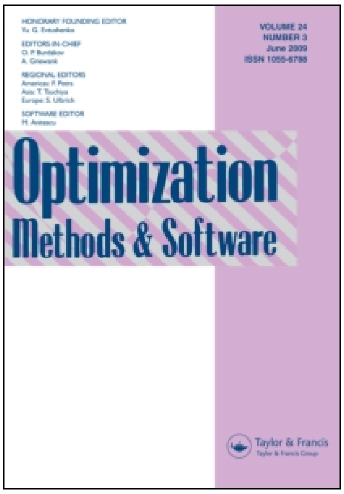
This article was downloaded by: *[CDL Journals Account]*
On: *10 August 2009*
Access details: *Access Details: [subscription number 912375050]*
Publisher *Taylor & Francis*
Informa Ltd Registered in England and Wales Registered Number: 1072954 Registered office: Mortimer House, 37-41 Mortimer Street, London W1T 3JH, UK



Optimization Methods and Software
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713645924

Country credit-risk rating aggregation via the separation-deviation model

Dorit S. Hochbaum [ab]; Erick Moreno-Centeno [a]
[a] Department of Industrial Engineering and Operations Research, University of California, Berkeley, CA, USA
[b] Walter A. Haas School of Business, University of California, Berkeley, CA, USA

Online Publication Date: 01 October 2008

To cite this Article Hochbaum, Dorit S. and Moreno-Centeno, Erick(2008)'Country credit-risk rating aggregation via the separation-deviation model',Optimization Methods and Software,23:5,741 — 762

To link to this Article: DOI: 10.1080/10556780802402432

URL: http://dx.doi.org/10.1080/10556780802402432

PLEASE SCROLL DOWN FOR ARTICLE

Full terms and conditions of use: http://www.informaworld.com/terms-and-conditions-of-access.pdf

This article may be used for research, teaching and private study purposes. Any substantial or systematic reproduction, re-distribution, re-selling, loan or sub-licensing, systematic supply or distribution in any form to anyone is expressly forbidden.

The publisher does not give any warranty express or implied or make any representation that the contents will be complete or accurate or up to date. The accuracy of any instructions, formulae and drug doses should be independently verified with primary sources. The publisher shall not be liable for any loss, actions, claims, proceedings, demand or costs or damages whatsoever or howsoever caused arising directly or indirectly in connection with or arising out of the use of this material.

Optimization Methods & Software
Vol. 23, No. 5, October 2008, 741–762

Taylor & Francis
Taylor & Francis Group

Country credit-risk rating aggregation via the separation-deviation model[†]

Dorit S. Hochbaum[a,b]* and Erick Moreno-Centeno[a]

[a]*Department of Industrial Engineering and Operations Research, University of California, Berkeley, CA, USA;* [b]*Walter A. Haas School of Business, University of California, Berkeley, CA, USA*

(*Received 30 September 2007; final version received 6 August 2008*)

Country credit-risk ratings are evaluated independently by several agencies. A common method of aggregating the ratings into a single rating is by taking their averages (the averaging method). We show here that an approach that captures the relative ranking of the countries given by each agency leads to an improved aggregate rating with respect to several criteria. The approach we use – the separation-deviation model – was proposed by Hochbaum. We compare the separation-deviation model with the averaging method for aggregating country credit-risk ratings provided by three different agencies. We show that the aggregate rating obtained by the separation-deviation model has fewer rank reversals (discrepancies in the rank ordering of the countries) than the aggregate rating obtained by the averaging method. We further prove several properties of the separation-deviation model, including the property that the aggregate rating obtained by the separation-deviation model agrees with the majority of agencies or reviewers, regardless of the scale used.

Keywords: network flow; aggregate ranking; country credit-risk ratings; group rating; group decision-making

Downloaded By: [CDL Journals Account] At: 19:51 10 August 2009

1. Introduction

Country credit-risk ratings quantify the risk associated with investing in a given country. Haque *et al.* [8] define country credit-risk rating as an estimate of the probability that a country will fail to pay back the debt it has acquired. To satisfy increasing investors' needs for information on countries' creditworthiness, several agencies periodically publish country credit-risk ratings. Often there are differences between the agencies' credit-risk ratings for a particular country. It is therefore of interest to aggregate those differing views into a coherent rating that represents a group consensus capturing the different expertise of the rating agencies.

Aggregating credit-risk ratings is a scenario within group decision-making. Group decision-making concerns the problem of finding a group consensus from the expressed evaluations of K reviewers (e.g. agencies) in relation to n objects (e.g. countries). The sense in which the aggregate preferences form a consensus is to be quantified by performance measures.

*Corresponding author. Email: hochbaum@ieor.berkeley.edu
[†]This paper is dedicated to the memory of Peter Hammer.

ISSN 1055-6788 print/ISSN 1029-4937 online
© 2008 Taylor & Francis
DOI: 10.1080/10556780802402432
http://www.informaworld.com

Most often the reviewers' evaluations are expressed as *point-wise scores* or *ratings*, which means that each reviewer assigns a scalar-valued score to each of the objects. Let r_i^k denote the evaluation of the ith object by the kth reviewer. We refer to a set of point-wise scores $\{r_i\}$ for $i = 1, \ldots, n$ as a *rating vector*. We differentiate between a *full-list* setting, where every reviewer assigns scores to *all* objects, and a *partial-list* setting, where each reviewer assigns scores only to a subset of the objects. This paper addresses the country credit-risk rating aggregation, which belongs to the full-list setting.

Alternatively, the reviewers' evaluations can be expressed as pairwise comparisons. That is, each reviewer gives pairwise intensity of preferences between object pairs. The intensity of preference of the kth reviewer for the ith object over the jth object, p_{ij}^k, may be used either in the additive (e.g. [2]) or multiplicative sense (e.g. [14]). Here we use intensities of preference in the additive sense. In this sense, the intensity of preference represents the difference between the strengths of the two objects compared. We will use the term *separation gap* to refer to the additive intensity of a preference.

The aggregation scheme considered here is the separation-deviation model proposed by Hochbaum [9,10]. To the best of our knowledge, the separation-deviation model is the only model that permits to combine both kinds of inputs: point-wise scores and pairwise comparisons. In this paper, we demonstrate that, even when the input is given *only* as point-wise scores, it is useful to consider also the implied separation gaps: $p_{ij}^k = r_i^k - r_j^k$. The separation gaps are scale-independent and are shown to mitigate the effect of inflated scores or shifts in evaluation scale.

The aggregate rating vector given by the separation-deviation model is the rating vector that minimizes the total sum of deviation penalties and separation penalties. A deviation penalty is a (convex) function of the difference between the aggregate score and a point-wise score, assuming a value of 0 for the argument 0. A separation penalty for a pair of objects i, j is a (convex) function of the difference between the separation gaps of the aggregate scores and the separation gaps of the point-wise scores. If this difference is 0 then the penalty is 0.

We prove several properties of the separation-deviation model, including the property that the aggregate rating obtained by the separation-deviation model agrees with the majority of agencies or reviewers, regardless of the scale used. The analysis of the separation-deviation model here is in the full-list setting. The details and assessment on how the separation-deviation model applies to the partial-list setting are described in a companion paper [12].

There are other rating-aggregation schemes, each resulting in a different outcome. The most commonly used method, of rating aggregation is the *averaging method*. In this method, the aggregate score of each country is the average of the point-wise scores that this country received from all of the reviewers. We assess the performance of the separation-deviation model and compare the model with the averaging method, using several performance measures. We demonstrate that in the full-list setting the aggregate rating vector, obtained by the separation-deviation model, better preserves the relative order of the objects induced by each of the input rating vectors as compared with the aggregate rating vector obtained by the averaging method.

The main contributions and results here are

(1) Illustrating the benefit of using the separation-deviation model in the credit-risk rating context.
(2) Proving that the aggregate rating vector obtained by the separation-deviation model with absolute value penalty functions agrees with the majority of reviewers. This demonstrates the model's robustness in the presence of individual reviewer's manipulations.
(3) Showing that the averaging method is a special case of the separation-deviation model with *uniform quadratic penalty functions*[1].
(4) Presenting an experimental study showing that the aggregate rating vector obtained by the separation-deviation model with absolute value penalty functions has fewer *rank reversals*

Downloaded By: [CDL Journals Account] At: 19:51 10 August 2009

than the aggregate rating vector obtained by the averaging method. Informally, a rank reversal is a discrepancy in the relative order between a pair of objects when comparing the aggregate rating vector with the input rating vectors.

(5) Using the separation-deviation model is shown to identify several outliers in the ratings of the agencies.

The paper is organized as follows: Section 2 gives a review of the separation-deviation model; Section 3 analyses the robustness properties of the model; Section 4 shows that the averaging method is special of the separation-deviation model; finally, Section 5 provides the details on the application of the separation-deviation model to country credit-risk aggregation.

2. Preliminaries and definitions

2.1 *Review of the separation-deviation model*

The separation-deviation model was proposed by Hochbaum [9,10] and Hochbaum and Levin [11]. The separation-deviation model can be applied in scenarios where the input is either point-wise scores or pairwise comparisons or any combination of both. Here we use it for the full-list setting and point-wise scores.

A set of separation gaps, p_{ij}, is said to be *consistent* if for all triplets i, j, k, $p_{ij} + p_{jk} = p_{ik}$. Consistency is equivalent to the existence of a set of weights ω_i for $i = 1, \dots, n$ so that $p_{ij} = \omega_i - \omega_j$. Such vector of weights is not unique, since for any consistent set of weights $\omega_1, \dots, \omega_n$ and a scalar c, the set $\omega_1 + c, \dots, \omega_n + c$ is also a consistent rating vector.

