Now, proceed with $n = 2, 3$, and 4 with similar arguments by iterating from lines 6 to 4. Finally, we obtain

$$\begin{aligned} b_{rk_pr}(4) &< 1 \\ s_{j_rr}(4) &> 0 \\ b_{k_tt}(4) &< 1 \\ s_{j_rt}(4) &> 0 \\ s_{j_rm}(4) &> 0 \end{aligned}$$

and

$$\begin{aligned} b_i(4) < b_i(3), \qquad & i = k_rr, rk_pp, k_rm, j_r, k_tm, j_p \\ s_i(4) > s_i(3), \qquad & i = rk_p, j_rr, j_pp, k_t, j_pm, k_r. \end{aligned}$$

The base case is proved. Assume now that $n > 0$

$$s_i(n) > s_k(n-1), \quad i \in I_s, \quad b_i(n) < b_i(n-1), \quad i \in I_b. \quad (26)$$

Then, from Lemma 2 ,we obtain

$$s_i(n+1) > s_k(n), \quad i \in I_s, \quad b_i(n+1) < b_i(n), \quad i \in I_b.$$

Therefore, $s_i(n)$ and $b_i(n)$ are monotonically increasing or decreasing, respectively. Since they are bounded by 0 and 1 [14], they are convergent.

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Productivity of Parallel Production Lines With Unreliable Machines and Material Handling

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***Abstract*—Using parallelism in bufferless production lines can improve productivity, with significant productivity gains achieved with crossover. However, including crossover in the line implies additional material-handling requirements that may reduce the availability of the system. This paper examines if parallel systems with crossover between the stages are more productive than parallel systems without crossover between the stages, when one considers the availability of the additional material handling required for the crossover. The minimum material-handling availability necessary for inclusion of crossover is determined for a given parallel line's configuration such that productivity can be maximized.**

***Note to Practitioners*—Two approaches in configuring parallel manufacturing lines are currently being used in industrial plants. These have been characterized as the Japanese approach of parallel independent cells of serial operations, and the European approach of a serial line with each operation being duplicated in parallel. The European approach has a productivity advantage over the Japanese approach when considering machine failures within each operation. However, the European approach requires more material handling which increases the configuration complexity and can reduce productivity. A math model is developed to determine which approach is best for a given line design when line length is defined by process planning and line balancing, and line width is determined by throughput requirements. The analysis is limited to cell configurations that do not use buffers internal to the cell.**

***Index Terms*—Availability, material handling, productivity, system analysis and design.**

I. INTRODUCTION

Configuration is an important, sometimes overlooked, aspect of the manufacturing-system design that can significantly effect its performance. Its effect has been studied by Koren *et al.* [1] who noted its impact on such parameters as reliability, productivity, quality, scalability, convertibility, and cost. For manufacturing-system design decisions involving capital expenditures, one of the most important parameters is productivity. Traditionally, system productivity is estimated from the availability of the system elements. In automated machining transfer lines, and to a lesser extent in assembly lines, productivity shortfalls due to equipment failures are customarily addressed by the inclusion

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