Department of Industrial Engineering & Operations Research

IEOR264 Computational Optimization Final Exam

Spring 2008 **Due: 5 PM, May 16, 2008**

Name: **Grade:**

This is an open book, open notes exam. Do not discuss the questions or your answers with anyone. Good luck!

Consider

$$(P) \quad \min \left\{ \sum_{i=1}^{n} c_i x_i : x \in X_r \right\},$$

where for $r \in \{1, 2, \ldots, n\}$

$$X_r := \left\{ x \in \{0,1\}^n : \sum_{i=1}^{n} a_i x_i + \max_{S \subseteq \{1,2,\ldots,n\}, |S|=r} \sum_{i \in S} d_i x_i \leq b \right\}$$

and

$$Y_r := \left\{ x \in \{0,1\}^n, y \in \mathbb{R}^n_+, z \in \mathbb{R}_+ : \sum_{i=1}^{n} (a_i x_i + y_i) + rz \leq b, \quad d_i x_i \leq y_i + z, \quad i = 1, \ldots, n \right\}.$$

1. Show that (P) is equivalent to $\min \left\{ \sum_{i=1}^{n} c_i x_i : (x, y, z) \in Y_r \right\}$.
2. Describe the appropriate decomposition methods that are covered in the course for solving (P) in detail. Explain the virtue of each alternative approach, discuss advantages and disadvantages. Write down the reformulations, i.e., master- and sub-problems. Discuss the structure of the solutions to these problems and the complexity of solving the sub-problem(s). Characterize upper and lower bounds on the optimal value for evaluating the progress of the decomposition algorithm. Your grade will be determined based on how thorough, specific, and precise your answer is.
3. Consider Y_r.
   (a) Characterize the extreme points of the LP relaxation of Y_r.
   (b) Find valid inequalities that use the knapsack budget b for Y_r. Prove validity.
   (c) Determine whether the inequalities in part (b) cut off the fractional extreme points in part (a).
   (d) Determine under what conditions the inequalities in part (b) define facets of $conv(Y_r)$.
   (e) Discuss how to find inequalities violated by a given point $(\bar{x}, \bar{y}, \bar{z}) \in \mathbb{R}^{2n+1}_+$.