Let the variable x_i be the aggregate score of the ith object, and the variable z_{ij} be the aggregate separation gap of the ith over the jth object. The mathematical programming formulation of the separation-deviation model, given in [11] is:

$$\text{(Sep–Dev)} \quad \min_{\mathbf{x},\mathbf{z}} \quad \sum_{k=1}^{K}\sum_{i=1}^{n}\sum_{j=i+1}^{n} f_{ij}^{k}(z_{ij} - p_{ij}^{k}) + \sum_{k=1}^{K}\sum_{i=1}^{n} g_i^k(x_i - r_i^k) \tag{1a}$$

$$\text{such that} \quad z_{ij} = x_i - x_j \quad (i = 1, \dots, n;\ j = i+1, \dots, n). \tag{1b}$$

In (Sep–Dev) the function, $f_{ij}^k(z_{ij} - p_{ij}^k)$ is the separation penalty function of the deviation from the separation gap on the pair (i, j) given by the kth reviewer. The function $g_i^k(x_i - r_i^k)$ is the deviation penalty function of the deviation from the point-wise score on the ith object given by the kth reviewer. The functions $f_{ij}^k()$ and $g_i^k()$ are convex functions that assume the value 0 for the argument 0. Constraint (1b) enforce the *consistency* of the aggregate separation gaps conforming to the aggregate rating vector.

One of the advantages of the (Sep–Dev) is its ability to incorporate *imprecise* beliefs, or less than full confidence in some of the point-wise scores or separation gaps. Higher (lower) confidence levels are implicit in the use of higher (lower) penalties for deviating from the scores or separation gaps. This allows differentiating between reviewers according to their expertise in evaluating specific objects or specific pairwise comparisons.

2.2 *The separation model*

We refer to the separation-deviation model with no deviation functions, or $g_i^k() \equiv 0$ for $k = 1, \dots, K$, as the *separation model*. In the separation model, for any feasible solution $\mathbf{x}$ and any constant c, $\mathbf{x} + c\mathbf{e}$ (where $\mathbf{e}$ is the vector of ones) is also a feasible solution with the same objective

Downloaded By: [CDL Journals Account] At: 19:51 10 August 2009

value. Therefore, the separation model has an infinite number of optimal solutions. To avoid this, we set the rating of an arbitrarily selected anchor node to zero, e.g. $x_1 = 0$. The other aggregate scores x_i for $i = 2, \ldots, n$ are then relative to this 'anchor' value. The mathematical representation of the separation model as optimization problem is:

$$\text{(Sep)} \quad \min_{\mathbf{x},\mathbf{z}} \quad \sum_{k=1}^{K} \sum_{i=1}^{n} \sum_{j=i+1}^{n} f_{ij}^k (z_{ij} - p_{ij}^k) \tag{2a}$$

$$\text{such that} \quad z_{ij} = x_i - x_j \quad (i = 1, \ldots, n;\ j = i+1, \ldots, n) \tag{2b}$$

$$x_1 = 0. \tag{2c}$$

2.3 *Existence and uniqueness of an optimal solution*

For both (Sep–Dev) and (Sep), it is easy to see that a feasible solution always exists. This holds since, e.g. for some k, the solution $x_i = r_1^k$ for $i = 1, \ldots, n$ and $z_{ij} = x_i - x_j$ for $i, j = 1, \ldots, n$, is obviously feasible. The uniqueness of the optimal solution is guaranteed when the functions $f_{ij}^k()$ and $g_i^k()$ are strictly convex functions. Otherwise the separation-deviation model might have multiple optimal solutions.

Downloaded By: [CDL Journals Account] At: 19:51 10 August 2009

3. Robustness of the separation-deviation model

3.1 *Robustness of the absolute value separation problem*

The *absolute value separation problem*, ($\|$, Sep) is formulated as follows:

$$(\|, \text{Sep}) \quad \min_{\mathbf{x},\mathbf{z}} \quad \sum_{k=1}^{K} \sum_{i=1}^{n} \sum_{j=i+1}^{n} u_{ij}^k |z_{ij} - p_{ij}^k| \tag{3a}$$

$$\text{such that} \quad z_{ij} = x_i - x_j \quad (i = 1, \ldots, n;\ j = i+1, \ldots, n) \tag{3b}$$

$$x_1 = 0. \tag{3c}$$

We denote an optimal solution to ($\|$, Sep) as $\mathbf{x}^{|S|}$.

In this section, we show that for the full-list setting ($\|$, Sep) is robust in that it resists manipulation by a minority of the reviewers. For this purpose, we prove that $\mathbf{x}^{|S|}$ agrees with the (weighted) majority of reviewers.

DEFINITION 3.1 *A rating vector* $\boldsymbol{x}$ *is said to be equivalent under translation to the rating vector* $\tilde{\boldsymbol{x}}$ *if there exists a constant c, such that* $x_i = \tilde{x}_i + c$ *for* $i = 1, \ldots, n$.

The relation of *equivalence under translation* is reflexive, symmetric, and transitive. As such, it partitions the set of rating vectors into equivalence classes.

The following lemma is needed in the proof of the property of 'resistance to manipulation by a minority of reviewers'. The problem analysed in the lemma is a special case of the weighted median on a line and the weighted median on a graph, which were studied extensively in [4,6].

Downloaded By: [CDL Journals Account] At: 19:51 10 August 2009

LEMMA 3.2 *Given the optimization problem* $y^* = \text{argmin} \sum_{k=1}^{K} w^k|y - a^k|$, *where* $w^k \geq 0$ *for* $k = 1, 2, \ldots, K$. *If there is a weight* w^i *such that* $w^i > 1/2 \sum_{k=1}^{K} w^k$, *then the optimal solution to the problem is* $y^* = a^i$.

Proof Suppose by contradiction that there exists an optimal solution of the form $y^{**} = a^i - \delta$ for some $\delta > 0$, then it follows from simple arithmetic calculations that $y^* = a^i$ has a strictly lesser objective value. The same holds for any solution of the form $y^{**} = a^i + \delta$ for some $\delta > 0$. ■

THEOREM 3.3 *For* $(\|, \text{Sep})$, *if a subset S of reviewers has rating vectors equivalent under translation, and S is a weighted majority for every pair i, j, i.e.* $\sum_{k\in S} u_{ij}^k > 1/2 \sum_{k=1}^{K} u_{ij}^k$, *then* $\mathbf{x}^{|S|}$ *is equivalent under translation to rating vector of the weighted majority, i.e.* $\mathbf{x}^{|S|}$ *is equivalent under translation to every* $\mathbf{r}^i$, $i \in S$.

Proof Omitting constraint (3b) decomposes the problem to several optimization problems, one for each z_{ij}. Each of these optimization problems, $z_{ij}^* = \text{argmin} \sum_{k=1}^{K} u_{ij}^k|z_{ij} - p_{ij}^k|$ for $i, j = 1, 2, \ldots, n$, is of the form described in Lemma 3.2. Since the reviewers in S have rating vectors equivalent under translation, we have that $p_{ij}^k = p_{ij}^S$ for all $k \in S$. Furthermore, since S is a weighted majority, then $u_{ij}^S = \sum_{k\in S} u_{ij}^k > \sum_{k=1}^{K} u_{ij}^k$. Therefore, by Lemma 3.2, $z_{ij}^* = p_{ij}^S$. Finally, since (by construction) the separation gaps p_{ij}^S are consistent in the additive sense, it follows that by setting $x_1 = 0$ and $x_i = z_{i1}^* + x_1$ for $i = 2, \ldots, n$, we obtain a rating vector satisfying constraints (3b) and (3c). In particular, this rating vector is equivalent under translation to all of the rating vectors of the reviewers in S. ■

COROLLARY 3.4 *For problem* $(\|, \text{Sep})$, *with two reviewers,* $K = 2$, *if all the penalty weights of reviewer* 1 *dominate the penalty weights of the reviewer* 2 (*i.e.* $u_{ij}^1 > u_{ij}^2$ *for every pair* i, j), *then any optimal solution to* $(\|, \text{Sep})$, *is an aggregate rating vector equivalent under translation to the rating vector of reviewer* 1.

Let the *unweighted absolute value separation problem*, $(\|, \text{Sep}, \mathbf{1})$, refer to $(\|, \text{Sep})$ with $u_{ij}^k = 1$, for $i, j = 1, \ldots, n$ and $k = 1, \ldots, K$.

From Theorem 3.3, we have the following corollaries.

COROLLARY 3.5 *For* $(\|, \text{Sep}, \mathbf{1})$, *if a simple majority of reviewers has rating vectors equivalent under translation, then any optimal solution to* $(\|, \text{Sep}, \mathbf{1})$ *is an aggregate rating vector equivalent under translation to every rating vector of each of the reviewers in the majority.*

COROLLARY 3.6 *The problem* $(\|, Sep, \mathbf{1})$ *with two reviewers,* $K = 2$, *has an infinite number of optimal solutions. Two of the solutions are equivalent under translation to the* (*input*) *rating vectors of reviewer* 1 *and reviewer* 2. *And any convex linear combination of these two rating vectors is an optimal solution as well.*

In contrast to $(\|, \text{Sep}, \mathbf{1})$, the solution to the averaging method does not have the property of agreeing with the majority. An example shown in Table 1 demonstrates that a single reviewer (reviewer 3) can dominate the aggregate rating vector solution of the averaging method by manipulating his/her rating scale. The aggregate rating obtained by the averaging method is denoted in Table 1 as $\mathbf{x}^{\text{Avg}}$.

Theorem 3.3 applies only when the penalty function used in (Sep) is the absolute value function (it applies exclusively for $(\|, \text{Sep})$) and cannot be extended to other convex penalty functions. It does not even hold for convex quadratic penalty functions, as shown in the example in Table 2, where the third reviewer dominates the aggregate ratings even though reviewers 1 and 2 had the same ratings.

Table 1. Aggregate rating vector $\mathbf{x}^{\text{Avg}}$ obtained by the averaging method.

	Reviewer 1	Reviewer 2	Reviewer 3	$\mathbf{x}^{\text{Avg}}$
Object 1	1	1	13	5
Object 2	2	2	10	4.67
Object 3	3	3	7	4.33
Object 4	4	4	4	4
Object 5	5	5	1	3.66

Table 2. Aggregate rating obtained by solving (Sep) with $f_{ij}^k(y) = y^2$ for all i, j, k.

