr>
<td>Avg. Mo. Usage Payment</td>
<td>$0.92</td>
<td>$2.19</td>
<td></td>
</tr>
<tr>
<td>Avg. Mo. Calls/Sub.</td>
<td>232</td>
<td>148</td>
<td>150</td>
</tr>
<tr>
<td>Total Call Volume/Yr.</td>
<td>3.17B</td>
<td>2.59B</td>
<td>3.75B</td>
</tr>
<tr>
<td>Total System Costs/Yr.</td>
<td>$109M</td>
<td>$140M</td>
<td>$189M</td>
</tr>
<tr>
<td>Total Revenues/Yr.</td>
<td>$385M</td>
<td>$524M</td>
<td>$618M</td>
</tr>
<tr>
<td>Total Net Revenue/Yr.</td>
<td>$276M</td>
<td>$384M</td>
<td>$499M</td>
</tr>
<tr>
<td>Total Cons. Surplus/Yr.</td>
<td>$253M</td>
<td>$192M</td>
<td>$157M</td>
</tr>
<tr>
<td>Total Surplus/Yr.</td>
<td>$529M</td>
<td>$576M</td>
<td>$606M</td>
</tr>
</tbody>
</table>

Notes: M = Million; B = Billion

![Fig. 4. Usage Distribution by Type of Subscriber.](image)

![Fig. 5. Total Surplus and Consumer Surplus by Type of Subscriber.](image)
Figs. 4 and 5 illustrate the volume, consumer surplus and net revenue distributions by customer type. An interesting feature of the surplus distribution for the nonlinear tariff is the fact that an entire segment of subscribers at the low end of the market are subsidized by the supplier. This is optimal since the increased revenues from the other subscribers who communicate with those subsidized, more than cover the subsidy and result in increased profit. This does not occur in this example for two part or flat rate tariffs. In general, however, such a subsidy is possible with two part tariffs.

8. CONCLUSIONS

The tabulated figures in the previous section reflect the specific assumptions and specifications used in deriving them. Nevertheless, the comparison of the different tariff structures reveals a fundamental relationship that one expects to find in a more general setting as well. The major difference between the three tariffs considered, lies in the relative allocation of consumer charges among subscription and usage fees. The nonlinear tariff, which consists mostly of usage charges, reduces total consumer surplus but increases supplier's profits and is more efficient in the sense that it increases social welfare. It provides an incentive to the supplier to lower the subscription fee thus making the network more affordable. Thus, it yields a larger network size and than either the flat rate or two part tariffs, and a smaller critical mass, which eases the start up of a new network.

REFERENCES


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