	Reviewer 1	Reviewer 2	Reviewer 3	Aggregate rating
Object 1	1	1	25	4
Object 2	2	2	20	3
Object 3	3	3	15	2
Object 4	4	4	10	1
Object 5	5	5	5	0

3.2 *Robustness of the absolute value separation-deviation problem*

We define the absolute value separation-deviation problem, (‖, Sep–Dev) as follows:

$$(\|, \text{Sep–Dev}) \quad \min_{\mathbf{x},\mathbf{z}} \quad \sum_{k=1}^{K} \sum_{i=1}^{n} \sum_{j=i+1}^{n} u_{ij}^k |z_{ij} - p_{ij}^k| + \sum_{k=1}^{K} \sum_{i=1}^{n} v_i^k |x_i - r_i^k| \tag{4a}$$

$$\text{such that} \quad z_{ij} = x_i - x_j \quad (i = 1, \ldots, n;\ j = i+1, \ldots, n). \tag{4b}$$

Let an optimal solution to (‖, Sep–Dev) by denoted by $\mathbf{x}^{|\text{SD}|}$.

In this section, we show that, for the full-list setting and under certain restrictions, any optimal solution to (‖, Sep–Dev) is an aggregate rating vector identical to the rating vector of the majority of reviewers.

THEOREM 3.7 *For* (‖, Sep–Dev)*, if a subset S of reviewers has identical rating vectors* $\mathbf{r}^S$ *and S is a weighted majority for both the deviation and the separation terms* (*i.e.* $\sum_{k\in S} u_{ij}^k > 1/2 \sum_{k=1}^{K} u_{ij}^k$ *for any pair* i, j *and* $\sum_{k\in S} v_i^k > 1/2 \sum_{k=1}^{K} v_i^k$ *for every* i)*, then* $\mathbf{x}^{|\text{SD}|}$ *is equal to* $\mathbf{r}^S$.

Proof Omitting constraint (4b) decomposes the problem into the following optimization problems:

$$z_{ij}^* = \operatorname{argmin} \sum_{k=1}^{K} u_{ij}^k |z_{ij} - p_{ij}^k| \quad \text{for } i = 1, \ldots, n;\ j = i+1, \ldots, n \tag{5}$$

$$x_i^* = \operatorname{argmin} \sum_{k=1}^{K} v_i^k |x_i - r_i^k| \quad \text{for } i = 1, 2, \ldots, n. \tag{6}$$

All of the problems are of the form described in Lemma 3.2. Since the reviewers in *S* have identical rating vectors, we have that $p_{ij}^k = p_{ij}^S$ and $r_i^k = r_i^S$ for all $k \in S$. Therefore by Lemma 3.2, we have that $z_{ij}^* = p_{ij}^S$ and $x_i^* = r_i^S$. Since the separation gaps were derived from the rating vectors by setting $p_{ij}^k = r_i^k - r_j^k$, it follows that $z_{ij}^* = x_i^* - x_j^*$, and so constraint (4b) is satisfied. Therefore,

Downloaded By: [CDL Journals Account] At: 19:51 10 August 2009

the optimal solution to the separation-deviation problem is an aggregate rating vector identical to all the rating vectors of the weighted majority of reviewers. ■

COROLLARY 3.8 *For problem* (‖, Sep–Dev) *with two reviewers,* $K = 2$*, if all the penalty weights of reviewer* 1 *dominate the penalty weights of the reviewer* 2 (*i.e.* $u^1_{ij} > u^2_{ij}$ *for every pair* i, j, *and* $v^1_i > v^2_i$ *for all* i)*, then the optimal solution to* (‖, Sep–Dev) *is an aggregate rating vector identical to the rating vector of reviewer* 1.

Let the *unweighted absolute value separation-deviation problem*, (‖, Sep–Dev, **1**), refer to (‖, Sep–Dev) with $u^k_{ij} = v^k_i = 1$, for $i, j = 1, \ldots, n$ and $k = 1, \ldots, K$.

From Theorem 3.7, we have the following corollaries.

COROLLARY 3.9 *For* (‖, Sep–Dev, **1**)*, if a simple majority of reviewers has identical rating vectors, then the optimal solution to* (‖, Sep–Dev, **1**) *is an aggregate rating vector identical to the rating vector of the majority.*

COROLLARY 3.10 *The problem* (‖, Sep−Dev,**1**) *with two reviewers,* $K = 2$*, has an infinite number of optimal solutions. Two of the solutions are identical to the* (*input*) *rating vectors of reviewer* 1 *and reviewer* 2*. And any convex linear combination of these two rating vectors is an optimal solution as well.*

Theorem 3.7 for (‖, Sep–Dev) is weaker than the corresponding Theorem 3.3 for (‖, Sep) in that it requires the rating vectors of the majority to be *identical* rather than just being *equivalent under translation*. Since there are $O(Kn^2)$ separation penalty terms and only $O(Kn)$ deviation penalty terms in the separation-deviation problem, one might think that it is possible to make Theorem 3.7 as strong as Theorem 3.3. The example shown in Table 3 proves that this is impossible.

Still, Table 3 data is a pathological instance of the problem. To demonstrate that, we show in Table 4 that with a minor perturbation in the data, $\mathbf{x}^{|\mathrm{SD}|}$ is equivalent under translation to the rating vectors of the majority (reviewers 1 and 2).

One might still prefer (‖, Sep–Dev) to (‖, Sep) since, even though it is only guaranteed to satisfy the weaker theorem, it tends to have an optimal solution on a 'similar' scale to the input rating vectors. An example illustrating this 'similarity' is shown in Table 5.

Table 5 provides an instance where $\mathbf{x}^{|S|}$ and $\mathbf{x}^{|\mathrm{SD}|}$ are equivalent under translation to the ratings given by the majority of reviewers (i.e. reviewers 1 and 2). The advantage of $\mathbf{x}^{|\mathrm{SD}|}$, is that its *scale* is closer to the scale used by the reviewers.

Table 3. $\mathbf{x}^{|S|}$ is equivalent under translation to the rating vector of the majority, but $\mathbf{x}^{|\mathrm{DS}|}$ is not.

	Reviewer 1	Reviewer 2	Reviewer 3	$\mathbf{x}^{\vert S\vert}$	$\mathbf{x}^{\vert\mathrm{SD}\vert}$
Object 1	1	4	517	0	4
Object 2	2	5	3	1	3

Table 4. Both $\mathbf{x}^{|S|}$ and $\mathbf{x}^{|\mathrm{SD}|}$ are equivalent under translation to the rating vector of the majority.

	Reviewer 1	Reviewer 2	Reviewer 3	$\mathbf{x}^{\vert S\vert}$	$\mathbf{x}^{\vert\mathrm{SD}\vert}$
Object 1	1	4	516	0	4
Object 2	2	5	3	1	5

Downloaded By: [CDL Journals Account] At: 19:51 10 August 2009

Table 5. $\mathbf{x}^{|\mathrm{DS}|}$ is closer to the input rating vectors than $\mathbf{x}^{|S|}$.

	Reviewer 1	Reviewer 2	Reviewer 3	$\mathbf{x}^{|\mathrm{SD}|}$	$\mathbf{x}^{|S|}$
Object 1	100	700	600	400	0
Object 2	200	800	500	500	100
Object 3	300	900	400	600	200

So far, we have shown that: (1) $\mathbf{x}^{|S|}$ is equivalent under translation to the majority rating; (2) (under stronger assumptions) $\mathbf{x}^{|\mathrm{SD}|}$ is identical to the majority rating; and (3) depending on the choice of anchoring (generally $\mathbf{x}_1 = 0$), $\mathbf{x}^{|\mathrm{SD}|}$ is closer than $\mathbf{x}^{|S|}$ to the scale of the input rating vectors. Next we show that, with a minor adjustment to (‖, Sep–Dev), we can obtain all of these desirable properties in a single model.

We note that (‖, Sep–Dev) is a multi-objective problem. The first objective is to minimize the separation penalty, and the second objective is to minimize the deviation penalty. So far we have minimized an unweighted sum of these two (possibly conflicting) objectives. However, we can obtain all of the desired properties by minimizing a weighted sum of the separation penalty and the deviation penalty. In particular, we propose the following problem:

$$(\|, M \cdot \text{Sep–Dev}) \quad \min_{\mathbf{x},\mathbf{z}} \quad M \cdot \sum_{k=1}^{K} \sum_{i=1}^{n} \sum_{j=i+1}^{n} u_{ij}^k |z_{ij} - p_{ij}^k| + \sum_{k=1}^{K} \sum_{i=1}^{n} v_i^k |x_i - r_i^k| \tag{7a}$$

$$\text{such that} \quad z_{ij} = x_i - x_j \quad (i = 1, \ldots, n;\; j = i+1, \ldots, n), \tag{7b}$$

where M is a large number so that the separation penalty is *lexicographically* more important than the deviation penalty. By lexicographically more important, we mean that the separation penalty is the dominant term in the optimization problem so that the deviation penalty is only used to choose among the feasible solutions with minimum separation penalty. In practice, it suffices to select M satisfying

$$M \geq n \cdot (\max_{ik} r_i^k - \min_{ik} r_i^k) \cdot \frac{\max_{ik} v_i^k}{\min_{ijk} u_{ij}^k}.$$

We denote an optimal solution to (‖, $M\cdot$ Sep–Dev) as $\mathbf{x}^{\mathrm{MSD}}$.

Observation 3.11 The optimal solution to (‖, $M\cdot$ Sep–Dev) is the rating vector that minimizes the deviation penalty among all the rating vectors in the set of all optimal solutions to (‖, Sep).

THEOREM 3.12 *An optimal solution to* (‖, $M \cdot$ Sep–Dev) *has the following properties:*

(1) *If a subset S of reviewers has identical rating vectors and S is a weighted majority, then* $\mathbf{x}^{\mathrm{MSD}}$ *is identical to the rating vectors of the majority.*
(2) *If a subset S of reviewers has rating vectors equivalent under translation, and S is a weighted majority, then* $\mathbf{x}^{\mathrm{MSD}}$ *is equivalent under translation to the rating vectors of the majority.*

Proof It is easy to see that, letting the weights of the separation terms to be $M \cdot u_{ij}^k$, property (1) follows from Theorem 3.7. Property (2) follows from Observation 3.11, Theorem 3.3, and the fact that if two rating vectors are identical, then they are also equivalent under translation. ■

Downloaded By: [CDL Journals Account] At: 19:51 10 August 2009

Downloaded By: [CDL Journals Account] At: 19:51 10 August 2009

4. The equivalence between uniform quadratic separation-deviation problem and the weighted averaging method

The mathematical formulation of the uniform quadratic separation-deviation problem, $(()^2$, Sep–Dev), is given in Equation (8).

$$(()^2, \text{Sep–Dev}) \quad \min_{\mathbf{x},\mathbf{z}} \quad \lambda \sum_{k=1}^{K} \sum_{i=1}^{n} \sum_{j=i+1}^{n} w^k (z_{ij} - p_{ij}^k)^2 + \sum_{k=1}^{K} \sum_{i=1}^{n} w^k (x_i - r_i^k)^2 \tag{8a}$$

$$\text{such that} \quad z_{ij} = x_i - x_j \quad (i = 1, \ldots, n; \; j = i+1, \ldots, n). \tag{8b}$$

Here λ is a parameter that allows to vary the relative importance of the separation penalty to the deviation penalty. Note that in $(()^2$, Sep–Dev), the weights w^k depend only on the reviewer and not on the object or object-pair as in (Sep–Dev). We denote an optimal solution to $(()^2$, Sep–Dev) as $\mathbf{x}^{(\text{SD})^2}$.

Let the *weighted averaging method* be the rating aggregation method where the aggregate score of each object is the weighted average of the point-wise scores of all reviewers for this object. The following theorem establishes that in the full-list setting $(()^2$, Sep–Dev) is equivalent to the weighted averaging method.

THEOREM 4.1 *The optimal solution to* $(()^2$, Sep–Dev) *is the same as the aggregate rating vector solution to the weighted average method, that is,* $x_i^* = \sum_{k=1}^{K} w^k r_i^k / W$ *for* $i = 1, 2, \ldots, n$, *and* $z_{ij}^* = x_i^* - x_j^*$, *where* $W = \sum_{k=1}^{K} w^k$.

Proof Omitting constraint (8b) decomposes the problem to separate optimization problems for each z_{ij} and each x_i. Each of this problems is of the form $\min_y \sum_{k=1}^{K} \alpha^k (y - r^k)^2$. It is easy to see that this unconstrained optimization problem achieves its minimum at $y_i^* = \sum_k \alpha^k r^k / \sum_k \alpha^k$.

Therefore, the optimal solution to the optimization problem obtained by omitting constraint (8b) is:

$$x_i^* = \frac{\sum_k w^k r_i^k}{W} \quad \text{for } i = 1, \ldots, n \tag{9}$$

$$z_{ij}^* = \frac{\sum_k \lambda w^k (r_i^k - r_j^k)}{\lambda W} = \frac{\sum_k w^k (r_i^k - r_j^k)}{W} \quad \text{for } i = 1, \ldots, n; \quad j = i+1, \ldots, n. \tag{10}$$

Since $z_{ij}^* = \left(\sum_k w^k (r_i^k - r_j^k)\right) / W = \sum_k w^k r_i^k / W - \sum_k w^k r_j^k / W = x_i^* - x_j^*$, constraint (8b) is satisfied by x_i^* and z_{ij}^* given in Equations (9) and (10). ■

The analogous separation model with uniform quadratic is formulated as follows:

$$(()^2, \text{Sep}) \quad \min_{\mathbf{x},\mathbf{z}} \quad \sum_{k=1}^{K} \sum_{i=1}^{n} \sum_{j=i+1}^{n} w^k (z_{ij} - p_{ij}^k)^2 \tag{11a}$$

$$\text{such that} \quad z_{ij} = x_i - x_j \quad (i = 1, \ldots, n; \; j = i+1, \ldots, n). \tag{11b}$$

COROLLARY 4.2 *The optimal solution to* $(()^2$, Sep) *is an aggregate rating vector identical to the aggregate rating vector obtained by the weighted averaging method,* $x_i^* = \sum_{k=1}^{K} w^k r_i^k / W$ *for* $i = 1, 2, \ldots, n$, *and* $z_{ij}^* = x_i^* - x_j^*$, *where* $W = \sum_{k=1}^{K} w^k$.

We conclude that, in the full-list setting, the optimal solution to $(()^2$, Sep–Dev) and (Sep) is the weighted average of the point-wise scores of each object. Therefore, the separation-deviation model offers no advantage compared with the weighted averaging method in this case. However, this equivalence does not carry to the partial-list setting. Indeed we show in [12] that, in the partial-list setting, $(()^2$, Sep–Dev) and $(()^2$, Sep) give a better aggregate rating vector than the weighted averaging method.

5. Experimental study

We set up an experimental study using the separation-deviation model for the purpose of aggregating country credit-risk rating vectors. We use the credit-risk ratings given by Standard and Poor's (S&P), Moody's (Mdy) and The Institutional Investor (InsI) in 1998 as the data. The input data[2] used is given in Table 6.

One challenge with this data is that each of the three agencies has its own rating scale: S&P uses an alphabetical rating scale (shown in the 'S&P scale' column in Table 7) ranging from low end at SD, CC through AAA; Mdy uses an alphanumerical rating scale (shown in the 'Moody's

Downloaded By: [CDL Journals Account] At: 19:51 10 August 2009

Table 6. Country credit-risk ratings by country and rating agency.

Country	S&P	Mdy	InsI
Argentina	BB	Ba3	42.70
Australia	AA+	Aa2	74.30
Austria	AAA	Aaa	88.70
Belgium	AA+	Aa1	83.50
Bolivia	BB−	B1	28.00
Brazil	B+	B2	37.40
Canada	AA+	Aa1	83.00
Chile	A−	Baa1	61.80
China	BBB	A3	57.20
China-HK	A	A3	61.80
Colombia	BB+	Baa3	44.50
Costa Rica	BB	Ba1	38.40
Croatia	BBB−	Baa3	39.03
Cyprus	A	A2	57.30
Czech Republic	A−	Baa1	59.70
Denmark	AA+	Aa1	84.70
Dominican Republic	B+	Ba2	28.10
Egypt	BBB−	Ba1	44.40
El Salvador	BB+	Ba2	31.20
Estonia	BBB+	Baa1	42.80
Finland	AA+	Aaa	82.20
France	AAA	Aaa	90.80
Germany	AAA	Aaa	92.50
Greece	A−	Baa1	56.10
Hungary	BBB	Baa2	55.90
Iceland	A+	Aa3	67.00
India	BB	Ba2	44.50
Indonesia	CCC+	B3	27.90
Ireland	AA+	Aaa	81.80
Israel	A−	A3	54.30
Italy	AA	Aa3	79.10
Japan	AAA	Aa1	86.50
Jordan	BB−	Ba3	37.30
Kazakhstan	B+	Ba3	27.90
Korea Republic	BBB	Ba1	52.70
Latvia	BBB	Baa2	38.00
Lebanon	BB−	B1	31.90
Lithuania	BBB−	Ba1	36.10

(*Continued*)

Downloaded By: [CDL Journals Account] At: 19:51 10 August 2009

Table 6. Continued.

Country	S&P	Mdy	InsI
Malaysia	BBB	Baa3	51.00
Malta	A	A3	61.70
Mexico	BB	Ba2	46.00
Morocco	BB	Ba2	43.20
Netherlands	AAA	Aaa	91.70
New Zealand	AA+	Aa2	73.10
Norway	AAA	Aaa	86.80
Pakistan	B−	Caa1	20.40
Panama	BB+	Ba1	39.90
Paraguay	B	B2	31.30
Peru	BB	Ba3	35.00
Philippines	BB+	Ba1	41.30
Poland	BBB	Baa3	56.70
Portugal	AA	Aa2	76.10
Romania	B−	B3	31.20
Russia	SD	B3	20.00
Singapore	AAA	Aa1	81.30
Slovak Republic	BB+	Ba1	41.30
Slovenia	A	A3	58.40
South Africa	BB+	Baa3	45.80
Spain	AA+	Aa2	80.30
Sweden	AA+	Aa2	79.70
Switzerland	AAA	Aaa	92.70
Thailand	BBB−	Ba1	46.90
Trin & Tobago	BBB−	Ba1	43.30
Tunisia	BBB−	Baa3	50.30
Turkey	B	B1	36.90
UK	AAA	Aaa	90.20
USA	AAA	Aaa	92.20
Uruguay	BBB−	Baa3	46.50
Venezuela	B	B2	34.40

Table 7. Conversion from S&P and Mdy's rating scales to a numeric scale.

S&P scale	Moody's scale	Converted scale
AAA	Aaa	100.00
AA+	Aa1	95.00
AA	Aa2	90.00
AA−	Aa3	85.00
A+	A1	80.00
A	A2	75.00
A−	A3	70.00
BBB+	Baa1	65.00
BBB	Baa2	60.00
BBB−	Baa3	55.00
BB+	Ba1	50.00
BB	Ba2	45.00
BB−	Ba3	40.00
B+	B1	35.00
B	B2	30.00
B−	B3	25.00
CCC+	Caa1	20.00
CCC	Caa2	15.00
CCC−	Caa3	10.00
CC	Ca	5.00
SD/D	C	0.00

scale' column in Table 7) ranging from low end at C, Ca through Aaa; and InsI uses a numeric scale ranging from high end at 100 through 0. Several authors (e.g. [5,7]) converted S&P and Mdy rating scales to the numeric scales shown in the 'converted scale' column of Table 4. This converted scale ranges from 0 to 100, where a lower numeric value denotes a higher probability of default. Note that in this scale the differences in the values assigned are constant for any pair of consecutive rating categories.

Let ($\|$, $M\cdot$ Sep–Dev, **1**), refer to ($\|$, $M\cdot$ Sep–Dev) with $u_{ij}^k = v_i^k = 1$, for all i, j, k. We use ($\|$, $M\cdot$ Sep–Dev, **1**), as no *a priori* estimates are available on the relative expertise of each agency.

We obtain the aggregate country-credit-risk rating vector by solving ($\|$, $M\cdot$ Sep–Dev, **1**). We refer to the solution of ($\|$, $M\cdot$ Sep–Dev, **1**) as the aggregate MSD1 rating, $\mathbf{x}^{\text{MSD1}}$, and to the aggregate rating vector obtained by the averaging method as the aggregate averaging rating, $\mathbf{x}^{\text{Avg}}$. The (converted) S&P's rating vector is denoted by $\mathbf{r}^{\text{SP}}$, the (converted) Mdy's rating vector by $\mathbf{r}^{\text{Mdy}}$ and the InsI's rating vector by $\mathbf{r}^{\text{InsI}}$. These rating vectors are given in Table 8.

Downloaded By: [CDL Journals Account] At: 19:51 10 August 2009

Table 8. Input and output for the country credit-risk aggregation problem.

Country	$\mathbf{r}^{\text{SP}}$	$\mathbf{r}^{\text{Mdy}}$	$\mathbf{r}^{\text{InsI}}$	$\mathbf{r}^{\text{MSD}}$	$\mathbf{x}^{\text{Avg}}$
Argentina	45	40	42.7	44.1	42.6
Australia	95	90	74.3	90.0	86.4
Austria	100	100	88.7	99.1	96.2
Belgium	95	95	83.5	94.1	91.2
Bolivia	40	35	28.0	36.5	34.3
Brazil	35	30	37.4	34.1	34.1
Canada	95	95	83.0	94.1	91.0
Chile	70	65	61.8	69.1	65.6
China	60	70	57.2	65.7	62.4
China-HK	75	70	61.8	70.3	68.9
Colombia	50	55	44.5	53.0	49.8
Costa Rica	45	50	38.4	46.9	44.5
Croatia	55	55	39.0	54.1	49.7
Cyprus	75	75	57.3	74.1	69.1
Czech Republic	70	65	59.7	68.2	64.9
Denmark	95	95	84.7	94.1	91.6
Dominican Republic	35	45	28.1	36.6	36.0
Egypt	55	50	44.4	52.9	49.8
El Salvador	50	45	31.2	45.0	42.1
Estonia	65	65	42.8	64.1	57.6
Finland	95	100	82.2	94.8	92.4
France	100	100	90.8	99.3	96.9
Germany	100	100	92.5	100.0	97.5
Greece	70	65	56.1	65.0	63.7
Hungary	60	60	55.9	60.0	58.6
Iceland	80	85	67.0	79.8	77.3
India	45	45	44.5	45.0	44.8
Indonesia	20	25	27.9	25.0	24.3
Ireland	95	100	81.8	94.8	92.3
Israel	70	70	54.3	69.1	64.8
Italy	90	85	79.1	87.6	84.7
Japan	100	95	86.5	95.0	93.8
Jordan	40	40	37.3	40.0	39.1
Kazakhstan	35	40	27.9	36.4	34.3
Korea Republic	60	50	52.7	58.3	54.2
Latvia	60	60	38.0	59.1	52.7
Lebanon	40	35	31.9	39.1	35.6
Lithuania	55	50	36.1	50.0	47.0
Malaysia	60	55	51.0	58.8	55.3
Malta	75	70	61.7	70.2	68.9

(*Continued*)

Downloaded By: [CDL Journals Account] At: 19:51 10 August 2009

Table 8. Continued.

Country	$\mathbf{r}^{\mathrm{SP}}$	$\mathbf{r}^{\mathrm{Mdy}}$	$\mathbf{r}^{\mathrm{InsI}}$	$\mathbf{r}^{\mathrm{MSD}}$	$\mathbf{x}^{\mathrm{Avg}}$
Mexico	45	45	46.0	45.0	45.3
Morocco	45	45	43.2	45.0	44.4
Netherlands	100	100	91.7	100.0	97.2
New Zealand	95	90	73.1	90.0	86.0
Norway	100	100	86.8	99.1	95.6
Pakistan	25	20	20.4	24.1	21.8
Panama	50	50	39.9	49.1	46.6
Paraguay	30	30	31.3	30.0	30.4
Peru	45	40	35.0	43.3	40.0
Philippines	50	50	41.3	49.8	47.1
Poland	60	55	56.7	59.1	57.2
Portugal	90	90	76.1	89.1	85.4
Romania	25	25	31.2	25.0	27.1
Russia	0	25	20.0	24.1	15.0
Singapore	100	95	81.3	95.0	92.1
Slovak Republic	50	50	41.3	49.8	47.1
Slovenia	75	70	58.4	70.0	67.8
South Africa	50	55	45.8	54.1	50.3
Spain	95	90	80.3	90.0	88.4
Sweden	95	90	79.7	90.0	88.2
Switzerland	100	100	92.7	100.0	97.6
Thailand	55	50	46.9	54.1	50.6
Trin & Tobago	55	50	43.3	51.8	49.4
Tunisia	55	55	50.3	55.0	53.4
Turkey	30	35	36.9	35.0	34.0
UK	100	100	90.2	99.1	96.7
USA	100	100	92.2	100.0	97.4
Uruguay	55	55	46.5	55.0	52.2
Venezuela	30	30	34.4	30.0	31.5

5.1 *Analysis of results*

In this section, we analyse the optimal solution to $(\|, M\cdot \text{Sep–Dev}, \mathbf{1})$, $\mathbf{x}^{\mathrm{MSD}\mathbf{1}}$. The analysis involves comparing the degree of agreement between $\mathbf{x}^{\mathrm{MSD}\mathbf{1}}$ and each agency's rating vectors.

For given penalty functions $f_{ij}()$, we define the *vector-separation distance* between two rating vectors $\mathbf{a}$ and $\mathbf{b}$ to be $\sum_{i=1}^{n}\sum_{j=i+1}^{n} f_{ij}(p_{ij}^{a} - p_{ij}^{b})$ and the *scalar-separation distance* between two rating-pairs $\{a_i, a_j\}$ and $\{b_i, b_j\}$ to be $f_{ij}(p_{ij}^{a} - p_{ij}^{b})$, where $p_{ij}^{a} = a_i - a_j$ and $p_{ij}^{b} = b_i - b_j$. Similarly, for given penalty functions $g_i()$ we define the *vector-deviation distance* between two rating vectors $\mathbf{a}$ and $\mathbf{b}$ to be $\sum_{i=1}^{n} g_i(a_i - b_i)$ and the *scalar-deviation distance* between two ratings a_i and b_i to be $g_i(a_i - b_i)$. When $f_{ij}(y) = |y|$ ($f_{ij}(y) = y^2$) will refer to the *absolute value (quadratic) vector-separation distance*. Finally, when $g_i(y) = |y|$ ($g_i(y) = y^2$) will refer to the *absolute value (quadratic) scalar-separation distance*.

The aggregate MSD1 rating vector $\mathbf{x}^{\mathrm{MSD}\mathbf{1}}$ is shown in the column 5 of Table 8. The absolute value vector-separation and vector-deviation distances between each agency's rating vector and $\mathbf{x}^{\mathrm{MSD}\mathbf{1}}$ are shown in Table 9.

Table 9. Distances between $\mathbf{x}^{\mathrm{MSD}\mathbf{1}}$ and each agency's rating vector.

	$\mathbf{r}^{\mathrm{SP}} - \mathbf{x}^{\mathrm{MSD}\mathbf{1}}$	$\mathbf{r}^{\mathrm{Mdy}} - \mathbf{x}^{\mathrm{MSD}\mathbf{1}}$	$\mathbf{r}^{\mathrm{InsI}} - \mathbf{x}^{\mathrm{MSD}\mathbf{1}}$	Total
Absolute value vector-separation	7540.00	6058.60	13952.00	27550.60
Absolute value vector-deviation	148.60	108.20	600.40	857.20
Total distance	7688.6	6166.8	14552.4	28407.8

Table 10. Distances between the rating vectors of each pair of agencies.

	S&P − Mdy	S&P − InsI	Mdy − InsI
Absolute value vector-separation	11440.00	19496.80	17994.60
Absolute value vector-deviation	230.00	703.60	618.60

The information in Table 9 demonstrates that InsI's country credit-risk ratings are the ratings which deviate the most from $\mathbf{x}^{\mathrm{MSD}\mathbf{1}}$. To explain why, we provide the absolute value vector-separation and vector-deviation distances for each pair of the three agencies in Table 10. These distances show that S&P and Mdy's rating vectors are, by far, the closest among the three pairs. We also note that Hammer *et al.* [7], found the correlation between the rating vectors of S&P and Mdy higher than both the correlation between the rating vectors of InsI and S&P and the correlation between the rating vectors of S&P and InsI. Therefore, S&P and Mdy form an 'almost' majority, and thus in the spirit of Theorem 3.7 should be closer to the aggregate MSD1 rating vector. This explains why $\mathbf{x}^{\mathrm{MSD}\mathbf{1}}$ is significantly closer to the rating vectors of S&P and Mdy.

5.1.1 *Analysis of the results via the absolute value scalar-deviation distance*

The absolute value scalar-deviation distance between each agency's country rating and the respective country rating of $\mathbf{x}^{\mathrm{MSD}\mathbf{1}}$ is given in Table 11.

Table 11. Countries sorted in descending absolute-value scalar-deviation distance per agency.

S&P		Moody's		Institutional Investor		Total	
Country	Dev	Country	Dev	Country	Dev	Country	Dev
Russia	24.1	Dominican Republic	8.4	Estonia	21.3	Russia	29.1
China	5.7	Korea Republic	8.3	Latvia	21.1	Estonia	23.1
Australia	5.0	Finland	5.2	New Zealand	16.9	Latvia	22.9
El Salvador	5.0	Iceland	5.2	Cyprus	16.8	New Zealand	21.9
Greece	5.0	Ireland	5.2	Australia	15.7	Australia	20.7
Indonesia	5.0	China	4.3	Croatia	15.1	Lithuania	18.9
Japan	5.0	Argentina	4.1	Israel	14.8	El Salvador	18.8
Lithuania	5.0	Brazil	4.1	Lithuania	13.9	Singapore	18.7
New Zealand	5.0	Chile	4.1	El Salvador	13.8	Cyprus	18.6
Singapore	5.0	Lebanon	4.1	Singapore	13.7	China	18.5
Slovenia	5.0	Pakistan	4.1	Ireland	13.0	Dominican Republic	18.5
Spain	5.0	Poland	4.1	Portugal	13.0	Ireland	18.4
Sweden	5.0	Thailand	4.1	Iceland	12.8	Iceland	18.2
Turkey	5.0	Malaysia	3.8	Finland	12.6	Finland	18.0
Malta	4.8	Kazakhstan	3.6	Norway	12.3	Croatia	16.9
China-HK	4.7	Peru	3.3	Slovenia	11.6	Israel	16.6
South Africa	4.1	Czech Republic	3.2	Canada	11.1	Slovenia	16.6
Bolivia	3.5	Costa Rica	3.1	Belgium	10.6	Korea Republic	15.6
Trin & Tobago	3.2	Egypt	2.9	Austria	10.4	Sweden	15.3
Colombia	3.0	Italy	2.6	Sweden	10.3	Portugal	14.8
Italy	2.4	Colombia	2.0	Spain	9.7	Spain	14.7
Egypt	2.1	Trin & Tobago	1.8	Denmark	9.4	Norway	14.1
Costa Rica	1.9	Bolivia	1.5	Panama	9.2	Greece	13.9
Czech Republic	1.8	Austria	0.9	Greece	8.9	Bolivia	13.5
Korea Republic	1.7	Belgium	0.9	UK	8.9	China-HK	13.5
Peru	1.7	Canada	0.9	Bolivia	8.5	Colombia	13.5
Dominican Republic	1.6	Croatia	0.9	China	8.5	Costa Rica	13.5
Kazakhstan	1.4	Cyprus	0.9	China-HK	8.5	Czech Republic	13.5

(*Continued*)

Downloaded By: [CDL Journals Account] At: 19:51 10 August 2009

Table 11. Continued.

S&P		Moody's		Institutional Investor		Total	
Country	Dev	Country	Dev	Country	Dev	Country	Dev
Malaysia	1.2	Denmark	0.9	Colombia	8.5	Egypt	13.5
Argentina	0.9	Estonia	0.9	Costa Rica	8.5	Italy	13.5
Austria	0.9	Israel	0.9	Czech Republic	8.5	Japan	13.5
Belgium	0.9	Latvia	0.9	Dominican Republic	8.5	Kazakhstan	13.5
Brazil	0.9	Norway	0.9	Egypt	8.5	Malta	13.5
Canada	0.9	Panama	0.9	France	8.5	Trin & Tobago	13.5
Chile	0.9	Portugal	0.9	Italy	8.5	Peru	13.3
Croatia	0.9	Russia	0.9	Japan	8.5	South Africa	13.3
Cyprus	0.9	South Africa	0.9	Kazakhstan	8.5	Canada	12.9
Denmark	0.9	UK	0.9	Malta	8.5	Malaysia	12.8
Estonia	0.9	France	0.7	Philippines	8.5	Belgium	12.4
Israel	0.9	China-HK	0.3	Slovak Republic	8.5	Chile	12.3
Latvia	0.9	Malta	0.2	Trin & Tobago	8.5	Austria	12.2
Lebanon	0.9	Philippines	0.2	Uruguay	8.5	Lebanon	12.2
Norway	0.9	Slovak Republic	0.2	Netherlands	8.3	Thailand	12.2
Pakistan	0.9	Australia	0.0	Peru	8.3	Denmark	11.2
Panama	0.9	El Salvador	0.0	South Africa	8.3	Panama	11.0
Poland	0.9	Germany	0.0	Malaysia	7.8	UK	10.7
Portugal	0.9	Greece	0.0	USA	7.8	France	9.9
Thailand	0.9	Hungary	0.0	Germany	7.5	Philippines	8.9
UK	0.9	India	0.0	Chile	7.3	Slovak Republic	8.9
France	0.7	Indonesia	0.0	Switzerland	7.3	Pakistan	8.7
Finland	0.2	Japan	0.0	Lebanon	7.2	Uruguay	8.5
Iceland	0.2	Jordan	0.0	Thailand	7.2	Brazil	8.3
Ireland	0.2	Lithuania	0.0	Romania	6.2	Netherlands	8.3
Philippines	0.2	Mexico	0.0	Korea Republic	5.6	Indonesia	7.9
Slovak Republic	0.2	Morocco	0.0	Tunisia	4.7	USA	7.8
Germany	0.0	Netherlands	0.0	Venezuela	4.4	Germany	7.5
Hungary	0.0	New Zealand	0.0	Hungary	4.1	Poland	7.4
India	0.0	Paraguay	0.0	Russia	4.1	Switzerland	7.3
Jordan	0.0	Romania	0.0	Pakistan	3.7	Turkey	6.9
Mexico	0.0	Singapore	0.0	Brazil	3.3	Argentina	6.4
Morocco	0.0	Slovenia	0.0	Indonesia	2.9	Romania	6.2
Netherlands	0.0	Spain	0.0	Jordan	2.7	Tunisia	4.7
Paraguay	0.0	Sweden	0.0	Poland	2.4	Venezuela	4.4
Romania	0.0	Switzerland	0.0	Turkey	1.9	Hungary	4.1
Switzerland	0.0	Tunisia	0.0	Morocco	1.8	Jordan	2.7
Tunisia	0.0	Turkey	0.0	Argentina	1.4	Morocco	1.8
USA	0.0	USA	0.0	Paraguay	1.3	Paraguay	1.3
Uruguay	0.0	Uruguay	0.0	Mexico	1.0	Mexico	1.0
Venezuela	0.0	Venezuela	0.0	India	0.5	India	0.5
Total	148.6		108.2		600.4		857.2
Maximum	24.1		8.4		21.3		29.1
Average	2.2		1.6		8.7		12.4
σ	3.3		2.0		4.4		5.7

Note: For each agency and each country column 'Dev' gives the distance between the respective agency's rating and the aggregate MSD1 rating. Column 'Total' shows the sum of the 3 absolute value scalar-deviations.

Downloaded By: [CDL Journals Account] At: 19:51 10 August 2009

As seen in Table 11, for each agency there is a set of countries where the absolute value scalar-deviation distance with respect to $\mathbf{x}^{\mathrm{MSD}\mathbf{1}}$ is considerably higher than the rest of the absolute value scalar-deviation distances. In particular, we note that:

(1) S&P's rating to Russia has an absolute value scalar-deviation distance with more than 6.6 σs from the mean, while all other S&P's ratings are within 1 σ from the mean.

(2) Mdy's ratings to the Dominican and Korean Republics have absolute value scalar-deviation distances with more than 3.3 σs from the mean, while all other Mdy's ratings are within 1.8 σs from the mean.
(3) InsI's rating to Estonia and Latvia have absolute value scalar-deviation distances with more than 2.7 σs from the mean, while all other S&P's ratings are within 1.9 σs from the mean.

We argue that these ratings are outliers with respect to the country credit-risk ratings given by this group of agencies.

In particular, Russia's rating by S&P has a scalar-deviation distance to the respective aggregate rating dramatically larger than all other scalar-deviation distances. We note that Russia appears to be an outlier in S&P's ratings as the 1998 ratings of S&P, Mdy, and InsI are SD (0), B3 (25) and 20.0, respectively. One possible explanation for this discrepancy in Russia's ratings is that S&P distinguishes between 'default' and 'selective default', whereas the other agencies don't do so. It should be pointed out that S&P upgraded Russia's rating from SD in 1998 and 1999 to B−, B and B+ in December 2000, June 2001, and December 2001, respectively [1].

For each agency and each country column 'Dev' gives the distance between the respective agency's rating and the aggregate MSD1 rating. Column 'Total' shows the sum of the 3 absolute value scalar-deviations.

Downloaded By: [CDL Journals Account] At: 19:51 10 August 2009

5.1.2 *Analysis of the results via the absolute value scalar-separation distance*

This section provides an analysis of the solution to ($\|$, $M\cdot$ Sep–Dev, $\mathbf{1}$) in terms of the absolute value scalar-separation distance analogous to the analysis provided in the previous section. The overall results are consistent with those of the previous section.

With ratings of 69 countries, we have 2346 pairwise comparisons and it is impossible to list all of the absolute value scalar-separation distances. Instead in Table 12, we list only the most significant pairwise comparisons in terms of largest absolute value scalar-separation distances between each agency's rating and $\mathbf{x}^{\mathrm{MSD}\mathbf{1}}$.

It is interesting to observe that the countries which have the highest scalar-deviation distances belong to the country-pairs which have the highest absolute value scalar-separation distances. Indeed, reviewing the ranked list of the pairwise comparisons which deviate the most from $\mathbf{x}^{\mathrm{MSD}\mathbf{1}}$, one observes that certain countries appear in country-pairs with high scalar-separation distances. These two observations are related to having derived the separation gaps from the ratings $p_{ij}^k = r_i^k - r_j^k$. Thus, any discrepancy in one score affects all pairwise comparisons with such score. Indeed, it is easy to see in Table 12 that for the case of S&P, the first 68 pairwise comparisons concern Russia. In the case of Mdy, the countries which dominate the results are the Dominican Republic and the Korean Republic. Finally, in the case of InsI, this clustering of countries is not as evident; however, one can still observe the predomination of Estonia and Latvia as the countries with the higher absolute value scalar-separation distances. Recall that these countries were the ones with the highest scalar-deviation distances.

5.2 *Comparison of the aggregate MSD1 rating to the aggregate averaging rating*

We now show that the aggregate MSD1 rating $\mathbf{x}^{\mathrm{MSD}\mathbf{1}}$ is in some sense closer than the aggregate averaging rating $\mathbf{x}^{\mathrm{Avg}}$ to the group consensus. The aggregate MSD1 rating and the aggregate averaging rating are shown in columns 5 and 6 of Table 8, respectively. We compare $\mathbf{x}^{\mathrm{MSD}\mathbf{1}}$ to $\mathbf{x}^{\mathrm{Avg}}$ by evaluating their respective distances to each of the agencies' rating vectors. For this purpose, we use the vector-separation and vector-deviation distances and the *number of reversals* which we define next.

Downloaded By: [CDL Journals Account] At: 19:51 10 August 2009

Table 12. Country-pairs with the highest absolute value scalar-separation distance per agency sorted in descending order.

S&P			Moody's			Institutional Investor		
Country 1	Country 2	Sep	Country 1	Country 2	Sep	Country 1	Country 2	Sep
Australia	Russia	29.1	Dominican Republic	Korea Republic	16.7	Estonia	Romania	27.5
El Salvador	Russia	29.1	Finland	Korea Republic	13.5	Latvia	Romania	27.3
Greece	Russia	29.1	Iceland	Korea Republic	13.5	Estonia	Venezuela	25.7
Japan	Russia	29.1	Ireland	Korea Republic	13.5	Latvia	Venezuela	25.5
Lithuania	Russia	29.1	China	Korea Republic	12.6	Brazil	Estonia	24.6
New Zealand	Russia	29.1	Argentina	Dominican Republic	12.5	Brazil	Latvia	24.4
Russia	Singapore	29.1	Brazil	Dominican Republic	12.5	Estonia	Indonesia	24.2
Russia	Slovenia	29.1	Chile	Dominican Republic	12.5	Indonesia	Latvia	24.0
Russia	Spain	29.1	Dominican Republic	Lebanon	12.5	Estonia	Turkey	23.2
Russia	Sweden	29.1	Dominican Republic	Pakistan	12.5	New Zealand	Romania	23.1
Malta	Russia	28.9	Dominican Republic	Poland	12.5	Cyprus	Romania	23.0
China-HK	Russia	28.8	Dominican Republic	Thailand	12.5	Latvia	Turkey	23.0
Bolivia	Russia	27.6	Dominican Republic	Malaysia	12.2	Estonia	Paraguay	22.6
Russia	Trin&Tob	27.3	Kazakhstan	Korea Republic	11.9	Latvia	Paraguay	22.4
Italy	Russia	26.5	Dominican Republic	Peru	11.7	Estonia	Mexico	22.3
Egypt	Russia	26.2	Czech Republic	Dominican Republic	11.6	Latvia	Mexico	22.1
Czech Republic	Russia	25.9	Costa Rica	Korea Republic	11.4	Australia	Romania	21.9
Korea Republic	Russia	25.8	Dominican Republic	Egypt	11.3	New Zealand	Venezuela	21.3
Peru	Russia	25.8	Dominican Republic	Italy	11.0	Croatia	Romania	21.3
Malaysia	Russia	25.3	Colombia	Korea Republic	10.3	Cyprus	Venezuela	21.2
Argentina	Russia	25.0	Dominican Republic	Trin&Tob	10.2	Israel	Romania	21.0
Austria	Russia	25.0	Bolivia	Dominican Republic	9.9	Estonia	India	20.8
Belgium	Russia	25.0	Argentina	Finland	9.3	India	Latvia	20.6
Brazil	Russia	25.0	Argentina	Iceland	9.3	Brazil	New Zealand	20.2
Canada	Russia	25.0	Argentina	Ireland	9.3	Australia	Venezuela	20.1
Chile	Russia	25.0	Brazil	Finland	9.3	Brazil	Cyprus	20.1

(*Continued*)

Downloaded By: [CDL Journals Account] At: 19:51 10 August 2009

Table 12. Continued.

S&P			Moody's			Institutional Investor		
Country 1	Country 2	Sep	Country 1	Country 2	Sep	Country 1	Country 2	Sep
Croatia	Russia	25.0	Brazil	Iceland	9.3	Lithuania	Romania	20.1
Cyprus	Russia	25.0	Brazil	Ireland	9.3	El Salvador	Romania	20.0
Denmark	Russia	25.0	Chile	Finland	9.3	Argentina	Estonia	19.9
Estonia	Russia	25.0	Chile	Iceland	9.3	Romania	Singapore	19.9
Israel	Russia	25.0	Chile	Ireland	9.3	Indonesia	New Zealand	19.8
Latvia	Russia	25.0	Finland	Lebanon	9.3	Argentina	Latvia	19.7
Lebanon	Russia	25.0	Finland	Pakistan	9.3	Cyprus	Indonesia	19.7
Norway	Russia	25.0	Finland	Poland	9.3	Estonia	Morocco	19.5
Pakistan	Russia	25.0	Finland	Thailand	9.3	Croatia	Venezuela	19.5
Panama	Russia	25.0	Iceland	Lebanon	9.3	Latvia	Morocco	19.3
Romania	19.2							
Portugal	Russia	25.0	Iceland	Poland	9.3	Portugal	Romania	19.2
Russia	Thailand	25.0	Iceland	Thailand	9.3	Israel	Venezuela	19.2
Russia	UK	25.0	Ireland	Lebanon	9.3	Australia	Brazil	19.0
France	Russia	24.8	Ireland	Pakistan	9.3	Iceland	Romania	19.0
Finland	Russia	24.3	Ireland	Poland	9.3	Estonia	Poland	18.9
Iceland	Russia	24.3	Ireland	Thailand	9.3	New Zealand	Turkey	18.8
Ireland	Russia	24.3	Austria	Korea Republic	9.2	Finland	Romania	18.8
Philippines	Russia	24.3	Belgium	Korea Republic	9.2	Latvia	Poland	18.7
Russia	Slovak Republic	24.3	Canada	Korea Republic	9.2	Cyprus	Turkey	18.7
Germany	Russia	24.1	Croatia	Korea Republic	9.2	Australia	Indonesia	18.6
Hungary	Russia	24.1	Cyprus	Korea Republic	9.2	Estonia	Jordan	18.6
India	Russia	24.1	Denmark	Korea Republic	9.2	Norway	Romania	18.5
Jordan	Russia	24.1	Estonia	Korea Republic	9.2	Brazil	Croatia	18.4
Mexico	Russia	24.1	Israel	Korea Republic	9.2	Jordan	Latvia	18.4
Morocco	Russia	24.1	Korea Republic	Latvia	9.2	Lithuania	Venezuela	18.3
Netherlands	Russia	24.1	Korea Republic	Norway	9.2	New Zealand	Paraguay	18.2
Paraguay	Russia	24.1	Korea Republic	Panama	9.2	El Salvador	Venezuela	18.2
Romania	Russia	24.1	Korea Republic	Portugal	9.2	Brazil	Israel	18.1

Downloaded By: [CDL Journals Account] At: 19:51 10 August 2009

Russia	Switzerland	24.1	Korea Republic	Russia	9.2	Singapore	Venezuela	18.1
Russia	Tunisia	24.1	Korea Republic	S. Africa	9.2	Cyprus	Paraguay	18.1
Russia	USA	24.1	Korea Republic	UK	9.2	Croatia	Indonesia	18.0
Russia	Uruguay	24.1	Finland	Malaysia	9.0	Mexico	New Zealand	17.9
Russia	Venezuela	24.1	France	Korea Republic	9.0	Romania	Slovenia	17.8
Kazakhstan	Russia	22.7	Iceland	Malaysia	9.0	Cyprus	Mexico	17.8
Dominican Republic	Russia	22.5	Ireland	Malaysia	9.0	Indonesia	Israel	17.7
Costa Rica	Russia	22.2	China-HK	Dominican Republic	8.7	Australia	Turkey	17.6
Colombia	Russia	21.1	Dominican Republic	Malta	8.6	Estonia	Pakistan	17.6
Russia	S. Africa	20.0	Finland	Peru	8.5	Ireland	Venezuela	17.4
Indonesia	Russia	19.1	Iceland	Peru	8.5	Latvia	Pakistan	17.4
Russia	Turkey	19.1	Ireland	Peru	8.5	Portugal	Venezuela	17.4
China	Russia	18.4	Korea Republic	Philippines	8.5	Canada	Romania	17.3
Australia	China	10.7	Korea Republic	Slovak Republic	8.5	Brazil	Lithuania	17.2
China	El Salvador	10.7	Czech Republic	Finland	8.4	Estonia	Russia	17.2
China	Greece	10.7	Czech Republic	Iceland	8.4	Estonia	Hungary	17.2
China	Japan	10.7	Czech Republic	Ireland	8.4	Iceland	Venezuela	17.2
China	Lithuania	10.7	Australia	Dominican Republic	8.4	Brazil	El Salvador	17.1
China	New Zealand	10.7	Dominican Republic	El Salvador	8.4	Brazil	Singapore	17.0
Total		7540.0			6058.2			13952.0
Maximum		29.1			16.7			27.5
Average		3.2			2.6			5.9
σ		4.4			2.6			5.0

Note: For each agency and each country-pair the column ‘Sep’ gives the scalar-separation distance between the respective agency’s separation gap and the aggregate MSD1 rating’s.

For a pair of vectors **a** and **b**, and $1 \le i < j \le n$, we define R_{ij} as follows:

$$R_{ij} = \begin{cases} 1 & \text{if } a_i > a_j \text{ and } b_i < b_j \text{ or } a_i < a_j \text{ and } b_i > b_j, \\ 1/2 & \text{if } a_i = a_j \text{ and } b_i \neq b_j \text{ or } a_i \neq a_j \text{ and } b_i = b_j, \\ 0 & \text{otherwise.} \end{cases}$$

The number of reversals between the two vectors is then $\sum_{i=1}^{n} \sum_{j=1,j>i}^{n} R_{ij}$. This quantity is also known as the Kemeny–Snell distance [13]. We note that finding the rating vector that minimizes the total number of reversals from given ratings vectors is an NP-complete problem [3].

Since $\mathbf{x}^{\mathrm{MSD}\mathbf{1}}$ is the optimal solution to ($\|$, $M\cdot$ Sep–Dev, **1**), it is the vector with minimum total sum of absolute value vector-separation and vector-deviation distances with respect to the agencies' rating vectors. Thus, $\mathbf{x}^{\mathrm{MSD}\mathbf{1}}$ tends to perform better than $\mathbf{x}^{\mathrm{Avg}}$ for the absolute value vector-deviation distance alone, and the absolute value vector-separation distance alone. This is shown in Tables 13 and 14.

From Theorem 4.1, we have that $\mathbf{x}^{\mathrm{Avg}}$ is the optimal solution to the separation-deviation problem with uniform quadratic penalty functions, i.e. it is the vector with minimum total sum of quadratic vector-separation and vector-deviation distances to the agencies' rating vectors. Thus, $\mathbf{x}^{\mathrm{Avg}}$ tends to perform better than $\mathbf{x}^{\mathrm{MSD}\mathbf{1}}$ for the quadratic vector-deviation distance alone, and the quadratic vector-separation distance alone. This is shown in Tables 15 and 16.

In Table 17, we show the number of reversals when comparing $\mathbf{x}^{\mathrm{MSD}\mathbf{1}}$ and $\mathbf{x}^{\mathrm{Avg}}$ with each of the agency's original ratings. As shown in column 4 of Table 17, $\mathbf{x}^{\mathrm{MSD}\mathbf{1}}$ has fewer total number of reversals from $\mathbf{r}^{\mathrm{SP}}$, $\mathbf{r}^{\mathrm{Mdy}}$, and $\mathbf{r}^{\mathrm{InsI}}$, as compared with $\mathbf{x}^{\mathrm{Avg}}$. Therefore, the solution to ($\|$, $M\cdot$ Sep–Dev, **1**) is closer to the ordering implied by the agencies' ratings than $\mathbf{x}^{\mathrm{Avg}}$. Furthermore,

Table 13. Absolute value vector-deviation distances between each aggregate rating vector and each agency rating vector.

	$\mathbf{r}^{\mathrm{SP}}$	$\mathbf{r}^{\mathrm{Mdy}}$	$\mathbf{r}^{\mathrm{InsI}}$	Total
$\mathbf{x}^{\mathrm{MSD}\mathbf{1}}$	148.6	108.2	600.4	857.2
$\mathbf{x}^{\mathrm{Avg}}$	280.8	210.8	432.2	923.8

Table 14. Absolute value vector-separation distances between each aggregate rating vector and each agency rating vector.

	$\mathbf{r}^{\mathrm{SP}}$	$\mathbf{r}^{\mathrm{Mdy}}$	$\mathbf{r}^{\mathrm{InsI}}$	Total
$\mathbf{x}^{\mathrm{MSD}\mathbf{1}}$	7540.0	6058.6	13952.0	27550.6
$\mathbf{x}^{\mathrm{Avg}}$	8990.4	8099.0	11839.0	28928.4

Table 15. Quadratic vector-deviation distances between each aggregate rating vector and each agency rating vector.

	$\mathbf{r}^{\mathrm{SP}}$	$\mathbf{r}^{\mathrm{Mdy}}$	$\mathbf{r}^{\mathrm{InsI}}$	Total
$\mathbf{x}^{\mathrm{MSD}\mathbf{1}}$	1049.76	453.69	6577.21	8080.66
$\mathbf{x}^{\mathrm{Avg}}$	1624.09	1043.29	3528.36	6195.74

Table 16. Quadratic vector-separation distances between each aggregate rating vector and each agency rating vector.

	$\mathbf{r}^{\mathrm{SP}}$	$\mathbf{r}^{\mathrm{Mdy}}$	$\mathbf{r}^{\mathrm{InsI}}$	Total
$\mathbf{x}^{\mathrm{MSD}\mathbf{1}}$	70596.49	31187.56	142129.00	243913.05
$\mathbf{x}^{\mathrm{Avg}}$	64566.81	43597.44	94740.84	202905.09

Downloaded By: [CDL Journals Account] At: 19:51 10 August 2009

Table 17. Number of reversals distance between each aggregate rating vector and each agency rating vector.

	$\mathbf{r}^{\mathrm{SP}}$	$\mathbf{r}^{\mathrm{Mdy}}$	$\mathbf{r}^{\mathrm{InsI}}$	Total
$\mathbf{x}^{\mathrm{MSD}\mathbf{1}}$	86	87.5	158.5	332
$\mathbf{x}^{\mathrm{Avg}}$	107	112.5	129.5	349

Table 18. Number of reversals between the rating vectors of each pair of agencies.

	$\mathbf{r}^{\mathrm{SP}}$	$\mathbf{r}^{\mathrm{Mdy}}$	$\mathbf{r}^{\mathrm{InsI}}$	Total
$\mathbf{r}^{\mathrm{SP}}$	0	133.5	214.5	348.0
$\mathbf{r}^{\mathrm{Mdy}}$	133.5	0	228.0	361.5
$\mathbf{r}^{\mathrm{InsI}}$	214.5	228	0	442.5

Downloaded By: [CDL Journals Account] At: 19:51 10 August 2009

$\mathbf{x}^{\mathrm{MSD}\mathbf{1}}$ is closer to both $\mathbf{r}^{\mathrm{SP}}$ and $\mathbf{r}^{\mathrm{Mdy}}$ than $\mathbf{x}^{\mathrm{Avg}}$. As noted previously, S&P and Mdy form a kind of 'majority'. We argue that even when there is no clear majority, as Theorem 3.12 requires, $\mathbf{x}^{\mathrm{MSD}\mathbf{1}}$ is closer to the rating vectors of the reviewers that show a 'high degree of agreement' than to the rating vectors of other reviewers. We conclude that $\mathbf{x}^{\mathrm{MSD}\mathbf{1}}$ is close to a group consensus.

We note that $\mathbf{r}^{\mathrm{SP}}$ has the fewest number of reversals from $\mathbf{x}^{\mathrm{MSD}\mathbf{1}}$, closely followed by $\mathbf{r}^{\mathrm{Mdy}}$. The $\mathbf{r}^{\mathrm{InsI}}$ rating vector has a far larger number of reversals. This contrasts with the observations in the previous sections, where Mdy had the closest agency-rating vector to $\mathbf{x}^{\mathrm{MSD}\mathbf{1}}$. This apparent contradiction might be explained by the following two observations:

(1) As shown in Table 18, $\mathbf{r}^{\mathrm{SP}}$ has the fewest total number of reversals when compared with the other two rating agencies. Furthermore, with respect to the number of reversals measure, $\mathbf{r}^{\mathrm{SP}}$ is closer to $\mathbf{r}^{\mathrm{InsI}}$ than $\mathbf{r}^{\mathrm{Mdy}}$ to $\mathbf{r}^{\mathrm{InsI}}$ and $\mathbf{r}^{\mathrm{SP}}$ is closer to $\mathbf{r}^{\mathrm{Mdy}}$ than $\mathbf{r}^{\mathrm{InsI}}$ to $\mathbf{r}^{\mathrm{Mdy}}$. So, when using this distance measure, $\mathbf{r}^{\mathrm{SP}}$ is closer to the group consensus than the other two rating vectors.

(2) Since the number of reversals distance is only relative to the (implied) ordering, rather than to the magnitude of the rating scores, it is less sensitive to outliers than the vector-separation and vector-deviation distances. In this regard, note that Russia's rating by S&P is the rating with the highest deviation and separation distances; while Russia contributes only one reversal when comparing $\mathbf{r}^{\mathrm{SP}}$ to $\mathbf{r}^{\mathrm{Mdy}}$ and only one reversal when comparing $\mathbf{r}^{\mathrm{Mdy}}$ to $\mathbf{r}^{\mathrm{InsI}}$.

6. Conclusions

In this paper, we demonstrate several properties of the separation-deviation model. Our main result is that the separation model has the property of resistance to manipulation by a minority. We also prove a similar, but weaker, result for the separation-deviation model. Additionally, we characterize the optimal solution to the model for certain classes of penalty functions.

The separation deviation model is used here to aggregate conflicting credit-risk ratings. We show that the aggregate MSD1 rating is closer to the group rating than the aggregate averaging rating. This is established here for the absolute value vector-deviation and vector-separation distances. Moreover, the aggregate MSD1 rating also has fewer reversals from the agencies' ratings than the aggregate averaging rating. We conclude that the aggregate MSD1 rating better reflects each of the agency's ratings than the aggregate averaging rating.

We anticipate that in more general scenarios, the separation-deviation model will prove to be a useful aggregation method. We believe that the separation-deviation model is a useful tool for aggregating disparate sources of information, and should be considered as an alternative to other group-decision-making methods.

Acknowledgements

The research of the first author is supported in part by NSF award No. DMI-0620677 and CBET-0736232. The second author is supported in part by NSF and by Consejo Nacional de Ciencia y Tecnología (CONACyT), México, under grant No. 160619. The authors are grateful to an anonymous referee who helped them to improve the presentation of this paper.

Notes

1. The concept of uniform quadratic functions is defined in Section 4.
2. We are grateful to Hammer *et al.* [7] for permission to use this data.

References

[1] S. Alexe, P.L. Hammer, A. Kogan, and M.A. Lejeune, *A non-recursive regression model for country credit risk rating*, RUTCOR Research Report, RRR 9-2003, Rutgers University, Piscataway, NJ, 2003.
[2] I. Ali, W.D. Cook, and M. Kress, *Ordinal ranking and intensity of preference: a linear programming approach*, Manage. Sci. 32 (1986), pp. 1642–1647.
[3] J. Bartholdi, C.A. Tovey, and M.A. Trick, *Voting schemes for which it can be difficult to tell who won the election*, Soc. Choice Welfare 6 (1989), pp. 157–165.
[4] A.E. Bindschedler and J.M. Moore, *Optimal location of new machines in existing plant layouts*, J. Ind. Eng. 12 (1961), pp. 41–48.
[5] G. Ferri, L.G. Liu, and J. Stiglitz, *The procyclical role of rating agencies: evidence from the east asian crisis*, Econ. Notes 28 (1999), pp. 335–355.
[6] R.L. Francis, *A note on the optimum location of new machines in existing facilities*, J. Ind. Eng. 15 (1963), pp. 106–107.
[7] P.L. Hammer, A. Kogan, and M.A. Lejeune, *Country risk ratings: statistical and combinatorial non-recursive models*, RUTCOR Research Report, RRR 8-2004, Rutgers University, Piscataway, NJ, 2004.
[8] N.U. Haque, M.S. Kumar, N. Mark, and D. Mathieson, *The economic content of indicators of developing country creditworthiness*, IMF Working Paper No. 96/9, 1996. Available at http://ssrn.com/abstract=882910.
[9] D.S. Hochbaum, *50th anniversary article: selection, provisioning, shared fixed costs, maximum closure, and implications on algorithmic methods today*, Manage. Sci. 50 (2004), pp. 709–723.
[10] ———, Ranking sports teams and the inverse equal paths problem. Second international workshop on internet and network economics, Greece, 2006.
[11] D.S. Hochbaum and A. Levin, *Methodologies for the group rankings decision*, Manage. Sci. 52 (2006), pp. 1394–1408.
[12] D.S. Hochbaum, E. Moreno-Centeno, R.A. Catena, and P. Yelland, *A new model for behavioral customer segmentation*, UC Berkeley Manuscript, 2008. Available at http://riot.ieor.berkeley.edu/ dorit/pub/customer-segment.pdf.
[13] J.G. Kemeney and L.J. Snell, *Mathematical Models in Social Science*, MIT Press, Boston, MA, 1962.
[14] T. Saaty, *A scaling method for priorities in hierarchical structures*, J. Math. Psychol. 15 (1977), pp. 234–281.

Downloaded By: [CDL Journals Account] At: 19:51 10 August 2009